

MEASURING HEIGHT DIFFERENCES WITH ATOMIC CLOCKS: AN OVERVIEW

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INTRODUCTION

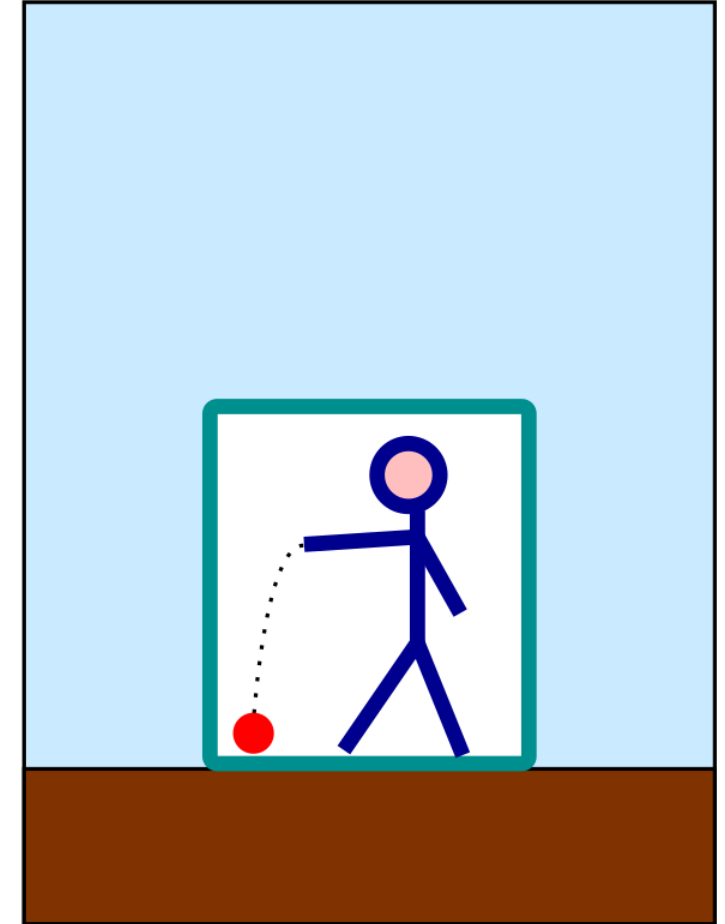
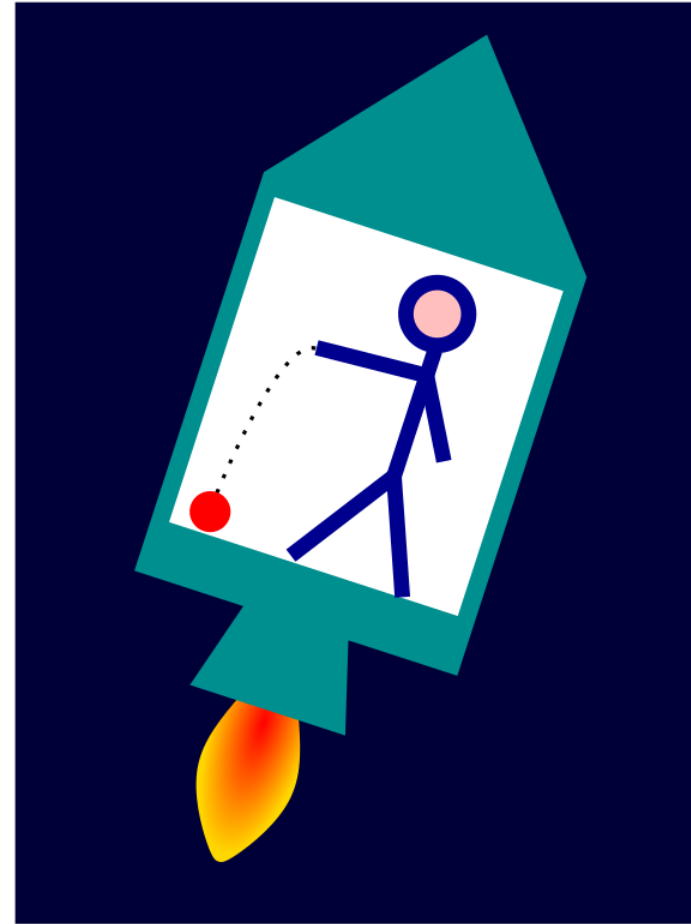
- A clock will tick at different speed depending on its location within a gravitational field (get slower deeper into the gravitational field)
- Thus, by comparing two clocks at different locations, the geopotential difference (i.e., the height difference) between the locations can be determined
 - This technique is called chronometric levelling
- With recent development in optical atomic clocks, especially transportable ones, this technique has now reached an accuracy where it is interesting for geodesy
- In this presentation I will introduce how chronometric levelling works and the recent results



AI's (Gemini) idea on how to measure height with clocks

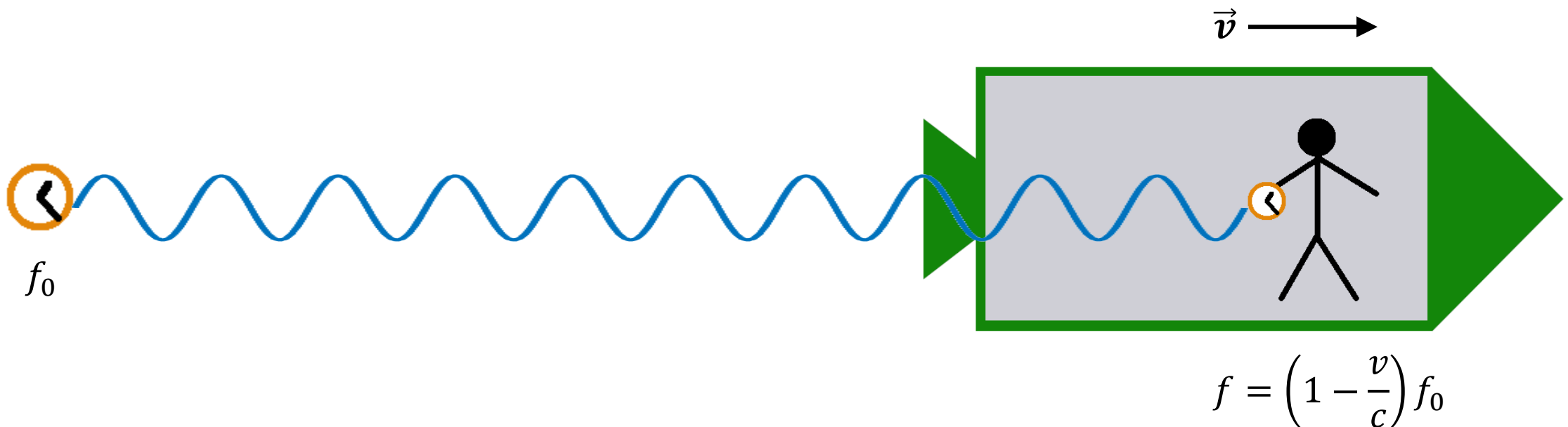
EQUIVALENCE PRINCIPLE

- Weak equivalence principle:
 - All uncharged, freely falling test particles follow the same trajectories, once an initial position and velocity have been prescribed
- Strong equivalence principle:
 - The outcome of any local, non-gravitational test experiment is independent of the experimental apparatus' velocity relative to the gravitational field and is independent of where and when in the gravitational field the experiment is performed
- A clock in a gravity field is affected in the same way as an accelerating clock



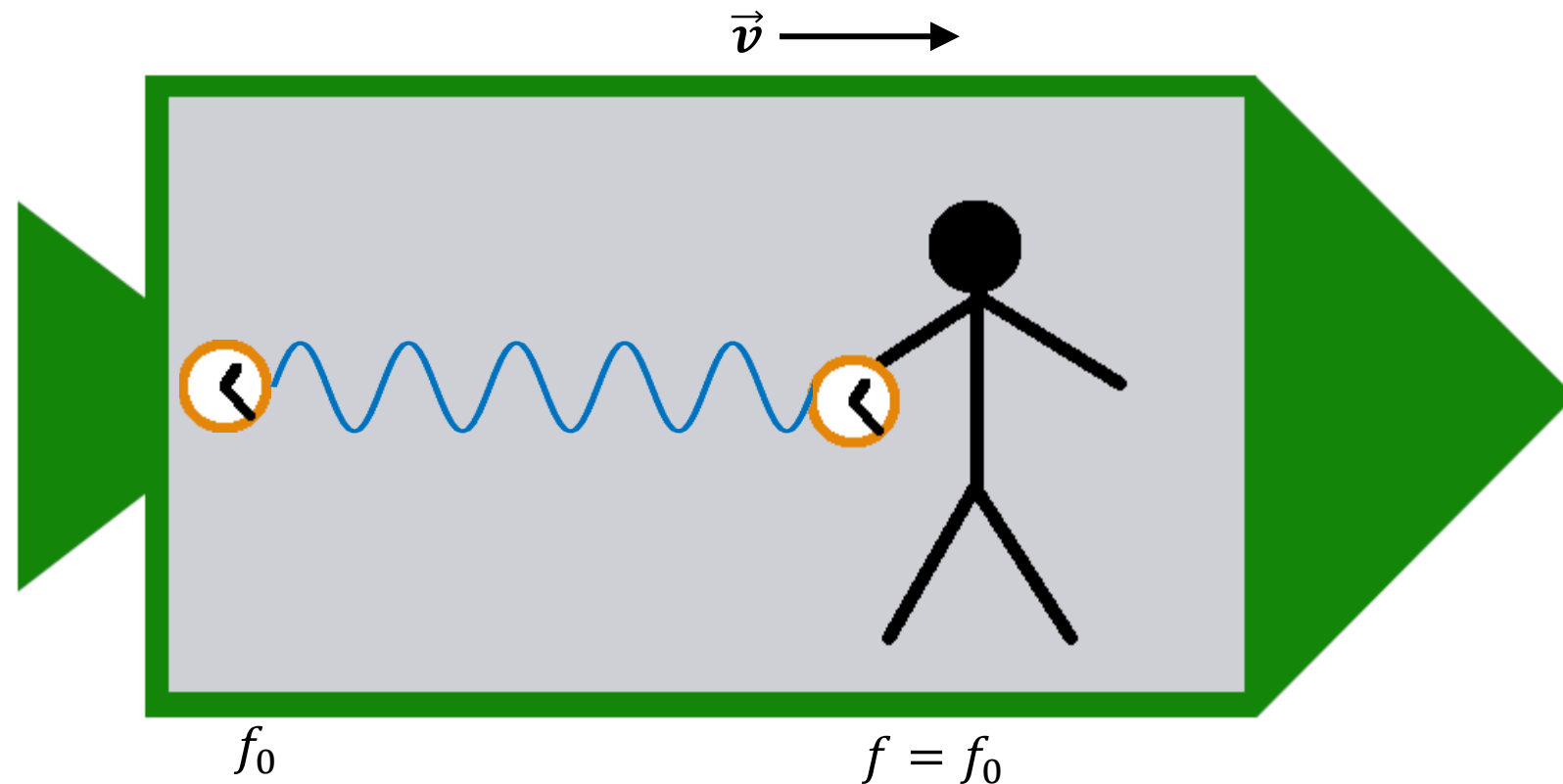
THOUGHT EXPERIMENT

- A clock is generating and transmitting a signal with frequency f_0
- The signal is received in a spaceship travelling with speed v relative to the clock and the frequency is measured
- Due to the Doppler effect, the measured frequency f is lower than f_0



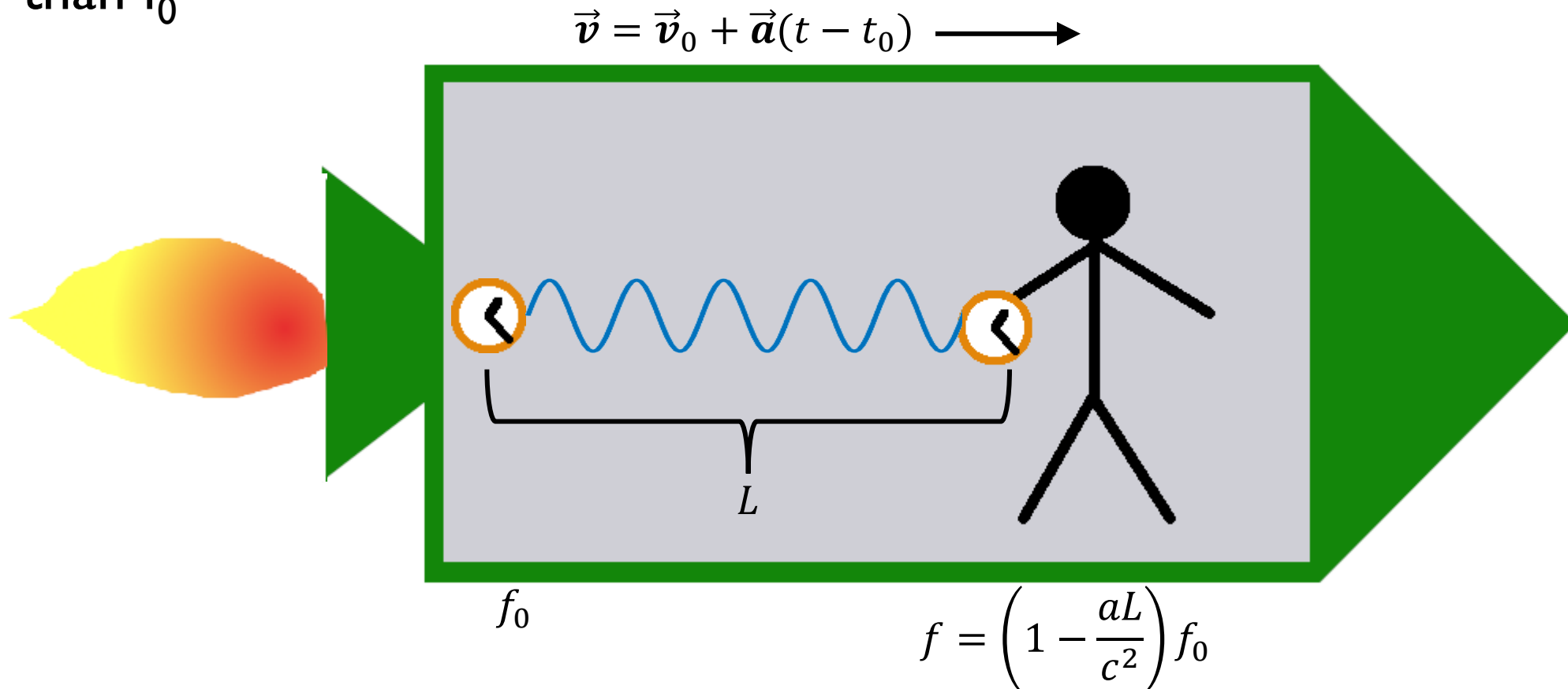
THOUGHT EXPERIMENT (II)

- If the clock would be onboard the spaceship, there would be no difference in frequency (no difference in velocity)



THOUGHT EXPERIMENT(III)

- What if the spaceship is accelerating?
- It will take time L/c for the signal to reach the receiver, in this time the velocity increase by $\Delta v = aL/c$. The observed frequency f is lower than f_0

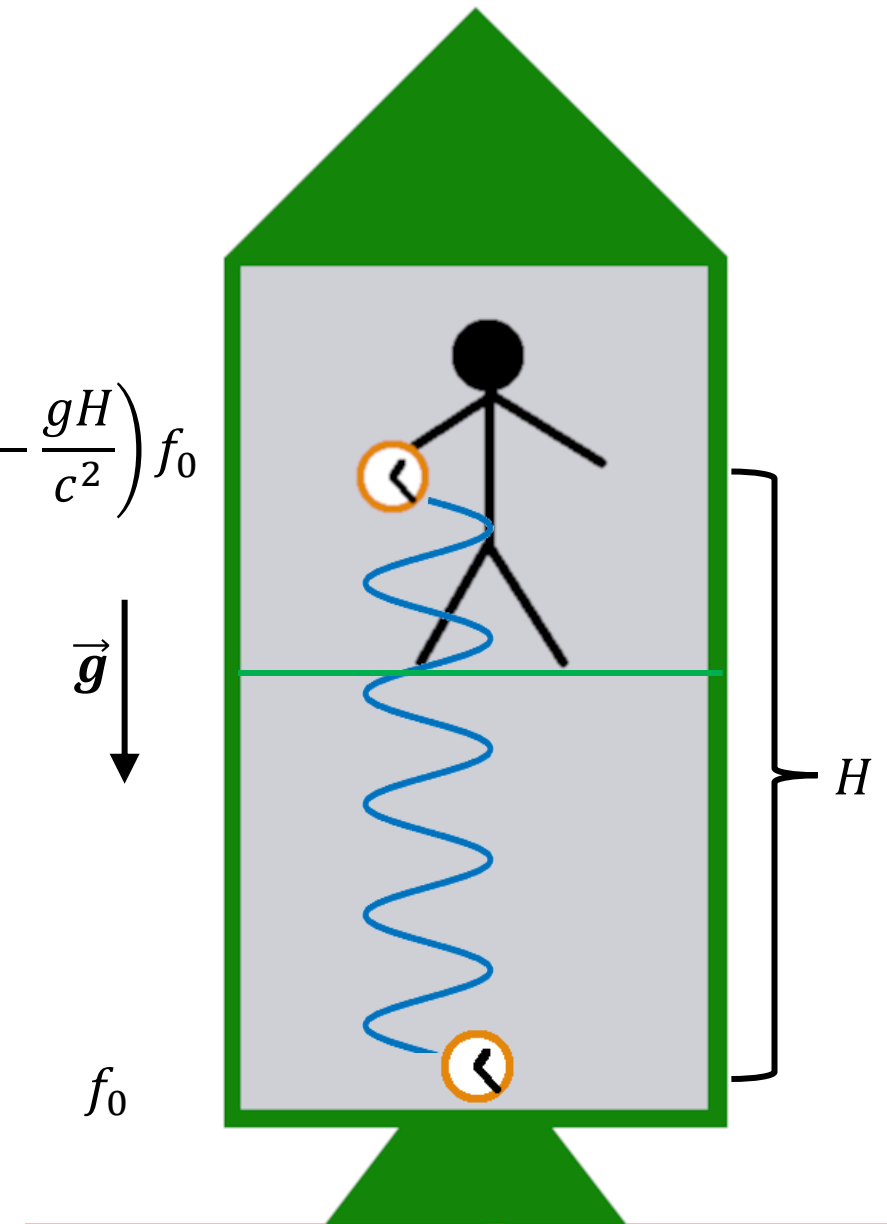


THOUGHT EXPERIMENT (IV)

- The spaceship is parked on a planet with gravity $\vec{g}$, receiver is at height H above the clock
- According to the equivalence principle the result will be the same as when the spaceship would be accelerating with acceleration g , i.e., the measured frequency is lower than f_0
- By measuring the frequency difference, the height H can be determined

$$H = \frac{f_0 - f}{f_0} \frac{c^2}{g}$$

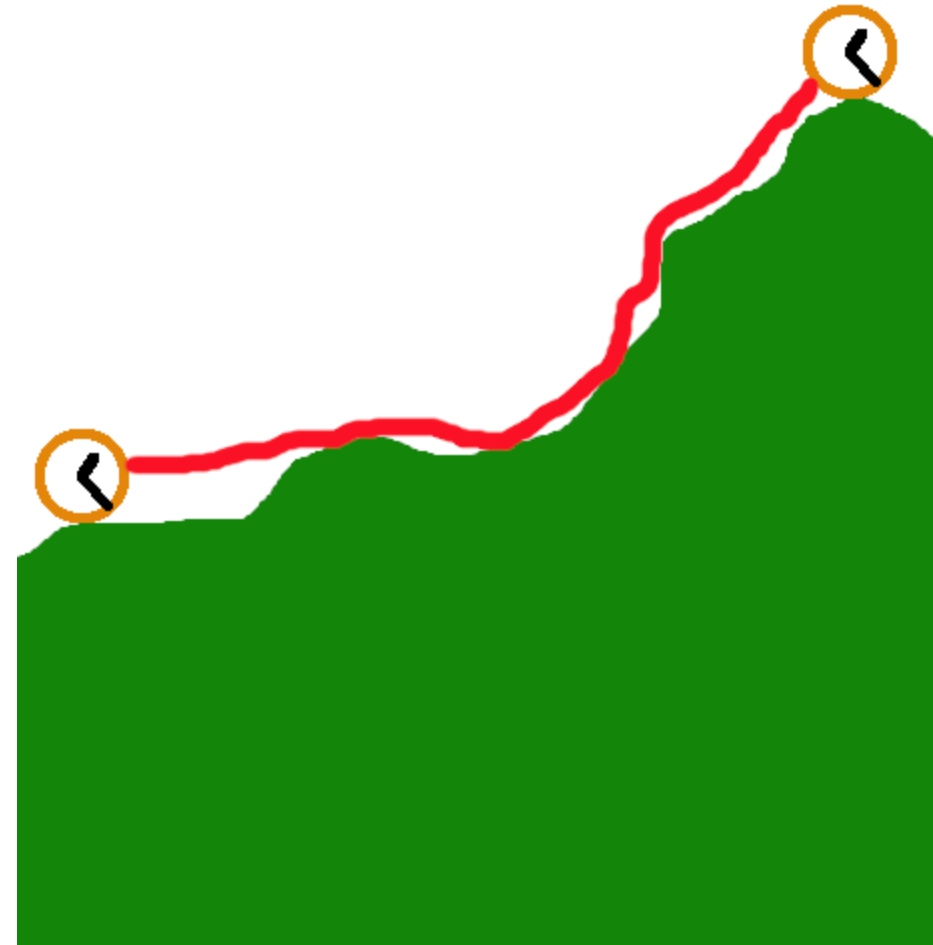
$$f = \left(1 - \frac{gH}{c^2}\right) f_0$$



MEASURING HEIGHTS USING CLOCKS

- It is possible to measure height with using clocks
 - Two clocks at two different locations
 - A signal is generated using the clock at the first location and transferred to the second location
 - At the second location, the frequency is measured using the second clock
 - From the frequency difference, the geopotential difference between the locations are obtained:

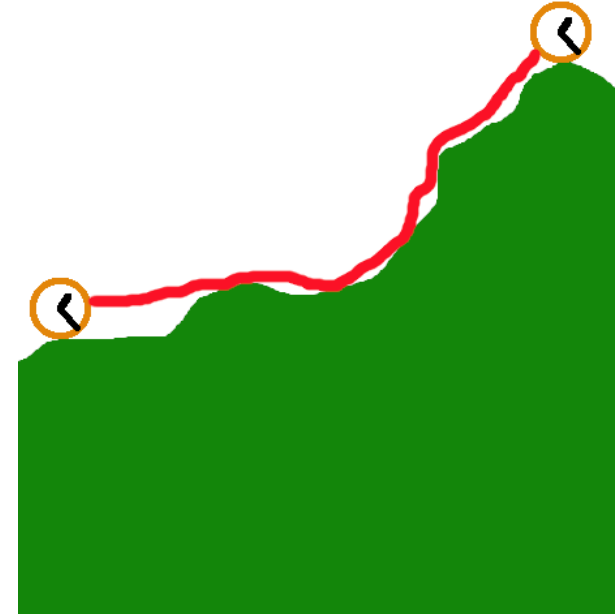
$$\Delta H \bar{\gamma} = \Delta W \approx \frac{\Delta f}{f_0} c^2$$



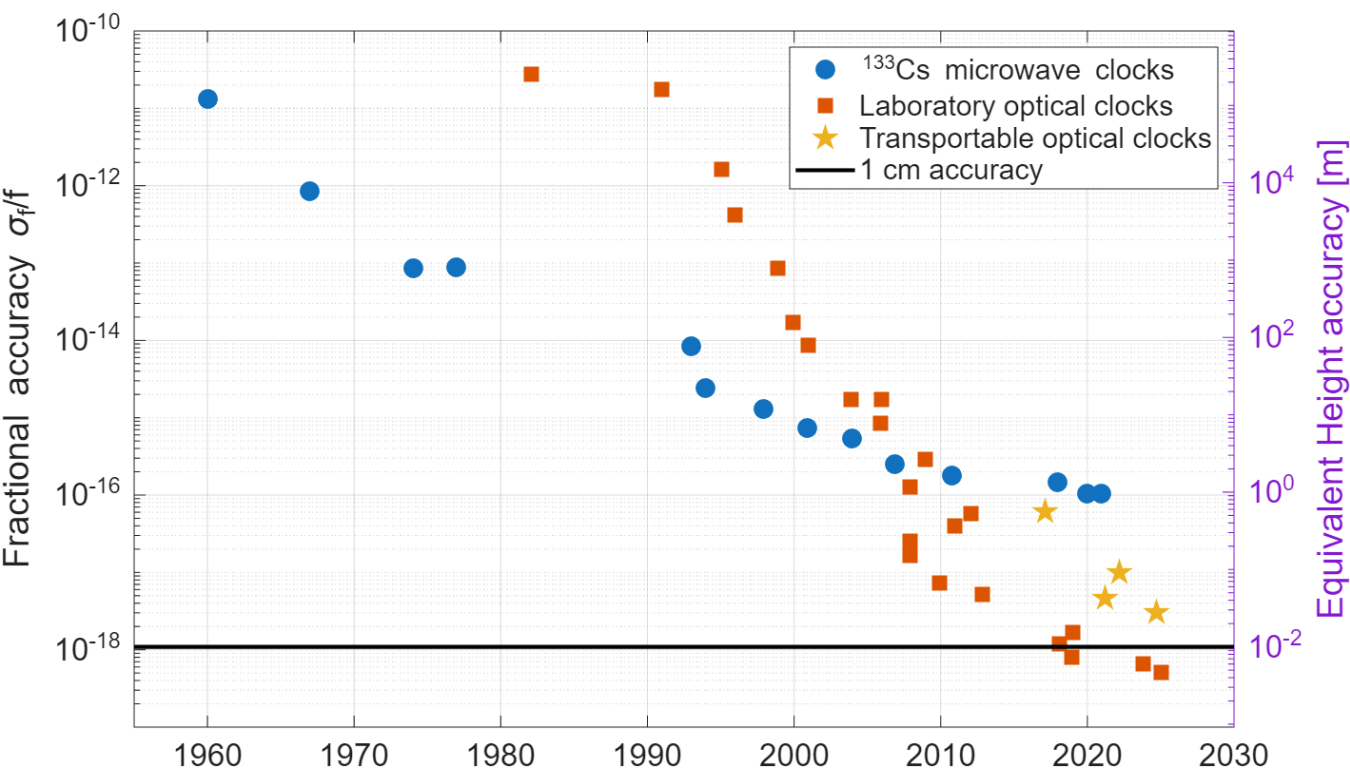
DIFFICULTIES IN MEASURING HEIGHTS WITH CLOCKS?

$$\Delta H \bar{\gamma} = \Delta W \approx \frac{\Delta f}{f_0} c^2$$

- There are always measurement errors!
- The factor c^2 is very big ($\sim 10^{17} \text{ m}^2/\text{s}^2$)
 - Δf must be measured very accurately
 - To reach 1 cm accuracy in height, $\Delta f/f$ must be measured with an accuracy of 10^{-18}
 - Requires extremely stable clocks
 - For 1 cm accuracy, the clock needs to be accurate to about 0.5 s over the age of the Universe
 - The frequency transfer must also be done with the same accuracy
- Is this possible?



ACCURACY OF ATOMIC CLOCKS AND FREQUENCY TRANSFER



Adopted from *Tara et al., Optica 13, 143-163, 2026*

Frequency transfer method	Uncertainty σ_f/f	Height uncertainty [m]
GNSS, VLBI etc.	10^{-16}	1
TWSTFT	10^{-16}	1
ACES (goal)	10^{-17}	0.1
Free space optical links	10^{-18}	0.01
Optical fiber links	10^{-19}	0.001

See next two presentations for more information about current accuracies of optical clocks and frequency transfer

HEIGHT MEASUREMENT WITH CLOCKS IN PRACTICE

Needed equipment:

- Two optical clocks, one needs to be transportable
- An accurate link for frequency transfer between the locations

1. Start with both clocks at a reference location (known height)

- Compare frequencies of the clocks, determine possible offsets

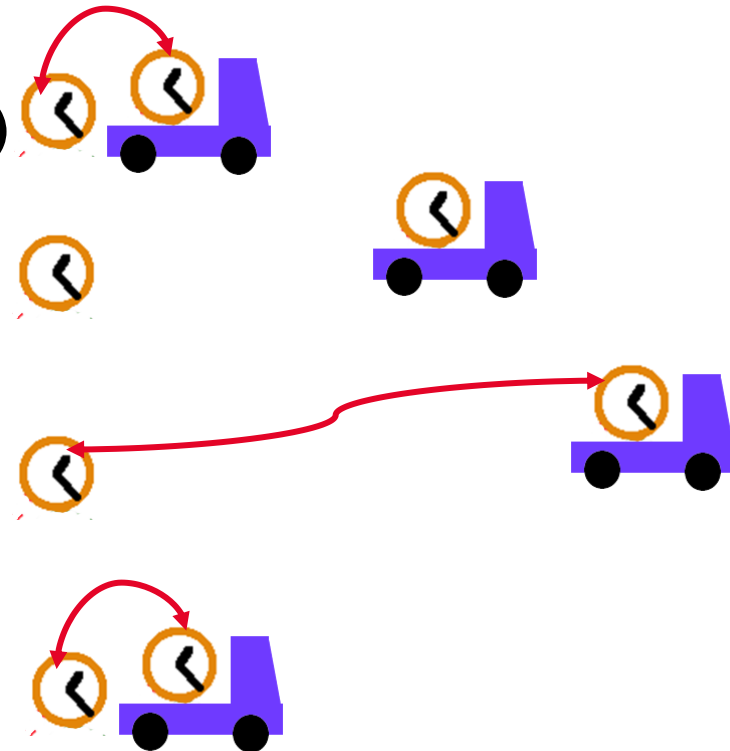
2. Move the transportable clock to the measurement location

3. Compare the frequencies of the clocks (via the link)

- From the frequency difference the geopotential difference (or height difference) can be determined

4. Move the transportable clock back to the reference location

- Compare frequencies of the clocks and check that it agrees with the results obtained in the beginning



HISTORICAL DEVELOPMENTS

- Experiments comparing clocks at different heights have been preformed since the 1970:ies
 - The goal of the earlier experiments was to validate the theory of general relativity rather than measuring heights
 - For example, *Briatore & Leschiutta (1977)* compared two clocks in Italy separated by 3250 m in height. Calculating a height difference from their results gives $\Delta H = 3600 \pm 700$ m
- The potential to use atomic clocks in geodesy for height measurements were first pointed out in the 1980'ies by, e.g., *Arne Bjerhammar* and *Martin Vermeer*
- In the last ten years, transportable optical clocks have been developed and first test of height measurements with these have been performed

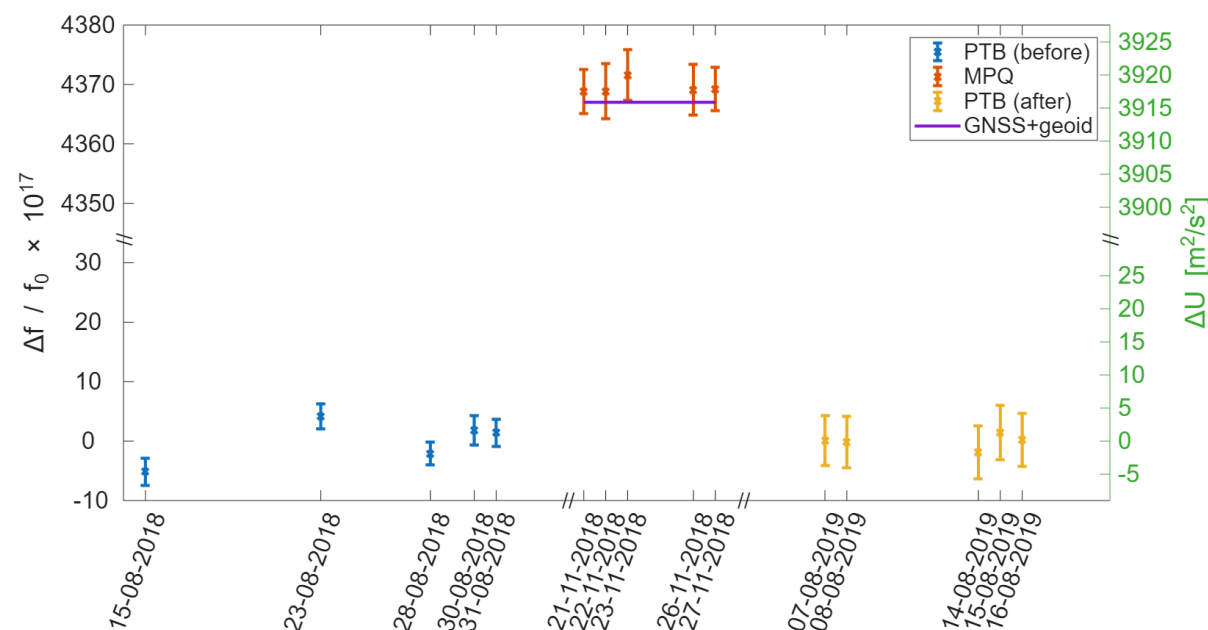
TOKYO SKYTREE

- In April 2019, two optical clocks were compared at the Tokyo Skytree, one at 450 m height and one on the ground
 - Height differences from clocks: 452.603 ± 0.039 m
 - Height difference from laser ranging: 452.596 ± 0.013 m
 - Some months before, the laser ranging measurements were compared to GNSS measurements. The agreement was on the 2 cm level



MEASUREMENTS BETWEEN PTB (BRAUNSCHWEIG, GEMANY) AND MPQ (GARCHING, GERMANY)

- A transportable optical clock from PTB, Germany, was used to measure the height difference between PTB, Braunschweig, and MPQ, Garching (457 km linear distance, connected by a 940 km long fiber-optic link), in 2018

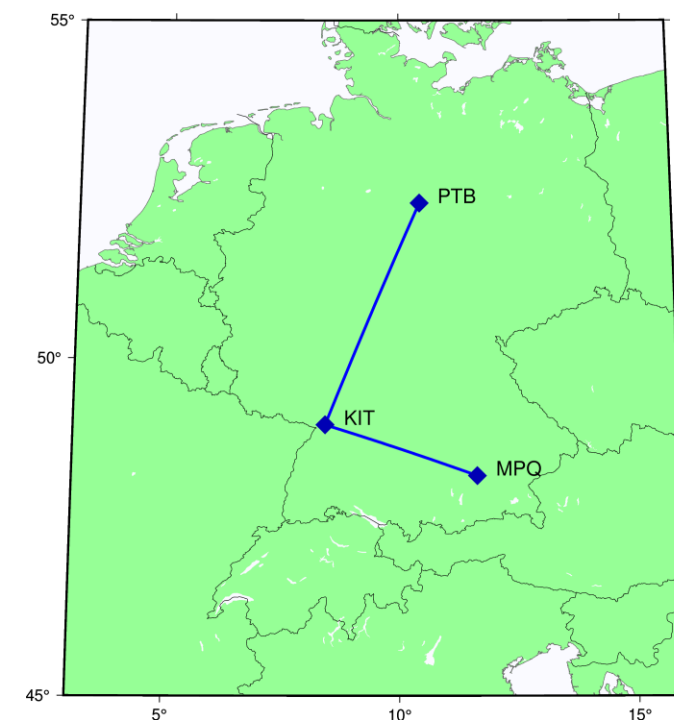


Difference in geopotential between the two locations:

- Clocks:
 - $3918.09 \text{ m}^2/\text{s}^2$
- GNSS/geoid:
 - $3915.88 \text{ m}^2/\text{s}^2$

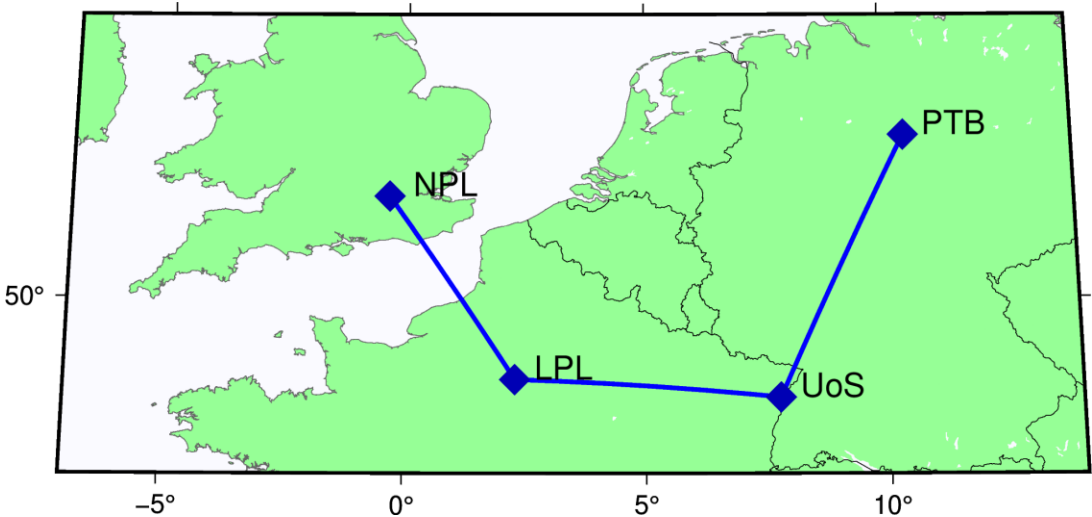
Difference: $2.21 \text{ m}^2/\text{s}^2$
(22 cm in height)

Uncertainty: $2.6 \text{ m}^2/\text{s}^2$
(27 cm in height)



MEASUREMENTS BETWEEN PTB (BRAUNSCHWEIG, GERMANY) AND NPL (TEDDINGTON, UK)

- Two transportable clocks, one from RIKEN, Japan and one from PTB, Germany
- Transportable clocks in Teddington, UK, compared to stationary clock in Braunschweig, Germany
- Distance 746 km, connected by a 2155 km long fiber-optic link
- March-April 2023
- Uncertainties in height of 5-6 cm, difference to GNSS+geiod of 7 cm



	Geopotential difference [m ² /s ²]	Uncertainty [m ² /s ²]
RIKEN Clock	668.18	0.58
PTB Clock	667.87	0.49
Clocks combined	668.00	0.37
GNSS+Geoid	667.30	0.29
EVRF2007	664.7	0.78
EVRF2019	666.3	0.28

International Clock and Oscillator Networking (ICON)
 Collaboration, arXiv:2410.22973, 2024

POSSIBLE APPLICATIONS OF CHRONOMETRIC LEVELLING

- The present accuracy of chronometric levelling is around 5 cm, it will likely increase to 1-2 cm in the near future
- Can be used where a good link for frequency transfer (like optical fiber) is available
- Possible applications:
 - Improve national and regional height systems
 - General improvement by adding an independent technique
 - Complementing where it is difficult to do traditional levelling (e.g., to islands)
 - Connecting height systems between different countries on the same continent (e.g., improve EVRF)
 - Connecting height systems between continents (requires a good link for frequency transfer). This will be important for improving IHRF

CONCLUSIONS

- Clocks can be used to measure height differences
 - Present accuracy ~5 cm, will probably improve to 1-2 cm soon
- Will be an important technique for connecting and improving height systems in the future
 - Especially on the continental and global level (like IHRF)
- Will require a good cooperation between the geodetic authorities and the national time and frequency laboratories
 - Optical clocks at the time laboratories may also become important reference points in national and international height systems

THANK YOU FOR YOUR ATTENTION!