On the relation between Moho and sub-crustal stress induced by mantle convection

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Abstract

The sub-crustal stress components due to the mantle convection have a direct relation with spherical harmonic coefficients of the Earth's disturbing potential so as those of the Moho model, developed by the Vening-Meinesz-Moritz theory. In this paper, the relation between the stress components and global and local models of Moho are mathematically developed in three different ways. An integral approach is developed for integration of a local Moho model for the stress recovery, but the kernels of this integral is not likely convergent and should be generated by their spectral forms to a limited degree. Another method is developed based on integral inversion, which is free of any mathematical problem and suitable to recover *S* from an existing model of Moho.

Introduction

The aim is to develop the mathematical relations between the Moho surface and the sub-crustal stress. Three methods are presented for this purpose. The first one considers the spherical harmonic coefficients of a Moho model, and relates it to those of the sub-stress components. The second approach is an integral formula to integrate an existing local Moho model and the topographic heights to deliver these components, and the last method is based on integral inversion of a local Moho model. Since the original spherical harmonic series of the sub-crustal stress of Runcorn (1967) is not likely convergent to high degrees, another method is used, which is asymptotically convergent to higher degrees than the Runcorn's formulae. This method named the *S* function (*S*) with numerical differentiation by Eshagh (2014) and is applied in this paper.

Mathematical relations between Moho surface and sub-crustal stress components

Proposition 1: *The mathematical relation between the spherical harmonic coefficients of a Moho model and the sub-crustal stress components is:*

$$S_x\mathbf{e}_\theta + S_y\mathbf{e}_\lambda = \frac{-g\Delta\rho}{s^2}\sum_{n=2}^{N}\frac{1}{s^{n+1}(n-1)}\sum_{m=-n}^{n}\left(-\frac{2n+1}{n+1}\frac{(\mu H)_{nm}}{2\Delta\rho}+T^0_{\mathrm{Moho},nm}\right)X^2_{nm}(\theta,\lambda) \qquad s=\left(\frac{R-D}{R}\right)$$

where S_x and S_y are the sub-crustal stress components, e_θ and e_λ the unit vectors towards the north and the east, *g* stands for the gravity attraction, $\Delta\rho$ is the density contrast between the crust and mantle, *R* is the mean radius of the Earth and *D* the mean Moho depth, $X^2_{nm}(\theta,\lambda)$ is the vector spherical harmonics of degree *n* and order *m* with arguments of co-latitude θ and longitude λ. $T^0_{\mathrm{Moho},nm}$ are the spherical harmonic coefficients of the Moho model and $(\mu H)_{nm}$ those of product topographic mass density μ and height (*H*).

Proposition 2: *The integral equation for recovering S from Moho depth is:*

$$\frac{-g\Delta\rho}{4\pi s^2}\iint_\sigma K(\psi)S'd\sigma = -\sum_{n=2}^{\infty}\frac{2n+1}{n+1}\frac{(\mu H)_n}{2\Delta\rho}+T^0_{\mathrm{Moho}}$$

$$K(\psi) = -\frac{s}{l}+\frac{s^2}{l^3}(\cos\psi-3s)+6\frac{s^3}{l^5}(s-\cos\psi)^2+s$$

where the argument of cos ψ the geocentric angle between the computation and integration points. σ is the unit sphere and $d\sigma$ the surface integration element. By solving this integral equation *S* is derived and by taking its derivatives towards the north and the east, we can compute the sub-crustal stress components. Eshagh (2014) showed that the spherical harmonic expansion of *S* function is convergent to higher degrees than that of Runcorn's stress.

Behaviour of integral kernels

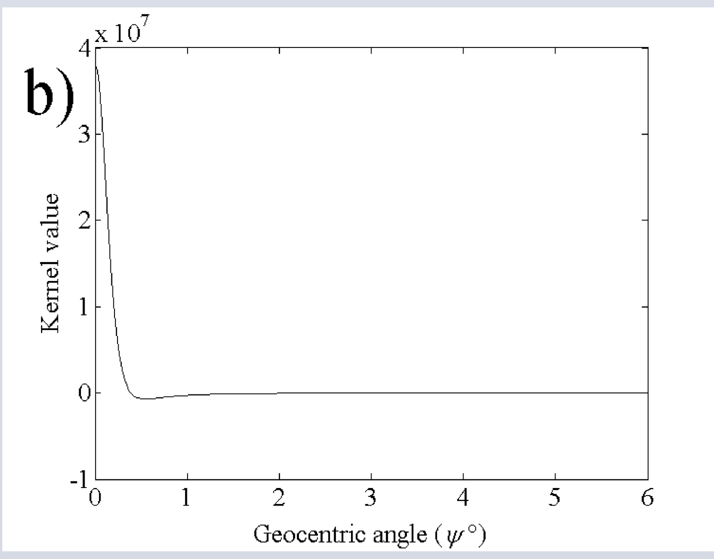

Figure 1. kernel of integral of Proposition 2

The isotropic parts of the kernels have the main role to show the significance of the far-zone data. Figure 1 illustrates the kernel of the integral formula presented in Proposition 2, which should be inverted to recover *S* and suitable for local studies. According to the figure, the kernel is well-behaving and suitable for the integral inversion of a local Moho model to *S*.

Applications

Now, EGM08 (Pavlis et al. 2008) to degree and order 360 and GRS80 (Moritz 2000) are used for generating the spherical harmonic coefficients of the VMM Moho model (Sjöberg 2009). Also, a density contrast of 600 gr / cm^3 is considered between the crust and mantle and an average value of 23 km is considered as the mean, or the normal, Moho depth. The digital terrain model of DTM2008 (Pavlis et al. 2006) is used for generating the topographic heights and the mean density of the crust is assumed to be 2.67 gr / cm^3. Figure 3a shows the global map of the gravimetric Moho model generated by the VMM theory on the background of the tectonic boundaries shown by the red lines. As expected, the gravimetric model of Moho has the deepest part under Himalaya and its slope has a very good agreement with the tectonic boundaries in the southern part.

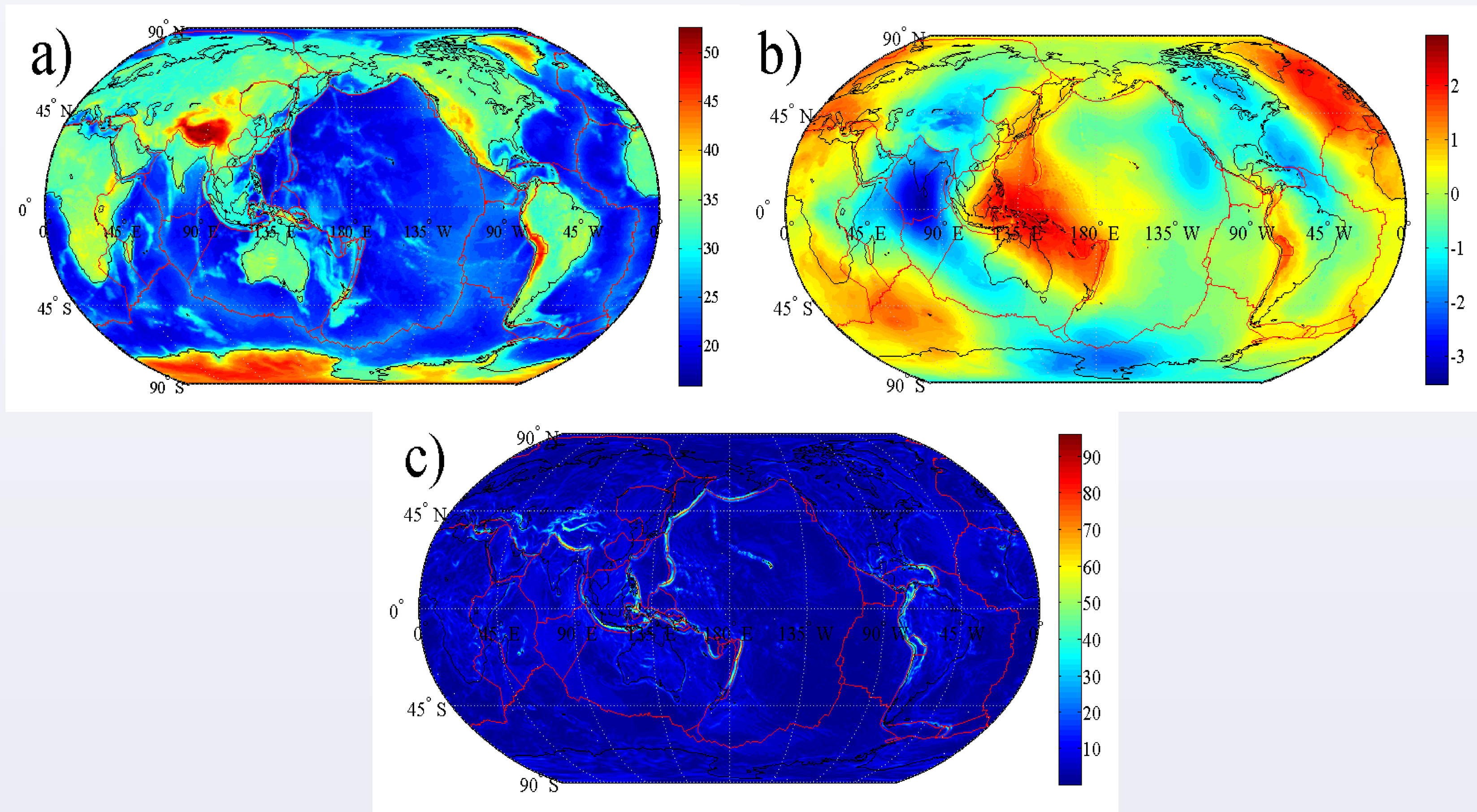

Figure 2. a) Gravimetric Moho model [km], b) *S* function [MPa] and c) sub-crustal stress [MPa]

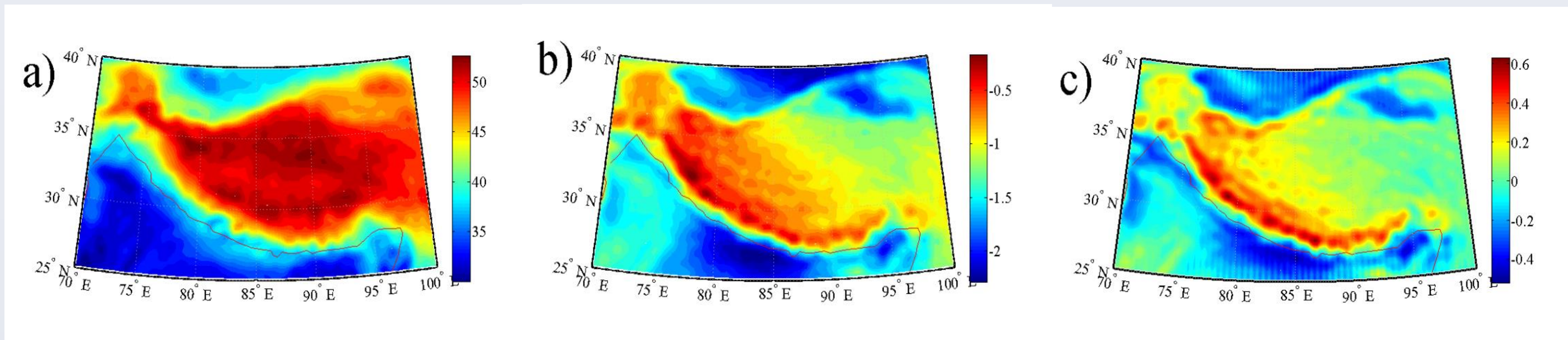

Figure 3. a) Moho model [km], b) *S* function generated by Proposition 1 [MPa] and c) *S* function generated by inverting the integral of Proposition 2 [MPa] over Himalaya

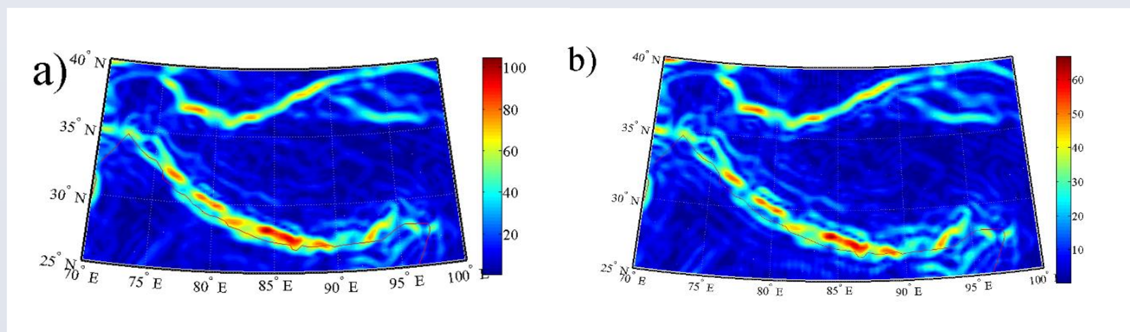

Figure 4. Sub-crustal stress derived by S-function with numerical differentiation method from a) global model, b) local model of Moho [MPa]

Figure 2a is the global map of the Moho surface and Figure 2b is the map of *S* function generated using the spherical harmonic coefficients of the global model of Moho. Figure 2c is the stress magnitudes generated numerically from the obtained *S* function. Figure 3a is the map of Moho in Himalaya and Figures 3b and 3c are the *S* functions generated based on spherical harmonics and the inversion of the integral equations presented in Proposition 2. Figures 4a and 4b are the maps of the stress magnitudes derived from spehrical harmonic series and integral inversion, respectively. They are very similar but the map presented in Figure 4a is slightly smoother than that in Figure 4b. Also, the magnitude of the stress is smaller when it is estimated locally, because in local inversion the long wavelength portion of the Moho signal becomes weaker than the global solution.

Conclusions

The spherical harmonic expansion of the *S* function (*S*) is asymptotically convergent to higher degrees than those of the original formulae of Runcorn because it does not contain the derivatives of the spherical harmonic coefficients. By organising the integral equation of Proposition 2, we solved the inverse problem of Runcorn locally, but for recovering *S* and not the stress components. Our numerical presentation shows that the global map of *S* is very similar to that of the global geoid model. The stress derived after taking numerical derivatives of *S* can very well present the mantle pressure beneath the tectonic boundaries. In addition, some stresses are seen in the places at which the curvature of the Moho surface changes. The maps of the sub-crustal stresses derived from the global and local models of Moho are similar and have similar interpretations, but the magnitude of that, computed from the local model, is smaller due to the weak contribution of Moho signal in the local inversion process.

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