

## Real Time Modelling of the Troposphere for Network RTK

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### ABSTRACT

GPS satellite signals are being refracted in the troposphere during transmission from GPS satellites to GPS receivers. When performing GPS positioning this refraction must be estimated in order to obtain positions with accuracies better than a few meters. In this paper the tropospheric influence on the satellite signals is initially described, and the network RTK positioning technique is briefly introduced. This is followed by a review of the various approaches that are presently being used for treating the tropospheric delay in GPS positioning. New approaches that will be suitable in connection with a network RTK are also described, and finally the use of numerical weather models in GPS positioning is discussed.

### THE NEUTRAL ATMOSPHERE

GPS satellite signals travel through various layers of the atmosphere during transmission from the satellites to the receivers on or near the surface of the Earth. As the signals travel through the neutral atmosphere, they are refracted by the medium. The signal refraction causes both a delay and a bending of the signal path and both effects cause the receiver-satellite distance determined from the satellite signals to be longer than the geometrical path the signal would have followed if it travelled through vacuum. In GPS terminology this combined effect is referred to as the tropospheric delay. In meteorological literature the effect is often referred to as the excess path length.

The neutral atmosphere is a part of the Earth atmosphere that is electrically neutral for propagation of electromagnetic waves with frequencies below 30 GHz, as for instance GPS signals. The neutral atmosphere includes the layers called the troposphere and the stratosphere, and it is the lowest part of the Earth atmosphere extending from the surface of the Earth up to approximately 50 km above.

The largest part of the delay of a satellite signal is experienced in the troposphere, and that is why the combined effect from the troposphere and the

stratosphere is normally referred to as the tropospheric delay or (more erroneous) the tropospheric error.

The delay experienced by a satellite signal received in the zenith,  $d_z$  is determined by integrating the refractivity along the signal path. For modelling purposes the refractivity,  $N$  is normally split into a hydrostatic part and a wet (or non-hydrostatic) part, denoted by the subscripts  $h$  and  $w$  in the formula below.

$$d_z = 10^{-6} \int_s^t N_h dz + 10^{-6} \int_s^t N_w dz$$

The integration is carried out in the zenith direction, which is indicated by  $dz$  in the formula, and the limits are given by the surface of the earth and the top of the neutral atmosphere.

The refractivity can be determined using the formula below:

$$N = k_1 \rho \frac{R}{M_d} + \left( k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) Z_w^{-1}$$

Where:

$k_1$ ,  $k_2$ ,  $k_3$  are constants,  $\Delta$  is the mass density of atmosphere (both dry and wet constituents),  $R$  is the gas constant,  $M_d$  is the molecular weight of dry air,  $T$  is temperature (in Kelvin),  $e$  is the partial pressure of water vapour (in hectopascal), and  $Z_w$  is the compressibility constant for water vapour.  $Z_w$  is accounting for the deviation of water vapour from an ideal gas, and it is a function of temperature and water vapour pressure.

The first term in the formula is thus the hydrostatic delay, and the term in the brackets is the wet delay. The wet delay is only present in the lowest part of the neutral atmosphere i.e. up to about 5 km above the Earth surface.

The effect of bending of the signal path is relatively small compared to the delay of the signals (about zero in the zenith direction and 1 cm at 15° elevation), and it is easily incorporated into the models of the tropospheric delay.

## MODELLING OF THE TROPOSPHERIC DELAY

Generally the tropospheric delay experienced by GPS signals is causing an error in the receiver-satellite range of 2 - 3 metres when measured in the zenith. The size of the error is increasing when the signals are received at lower elevation angles. A number of mapping functions have been developed, and they can be used for mapping the tropospheric zenith delay down to lower elevation angles. For a review refer to Mendes (1999).

The hydrostatic part of the delay is approximately 90 % of the total tropospheric delay. The hydrostatic delay is modelled quite well, and the models can be used for estimating the hydrostatic delay with accuracies down to a few mm in zenith (Mendes, 1999).

The wet part of the delay is approximately 10 % of the total delay. The size of the wet delay is only a few mm in dry arctic areas or in deserts, but it can be up to 35 cm in tropical regions (Bevis et al., 1992). The behaviour and distribution of water vapour in the atmosphere is difficult to predict, and therefore it is also difficult to model the wet part of the delay. Present models can estimate the wet delay within a few cm in zenith (Mendes, 1999).

Important in relation to most tropospheric delay models is that a horizontal spherical symmetry is anticipated. However the atmosphere and especially the wet part is not isotropic. Lately a number of papers have been published on this subject, where various methods for treating the horizontal gradients are described. One example is Emarson and Jarlemark (1999). They are obtaining considerable improvements in the wet delay estimation when incorporating horizontal gradients into the modelling.

## HIGH ACCURACY GPS POSITIONING

When using GPS for high accuracy differential positioning, at least two GPS receivers must be used. One receiver is located at a reference point with known coordinates (the reference) and the other receiver (the rover) is located at a point with unknown coordinates. The position of the rover is then determined relative to the reference station and it is based on both code and phase observations collected by the two receivers. This can be carried out in real time utilising a data link, or in a post processing mode whereby the data is processed on a computer after the data collection has been carried out.

Generally GPS positioning using phase data is based on the phase observation equation as described, for instance, in Kleusberg and Teunissen (1996) or in Blewitt (1997). With observations from more receivers and satellites at the same time epoch, the observation equation can be generated for each receiver-satellite combination, and the

equations can be twice differenced (double differenced). The result is the double differenced phase observation given below, where  $\Delta\nabla$  indicates the double difference operator.

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \lambda\Delta\nabla N + \Delta\nabla\epsilon$$

The satellite and receiver clock errors cancel out, and the size of the error in the satellite position,  $d\rho$ , the tropospheric delay,  $d_{trop}$ , and the ionospheric delay,  $d_{ion}$ , are reduced. The remaining part of these errors is correlated with the distance between the two GPS receivers used for the double difference. The size of the residual error is growing when the distance between the receivers is increasing.

The double differenced ambiguity,  $\Delta\nabla N$ , is by nature an integer number and to obtain position accuracies of a few cm or mm the correct integer numbers for the double difference ambiguities must be determined. Different ambiguity resolution techniques do exist and generally the processes are improved if the size of the spatially correlated errors is decreased.

## NETWORK RTK

Real Time Kinematic positioning (RTK) is a real time differential technique providing positions with cm accuracy in real time. The technique requires a data link between the reference and the roving receiver and the positioning process is often based on double differencing. The ambiguities for the baseline are solved "on the fly" or during a short static observation span where phase data from the reference station is transmitted to the rover.

The technique generally works well with shorter distances (10 – 20 km) between the reference and the rover, when it can be anticipated that the remaining part of the orbit and atmospheric errors are negligible after double differencing. As the distance increases, the residual errors increase as well causing problems with the ambiguity resolution and thereby the accuracy of the position is degraded. Using a network of reference stations can however ease this problem.

For network RTK, observations from multiple reference stations are gathered and processed in a common adjustment process where corrections for code and phase observations logged at the reference stations and in the rover are determined.

The purpose of the corrections is to reduce the influence of the spatially correlated errors. This means that when the corrections are applied to the rover code and phase observations, the position is determined using the

corrected observations. The influence from the satellite position error and the atmospheric errors will be smaller, which results in an improved positioning performance. See for instance Raquet (1998) or Wübbena et al. (1996).

### **SPATIALLY CORRELATED ERRORS AND NETWORK RTK**

When using dual frequency GPS receivers in connection with network RTK, the ionospheric error can be handled by introducing the ionosphere free linear combination of the phase observations into the processing. Hereby the first order ionospheric effects are removed. The higher order effects of the ionosphere will still be present. For baselines of a 50 – 100 km their influence is relatively small, but for longer baselines the higher order effects should be considered (Brunner and Gu, 1991).

The orbit error can be reduced by using orbit parameters with a better quality than the broadcast orbits. The best orbit product presently available is the Ultra Rapid Precise Orbits from the IGS. These orbits have been available since March 2000. The orbits are determined twice a day (03 and 15 UTC) based on data collected up to 3 hours before the processing is done. The orbit files cover 48 hours where the first 24 hours are based on actual GPS observations and the next 24 hours are the predicted (extrapolated) orbits (Springer, 2000). Daily estimates of the accuracy can be found on the web site of the University of Bern.

The largest remaining error in the positioning process is now the tropospheric delay. Good modelling of the tropospheric delay is important in order to reduce the residual part of the tropospheric delay, which is the part of the delay that the RTK corrections must be accounting for.

### **TROPOSPHERIC DELAY IN POST PROCESSING**

When performing traditional GPS positioning based on code and/or phase observations the raw observations are normally corrected for the tropospheric delay by means of a global tropospheric delay model. The models are often based on global climate models, and the accuracy of the tropospheric models is limited by the accuracy of the climate models.

After the GPS observations have been corrected by a tropospheric delay model the remaining tropospheric delay is of the size of a few cm in the zenith and larger for lower elevation angles. The remaining delay is mainly caused by difficulties with modelling of the wet part of the delay.

If performing high accuracy GPS positioning the GPS observations can now be double differenced. The size of the residual tropospheric delay will be a few mm for a medium GPS baseline (50 – 100 km). For shorter baselines the tropospheric delay will be practically eliminated by double differencing.

For longer baselines the residual tropospheric delay will have a considerable size, and it has to be considered in the further data processing e.g. by estimating the delay as an element in the adjustment process.

A rule of thumb for relative GPS positioning says that the troposphere is causing an error in the height difference between the two GPS stations that is 3 times the size of the relative residual tropospheric delay at 15°. This means that a relative residual tropospheric error of 1 cm, can cause an error of 3 cm in the height of the rover (Brunner and Welsh, 1993).

This “rule” indicates how important handling of the troposphere is for high accuracy GPS positioning.

### **TROPOSPHERIC DELAY IN REAL TIME POSITIONING**

When performing real time positioning the computation process in the rover must be fairly simple due to the time constraints, and estimation of any residual tropospheric error can not be carried out using the same adjustment procedures as for post processing. Other procedures like improved modelling must therefore be considered.

For real time differential GPS positioning (DGPS), where positions with accuracies within 1 – 5 meter are obtained, global models for the tropospheric delay are used.

For wide area DGPS systems (WADGPS), where an improved accuracy (down to 50 cm) and reliability is aimed at, a slightly different approach has been selected. For instance for the European EGNOS<sup>1</sup> project where a global tropospheric delay model is combined with meteorological data for the site of the roving GPS receiver. The meteorological data is provided from a regional climate model, and the parameters are determined based on an interpolation within this model.

Dodson et al. (1999) tested this approach for two different sites in England. When analysing the difference between tropospheric zenith delays determined based on GPS observations and based on the EGNOS approach described above, they concluded that the latter can be used to estimate the tropospheric zenith delay within 2 cm. However for some rapid weather changes the model

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<sup>1</sup> European Geostationary Navigation Overlay Service

approach did not perform as well and errors up to 15 cm were found for the zenith tropospheric delay. This approach is therefore not ideal for network RTK and another solution for obtaining real time estimates of the tropospheric delay must be found.

A few groups are achieving promising results with different approaches for estimating the tropospheric delay in real time based on the use of GPS data.

## REAL TIME ESTIMATION OF TROPOSPHERIC DELAY

At Onsala Space Observatory real time tropospheric delays are estimated based on real time processing of GPS data. The processing is presently running in a test mode. GPS data is retrieved from 8 SWEPOS stations distributed over an area of approximately 400 x 400 km in Southern Sweden. Known station coordinates are corrected for solid Earth tide and used in the processing. The tropospheric zenith delay is then estimated using a Kalman filter and by assuming a random walk process for each station. The delay is estimated every 15 seconds with a delay of 3 – 4 seconds.

26 hours of zenith delays have been compared to a final solution (i.e. data processed with a 30 day delay using the GIPSY software and precise orbits from JPL). The estimated real time tropospheric zenith delay lies within 3 cm of the final solution, which is less than the residual estimates from the Kalman filter. See Stoew et al. (2000).

With the purpose of improving weather forecasts and to validate forecasts based on traditional meteorological data the MAGIC<sup>2</sup> project provides near real time (3 hour delay) estimates of the tropospheric zenith delay based on GPS data.

The processing is carried out by CNRS<sup>3</sup> in France and it is described in Ge et al. (2000). The processing is based on data from 15 EUREF stations, and the data is processed using the GAMIT software and the IGS ultra precise rapid orbits.

One of the plans for the MAGIC project is that the other partners in the project shall participate in this near real time estimation of tropospheric delays. Other groups are

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<sup>2</sup> MAGIC – *Meteorological Applications of GPS Integrated Column Water Vapour Measurements in the Western Mediterranean*. A cooperation between meteorological organisations in Spain, Italy, France and Denmark.

<sup>3</sup> CNRS - *Centre National de la Recherche Scientifique*, Université de Montpellier, France.

using the Bernese and the Gipsy software packages and a comparison between the three different processing scenarios will thus be an output of the project. More information as well as the near real time products is available on the MAGIC web site.

NOAA Forecast Systems Laboratory is estimating near real time (about 55 minutes delay) water vapour based on 59 GPS stations in the USA. They obtain a difference of 1.2 mm in integrated precipitable water vapour when comparing hourly results with final daily solutions. The integrated precipitable water vapour values can be found on NOAA's web site along with other more or less real time meteorological observations from the stations (<http://www-dd.fsl.noaa.gov/gps.html>).

## TROPOSPHERIC DELAY FROM NUMERICAL WEATHER PREDICTION MODELS

The tropospheric delay can also be estimated or predicted using numerical weather prediction (NWP) models. NWP's form the basis for weather forecasts and they are based on numerous meteorological observations collected world wide. The observations are mainly collected by ground based equipment, and a few radiosondes (weather balloons), transatlantic airplanes, and microwave radiometers are providing data about conditions higher up in the atmosphere.

If more information about the distribution of water vapour in the atmosphere was available weather forecasts would be improved considerably. Therefore an extensive effort is carried out within the meteorological community in order to develop procedures for using GPS derived information in weather predictions. The research activities within this field have led to a number of tests where GPS derived tropospheric zenith delays have been compared to delays determined from integration in numerical weather prediction models. These tests have been carried out by meteorologists in order to investigate how good GPS is. However in this paper the tests are introduced to describe how good the NWP's are.

The HIRLAM model from the Danish Meteorological Institute (DMI) is an example of an NWP. It is initiated by the states of the last prediction, by boundary conditions given by the ECMWF-model<sup>4</sup>, and by observations of current meteorological conditions collected world wide (DMI's web site).

Haase et al. (2000) describe a comparison between tropospheric zenith delays determined with GPS and by 6 hour predictions using the HIRLAM model. Since GPS

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<sup>4</sup> ECMWF is the *European Centre for Medium-Range Weather Forecasts*

data is presently not used in the HIRLAM model this comparison is between two independent data sets.

Based on the GPS data the tropospheric zenith delay was determined using an automated post processing set up described by Ge et al. (2000). The HIRLAM model was run with a resolution of  $0.3^\circ$  in the horizontal, and 31 vertical levels. The model was generated for an area covering the western part of the Mediterranean.

Based on a numerical integration in the HIRLAM model the tropospheric zenith delay, zenith wet delay, and the integrated water vapour were determined for the GPS sites.

When determining the difference between the tropospheric zenith delay determined with GPS and the HIRLAM model, the bias of the results from the 41 stations was generally within  $-10$  to  $+15$  mm and the standard deviation was less than 25 mm in most of the stations. The comparison was carried out on  $1\frac{1}{2}$  years of the data. There is a clear seasonal trend in the differences, showing a larger scatter during the summer months (where the humidity is higher).

Schueler et al. (2000) maintain a database of daily tropospheric zenith delays based on GPS data from 100 IGS stations. In the database the total delay values are split into the hydrostatic and wet part. For an 8 month period the GPS derived wet zenith delays have been compared to delays determined by integration in the NOAA/NCEP global weather model (a medium resolution model with a  $1^\circ \times 1^\circ$  horizontal resolution and 26 vertical layers). The tests show an RMS of 14 mm on the differences.

There is a correlation between latitude and RMS of the wet delays. Lower latitude (and higher humidity) gives a higher RMS.

#### **NWP IN RELATION TO NETWORK RTK**

The tests described above show that it is possible to use NWP's to determine total or wet tropospheric zenith delays with an accuracy that is close to delays determined based on GPS. Also the NWP derived zenith delays are better than if global tropospheric delay models were used. The best of these models are giving accuracies in the tropospheric zenith delay estimates of about 3 cm (Schueler et al. 2000).

However most of the tests carried out so far are done in post mission. In order to effectively use information about the tropospheric zenith delay for real time positioning the delay must be predicted so it is available at the epoch of observation.

This can be done using an NWP in a prediction mode for predicting the tropospheric delay for a given GPS station, and considering the tests described above the quality of such an NWP estimate will be better than if a global tropospheric delay model is used.

An even better estimation of the total tropospheric delay for network RTK might be obtained if an NWP model is used in combination with GPS estimates for the tropospheric delay as determined for instance by the Onsala or MAGIC real time approaches. New procedures thus have to be developed in order to successfully combine these two sources of information about the troposphere in order to improve the present procedures for network RTK.

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World Wide Web sites:

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Danish Meteorological Institute:  
<http://www.dmi.dk/>

Forecast Systems Laboratory, National Oceanic and Atmospheric Administration (NOAA):  
<http://www.fsl.noaa.gov/>

MAGIC:  
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