



Investigation of systematic quasigeoid differences in mountain areas due to the handling of topographic corrections and analytical continuation (in the old NKG2015 geoid project)

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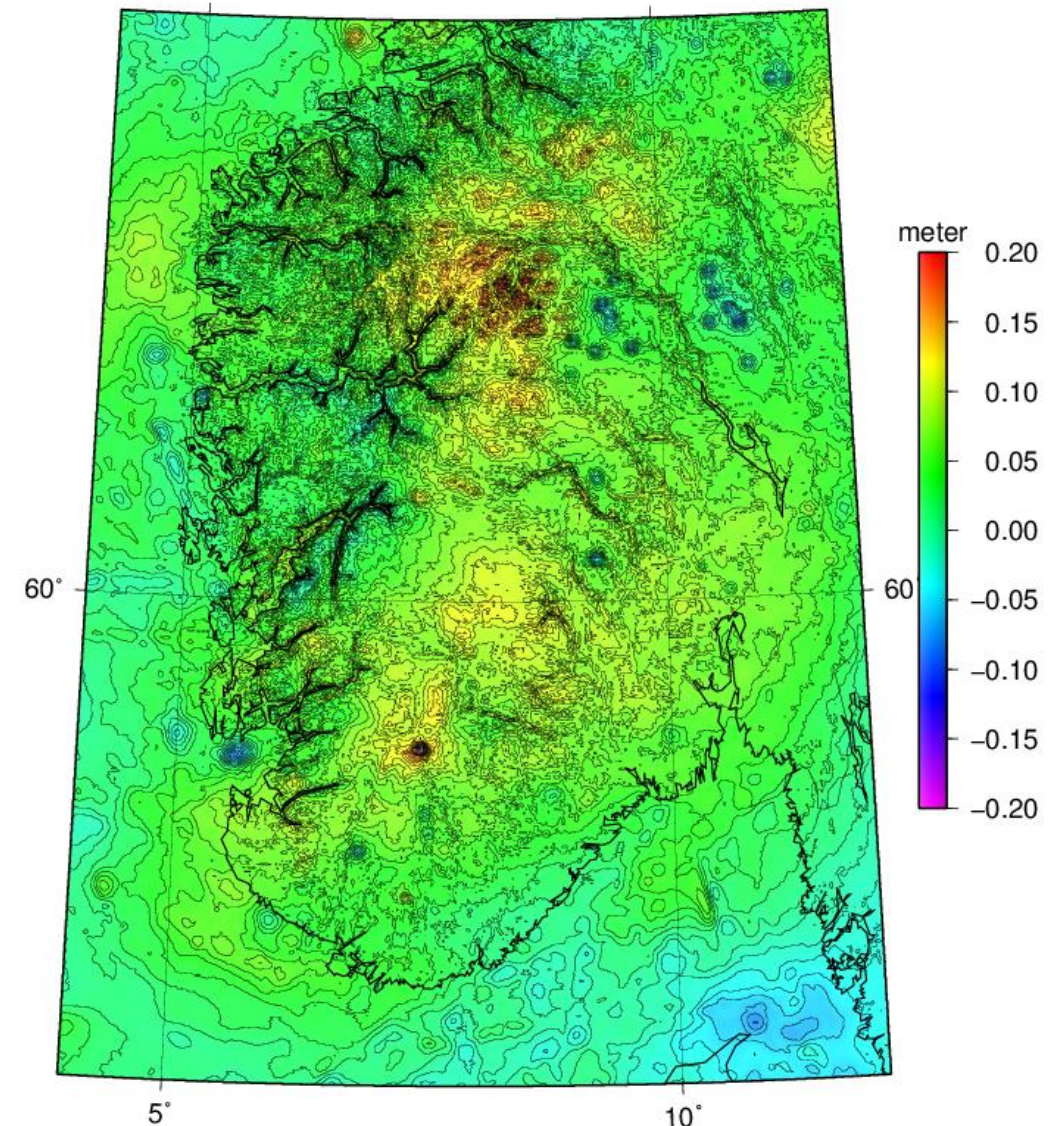
Introduction (I)

- There is a long tradition of quasigeoid determination within the NKG
- So far, this has resulted in the following sequence of models: NKG1986, NKG1989, NKG1996, NKG2002, NKG2004, and NKG2015
- Currently, the NKG Working Group of Height and Geoid (WGHG) is working on the next official model with the working name NKG2026
- In the old NKG2015 project, basically two different methods were used by the computation centres
 - Least squares modification of Stokes' formula with additive corrections (here called **KTH method**)
 - Remove-compute-restore (R-C-R) method with residual terrain model (RTM) corrections and Wong and Gore kernel modification (here called **Standard R-C-R method**)
- It should be pointed out, though, that the different computation centres applied these two main methods somewhat differently
- It is planned to use mainly the same methods also in the NKG2026 project

Introduction (2)

- When analysing the NKG2015 results, it was observed that the methods in question resulted in systematically different quasigeoid models in some areas, especially in the mountains in Norway
- See the plot to the right, which is taken from the IUGG2015 presentation in Prague)
- It has been suspected that this, at least partly, depended on how the topography and analytical continuation are dealt with by the two methods, but this was never considered in the project
- The **purpose** of this presentation is to investigate to what extent the systematic differences in question are caused by the ways topographic corrections and analytical continuation are handled by the KTH and standard RCR methods

Height anomaly difference between the standard RCR and KTH methods (DTU3a–LM1a) (Ågren et al. 2015, IUGG2015)



The KTH method

- Least Squares Modification of Stokes' formula with Additive corrections, LSMSA (Sjöberg 1991, 2003,...)
- The height anomaly is computed as

$$\zeta = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \Delta g_{surface}^{grid} d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M (s_n + Q_n^M) \Delta g_n^{GGM} + \zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL}$$

$S^M(\psi)$ is the modified Stokes' function chosen according to Sjöberg (1991).

$\delta\zeta_{DWC}$ includes analytical continuation to point-level of both the gravity anomalies (Moritz 1980) and the spherical harmonic expansion; cf. Sjöberg (2003) and Ågren (2004).

$\delta\zeta_{ATM}$ is the atmospheric correction (Sjöberg and Nahavandchi 2000).

$\delta\zeta_{ELL}$ is the ellipsoidal correction (Sjöberg 2004).

- Least squares (stochastic) kernel modification (Sjöberg 1991,...).
- Surface gravity anomalies are gridded using a **remove-interpolate-restore** technique with interpolation by Least Squares Collocation (LSC) on the EGM and RTM reduced gravity anomaly (at LM)

$$\Delta g_{surface}^{grid} = LSC(\Delta g_{surface}^{point} - \Delta g_{EGM}^{point} - \Delta g_{RTM}^{point}) + \Delta g_{EGM}^{grid} + \Delta g_{RTM}^{grid}$$

Analytical continuation to point level by the g_1 -correction (Moritz 1980) is included in the *additive correction* ζ_{DWC}

$$\zeta_P \approx \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) (\Delta g + g_1) d\sigma \quad g_1 = -\frac{\partial \Delta g}{\partial r} (H - H_P)$$

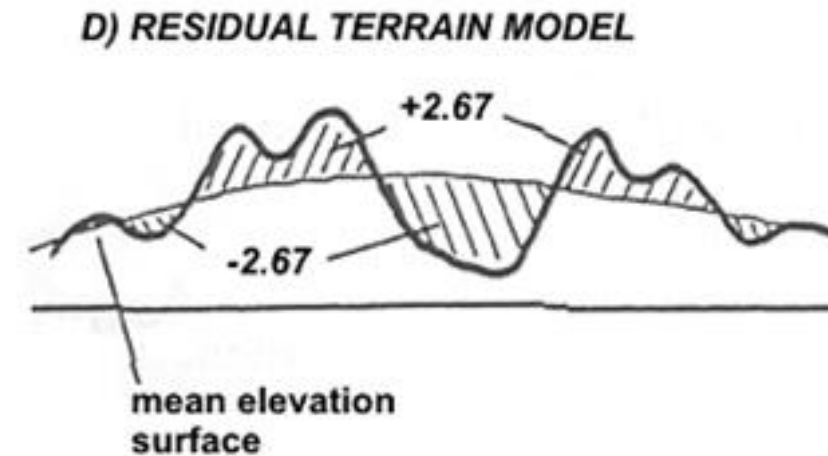
The standard RCR method

The height anomaly is computed as (Forsberg 1984, 1997, etc., etc.)

$$\zeta_P = \frac{R}{4\pi\gamma} \iint_{\sigma_0/c} S^L(\psi) (\Delta g - \Delta g_{EGM}^M - \Delta g_{RTM} + g_{1,red}) d\sigma + \zeta_{EGM}^M + \zeta_{RTM}$$

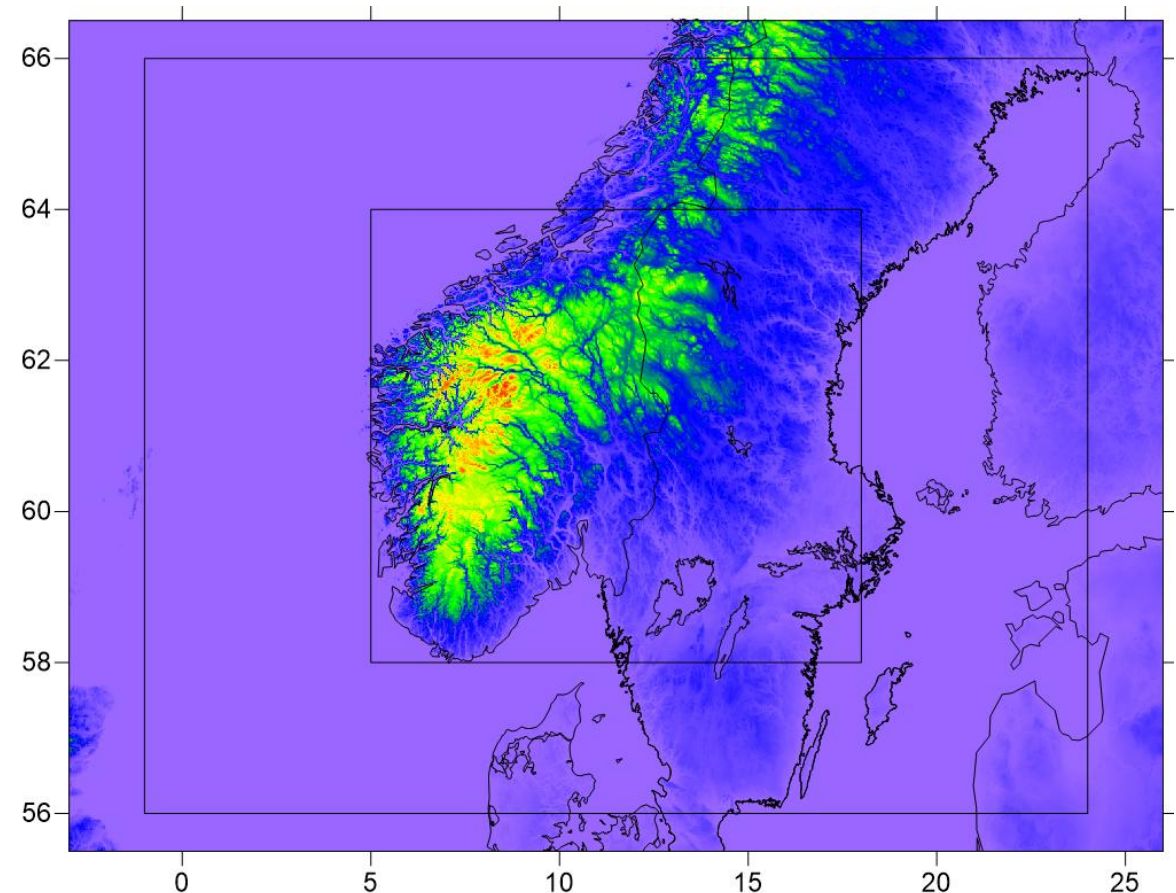
- Modified Stokes' formula is truncated to a cap σ_0 or a fixed rectangular area
- Stokes' kernel is modified according to the **Wong & Gore** (1969) method, usually with linear blending between degree N_1 and N_2
- The topographic effects are computed by the **Residual Terrain Modelling (RTM)** technique using rectangular prisms in TC, condensation approximation in SPFOUR, etc.
- The smooth **mean elevation surface** is often computed using a **moving average** filter using the Gravsoft program TCGRID. Sometimes other filters are used
- The resolution of the mean elevation surface is often chosen by to optimise the fit to GNSS/levelling
- Harmonic corrections are nowadays applied both for the RTM effect on gravity (Forsberg, 1984) and for the height anomaly (Klees et al., 2023)

The $g_{1,red}$ -term is small for the residual gravity anomaly in question. It is more or less always neglected



Test area and data

- To investigate whether there is some kind of systematic difference between the KTH and RCR methods in the mountains, a test area is selected over southern Norway and central Sweden
- The following data is used
 - Gravity data from the NKG2015 project (as NKG2026 DB are still not ready and to simplify things)
 - NKG_DEM2014 (Bilker, 2014)
 - GOCO06S (M=300) and XGM2019 (M=760)
 - GNSS/levelling data from NKG2026 (version 0.1), list B



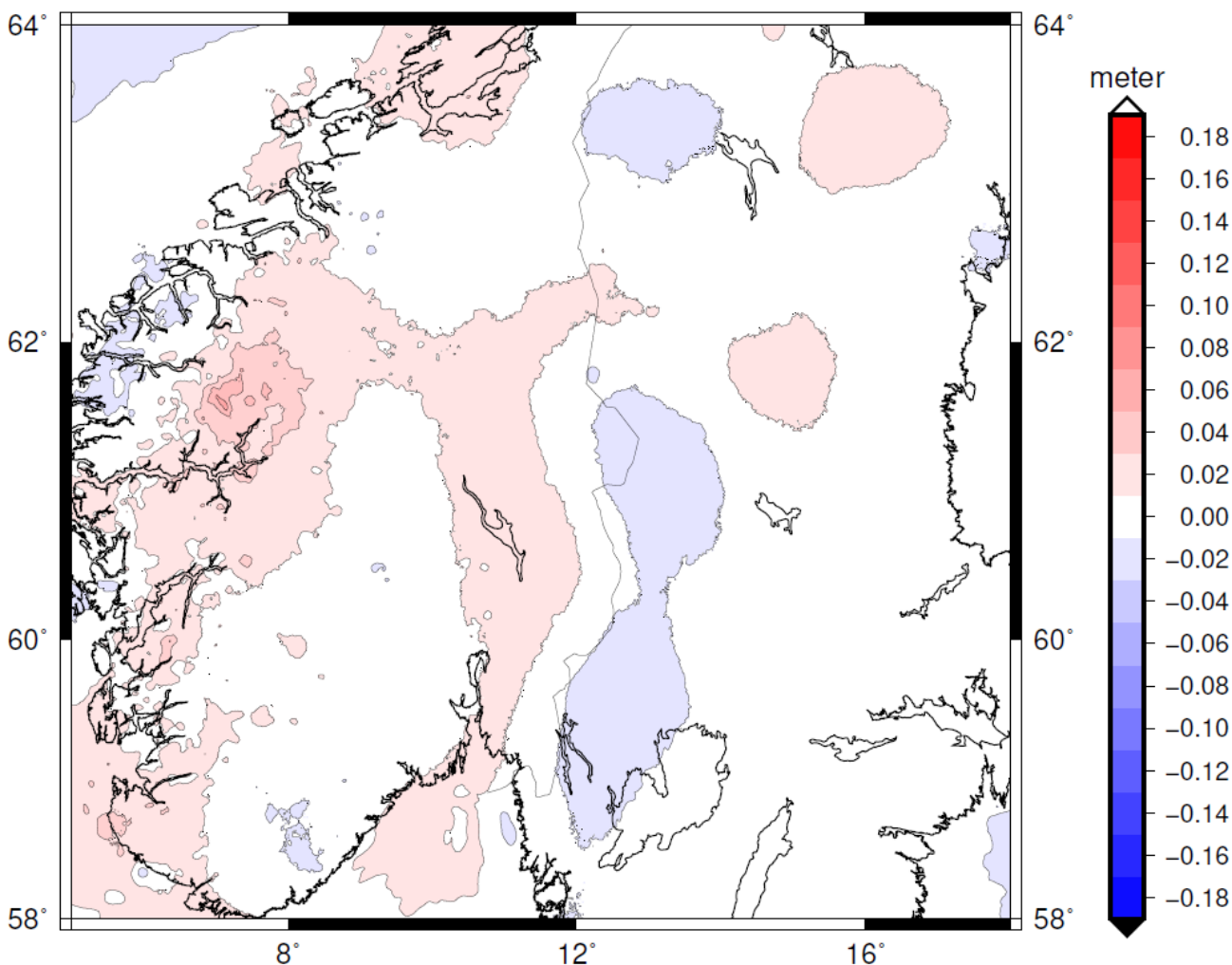
Numerical tests

- There are many different error sources that affect both the KTH and RCR methods
- As the purpose of this presentation is to investigate only how the different methods deal with topographic corrections and analytical continuation, the **same kind of kernel modification will be used by both methods**, namely the least squares modification of Stokes' formula
- Besides what is explicitly mentioned here, the same settings as for the final NKG2015 model are applied
- The following quasigeoid computations were made:

Method	EGM	M, max. degree of the EGM	Resolution of the mean elevation surface for RTM (TCGRID)	K, approximate max. degree of mean elevation surface
KTH_A	GOCO06S	240	~82 km x 82 km	~240
KTH_B	XGM2019	720	~27 km x 27 km	~720
RCR_A	GOCO06S	240	~82 km x 82 km	~240
RCR_B	GOCO06S	240	~55 km x 55 km	~360
RCR_C	GOCO06S	240	~21 km x 21 km	~900
RCR_D	XGM2019	720	~21 km x 21 km	~900

Comparison of KTH_A and NKG2015

Height anomaly difference [m]



KTH_A

GOCO06S

240

~82 km x 82 km

~240

Statistics of GNSS/levelling I-parameter fit in **Norway**[m]

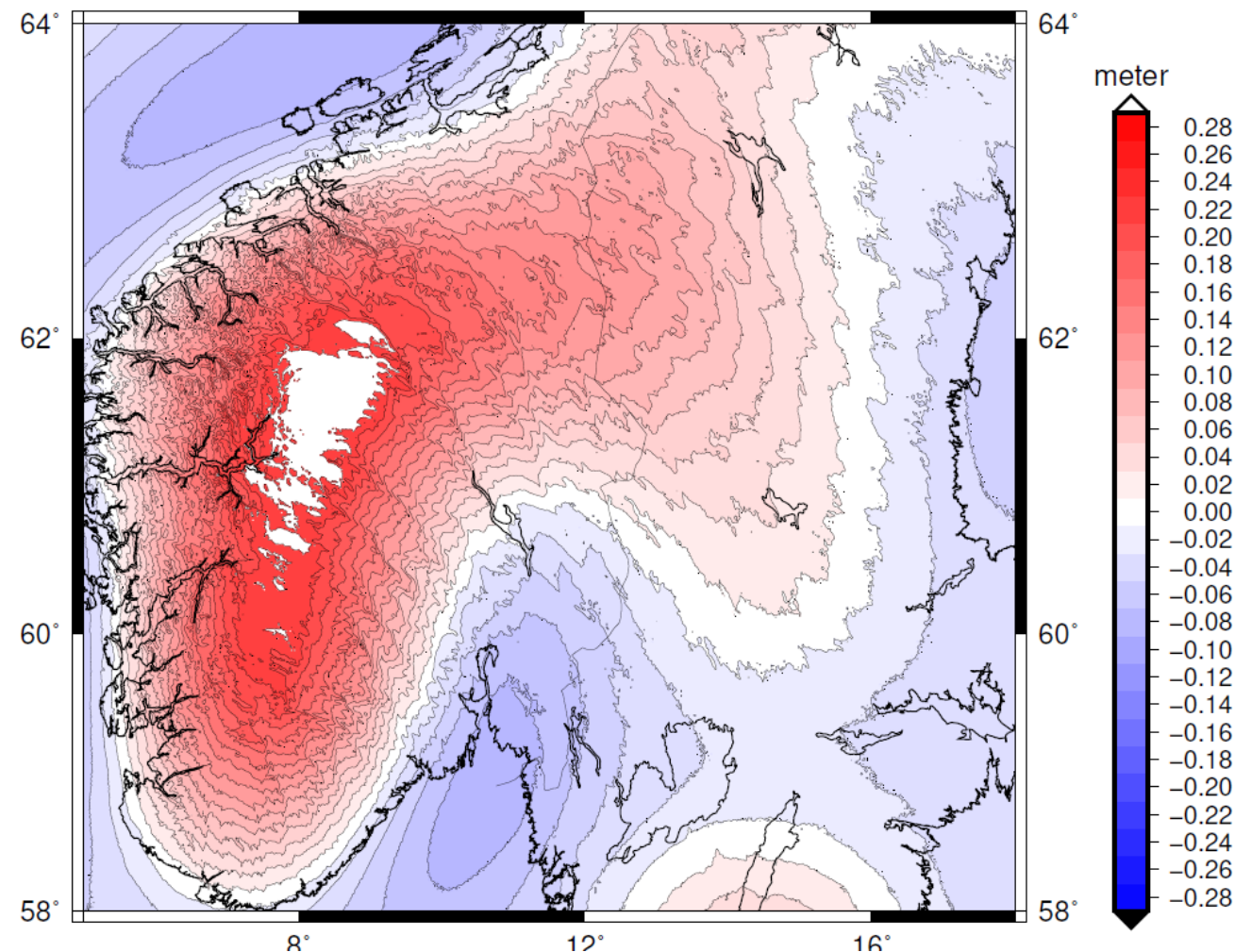
Method	Min	Max	Mean	StdDev
KTH_A	-0.1430	0.0880	0.0000	0.0301
NKG2015	-0.1410	0.0870	0.0000	0.0293

Statistics of GNSS/levelling I-parameter fit in **Sweden**[m]

Method	Min	Max	Mean	StdDev
KTH_A	-0.0580	0.0310	0.0000	0.0136
NKG2015	-0.0570	0.0410	0.0000	0.0135

Comparison of RCR_A and KTH_A

Height anomaly difference [m]



RCR_A	GOCO06S	240	~82 km x 82 km	~240
KTH_A	GOCO06S	240	~82 km x 82 km	~240

Statistics of GNSS/levelling I-parameter fit in **Norway**[m]

Method	Min	Max	Mean	StdDev
RCR_A	-0.2600	0.1940	0.0000	0.1085
KTH_A	-0.1430	0.0880	0.0000	0.0301

Statistics of GNSS/levelling I-parameter fit in **Sweden**[m]

Method	Min	Max	Mean	StdDev
RCR_A	-0.1720	0.0900	0.0000	0.0531
KTH_A	-0.0580	0.0310	0.0000	0.0136

- The RCR_A solution performs very badly in the Norwegian mountains

GNSS/levelling statistics for all solutions

Statistics of GNSS/levelling I-parameter fit in **Norway** [m]

Method	Min	Max	Mean	StdDev
KTH_A	-0.1430	0.0880	0.0000	0.0301
KTH_B	-0.0146	0.0740	0.0000	0.0284
RCR_A	-0.2600	0.1940	0.0000	0.1085
RCR_B	-0.1550	0.1110	0.0000	0.0524
RCR_C	-0.1370	0.0860	0.0000	0.0309
RCR_D	-0.1330	0.0710	0.0000	0.0300

Statistics of GNSS/levelling I-parameter fit in **Sweden** [m]

Method	Min	Max	Mean	StdDev
KTH_A	-0.0580	0.0310	0.0000	0.0136
KTH_B	-0.0560	0.0360	0.0000	0.0141
RCR_A	-0.1720	0.0900	0.0000	0.0531
RCR_B	0.0890	0.0490	0.0000	0.0272
RCR_C	-0.0640	0.0350	0.0000	0.0148
RCR_D	-0.0620	0.0410	0.0000	0.0149

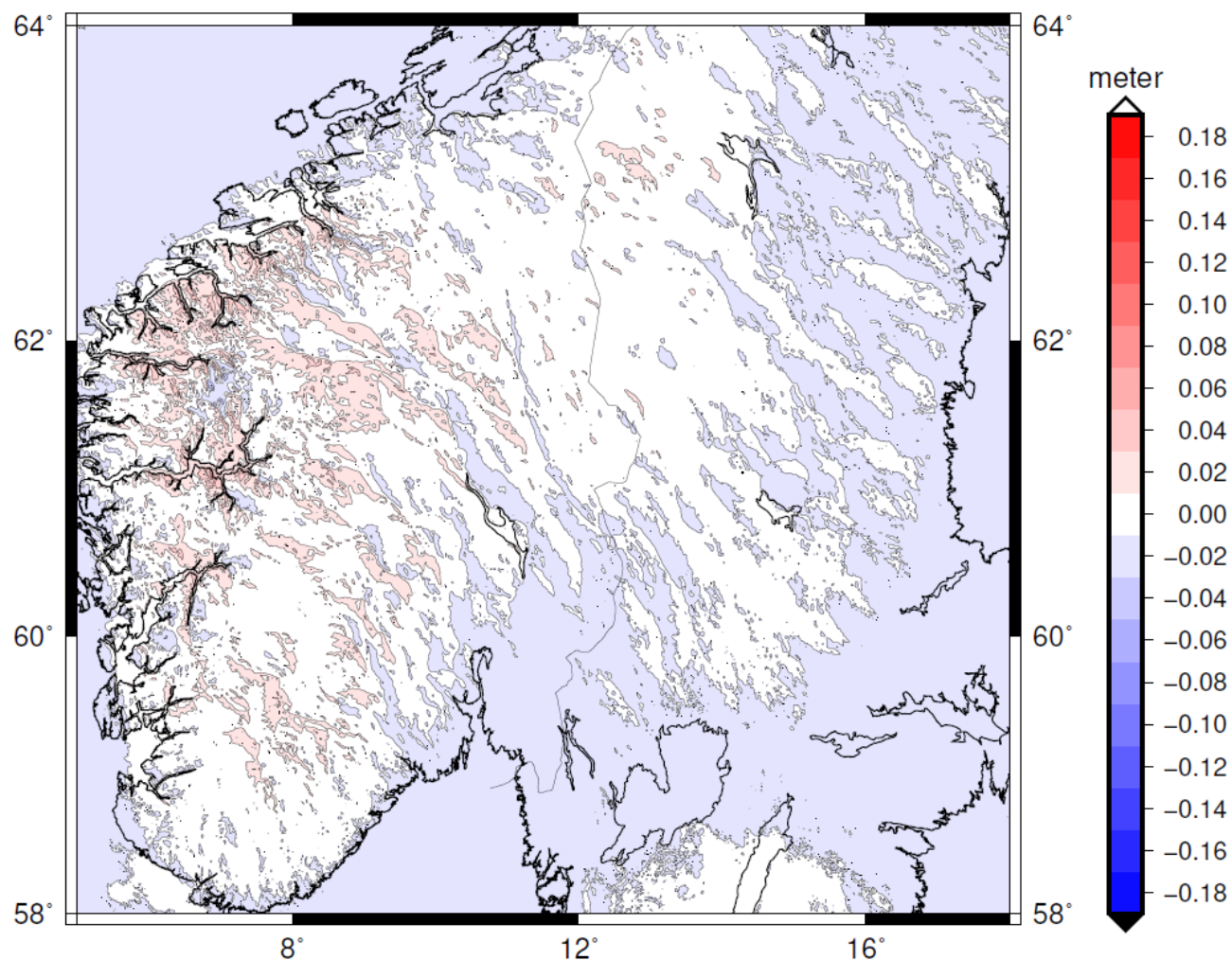
- In the RCR/satellite-only case, the resolution of the mean elevation surface must be chosen sufficiently high. The corresponding degree K should be much higher than M. In our case K=900 gives good results
- This is not so important for a combined EGM with higher maximum degree, like XGM2019 with M=760
- The resolution of the mean elevation surface is not crucial for the KTH method, since the RTM effect on gravity anomaly is both removed and restored)

Numerical tests (same table again)

Method	EGM	M, max. degree of the EGM	Resolution of mean elevation surface for RTM (TCGRID)	K, approximate max. degree of the mean elevation surface
KTH_A	GOCO06S	240	~82 km x 82 km	~240
KTH_B	XGM2019	720	~27 km x 27 km	~720
RCR_A	GOCO06S	240	~82 km x 82 km	~240
RCR_B	GOCO06S	240	~55 km x 55 km	~360
RCR_C	GOCO06S	240	~21 km x 21 km	~900
RCR_D	XGM2019	720	~21 km x 21 km	~900

Comparison of RCR_C and KTH_A

Height anomaly difference [m]



RCR_C	GOCO06S	240	~21 km x 21 km	~900
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KTH_A	GOCO06S	240	~82 km x 82 km	~240
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Statistics of GNSS/levelling I-parameter fit in **Norway**[m]

Method	Min	Max	Mean	StdDev
RCR_C	-0.1370	0.0860	0.0000	0.0309
KTH_A	-0.1430	0.0880	0.0000	0.0301

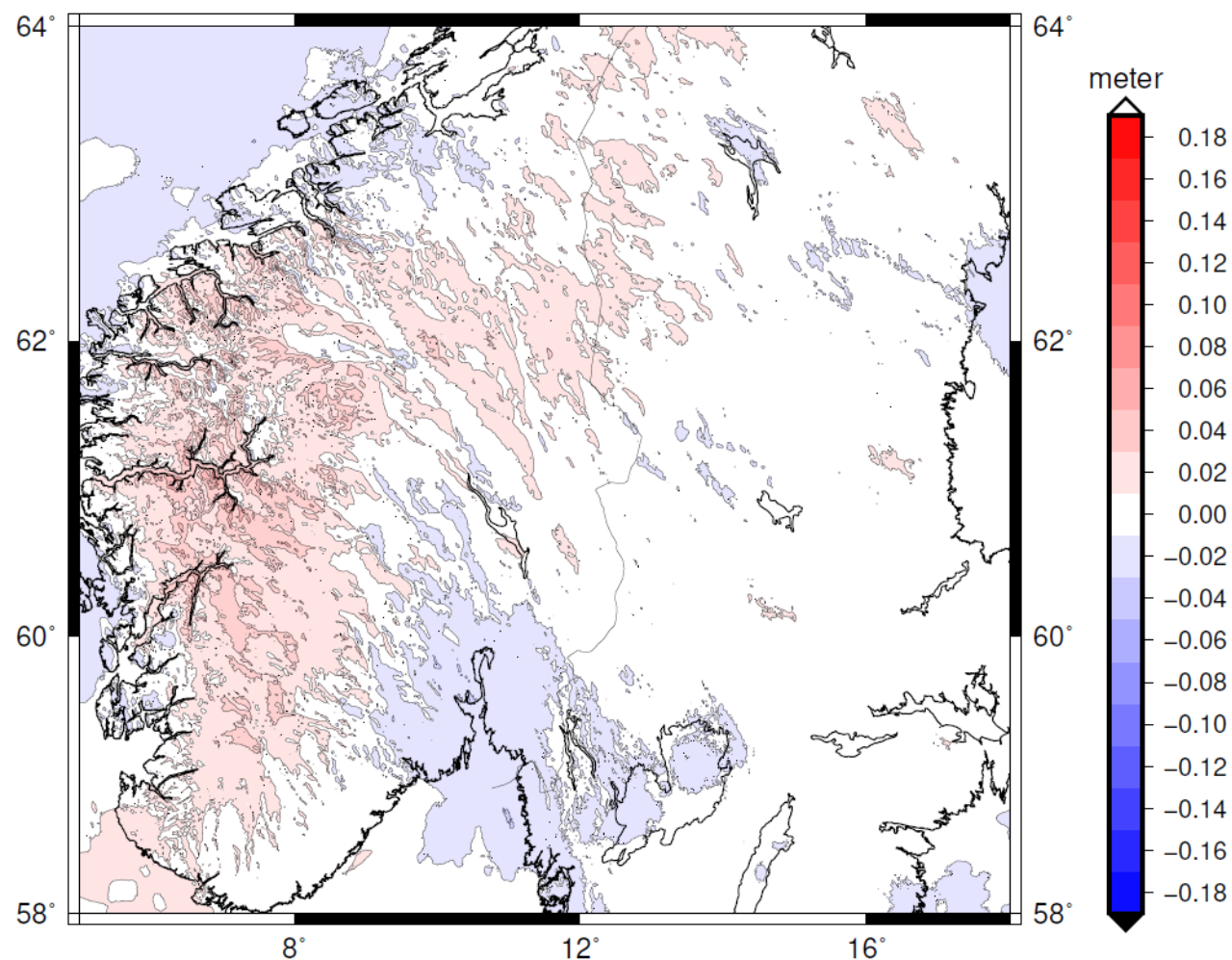
Statistics of GNSS/levelling I-parameter fit in **Sweden**[m]

Method	Min	Max	Mean	StdDev
RCR_C	-0.0640	0.0350	0.0000	0.0148
KTH_A	-0.0580	0.0310	0.0000	0.0136

- When $K=900$, ($\gg M$) the RCR method gives excellent results with a satellite-only model

Comparison of RCR_D and KTH_A

Height anomaly difference [m]



RCR_D	XGM2019	720	~21 km x 21 km	~900
KTH_A	GOCO06S	240	~82 km x 82 km	~240

Statistics of GNSS/levelling I-parameter fit in **Norway**[m]

Method	Min	Max	Mean	StdDev
RCR_D	-0.1330	0.0710	0.0000	0.0300
KTH_A	-0.1430	0.0880	0.0000	0.0301

Statistics of GNSS/levelling I-parameter fit in **Sweden**[m]

Method	Min	Max	Mean	StdDev
RCR_D	-0.0620	0.0410	0.0000	0.0149
KTH_A	-0.0580	0.0310	0.0000	0.0136

- The RCR method performs well also with a combined EGM
- In this case K can be chosen equal to or close to M

Conclusions

- For the RCR method, it is found that very significant systematic errors are introduced in the Norwegian mountains in case
 - A satellite-only EGM is used
 - The mean elevation surface for RTM is computed with a moving averaging filter, using for instance TCGRID
 - The mean elevation surface has comparatively low resolution (K in the vicinity of M)
- Furthermore, if a mean elevation surface with very high resolution is used ($K \gg M$), then very similar results are obtained by the tested versions of the RCR and KTH methods, and a good fit to GNSS/levelling is obtained
- The same systematic behaviour does not arise when a combined EGM with high maximum degree (like 720) is used. K can then be close to M
- The reason for the systematic behaviour is believed to be that the moving average filter results in significant low degree information below degree K in the RTM correction (cf. the degree transfer function of the moving average filter).
- This low degree effect is not cancelled when adding the indirect RTM effect on the height anomaly in the RCR method (with a modified kernel).
- However, this must be more properly studied in the future