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Determination of Earth's Crustal Thickness by Vening Meinesz-Moritz Hypothesis and Its Geodetic Applications

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Introduction

The crustal thickness or *Mohorovičić discontinuity,* usually called the *Moho*, is the boundary between the Earth's crust and mantle. Several isostatic hypotheses exist for estimating the crustal thickness/Moho and density of the Earth's crust, and it is not clearly found which one is the most suitable to use in geophysical and geodynamical applications.
The main objective of this study is to develop an isostatic Moho model for using in many essential aspects of geodetic, geophysical and geodynamical applications. Moritz (1990) presented the inverse isostatic problem based on the Vening Meinesz hypothesis in a global spherical approximation, and we hereafter call this method Vening Meinesz-Moritz' (VMM) problem/method. Recently Sjöberg (2009) formulated this problem as that of solving a non-linear Fredholm integral equation of the first kind.

Objectives

- Crustal thickness by Vening Meinesz-Moritz hypothesis.
- Modelling Moho density contrast based on the VMM model.
- Local gravity field recovery from satellite gravity gradiometry data by the topographic-isostatic models.
- Recovery of Moho from the satellite gravity gradiometry data.
- Synthetic Earth Gravity Model (SEGM) by an isostatic hypothesis.

Input data

- The global geopotential model EGM08 to degree and order 2160 (Pavlis et al. 2008).
- The global topographic heights from DTM2006 (Pavlis et al. 2006) elevation model to degree and order 2160.
- The 2 2 discrete data global crustal model CRUST2.0 (Bassin et al. 2000).

Methodology for Vening Meinesz-Moritz Inverse Problem in isostasty

Inverse problems in isostasy will consist in making the isostatic anomalies to be zero under a certain isostatic hypothesis. On the other hand, the VMM problem is to determine the Moho depth *T*(*P*) such that the compensating attraction A_C totally compensates the Bouguer gravity anomaly Δg_B on Earth's surface, implying that the isostatic anomaly Δg_I vanishes for point *P* on the Earth's surface. The main idea is easy but the theoretical analysis is somewhat difficult.

a) Moho estimation

$$\Delta g_I = \Delta g_B + A_C(T) = 0 \qquad s = 1 - T/R$$

$$R\iint_\sigma K \;\; r_P,\psi,s \; d\sigma = -\;\; \Delta g_B(P) + A_{C0}(P) \;/\; G\Delta\rho$$

$$T_P = (T_1)_P + \frac{(T_1)_P^2}{R} - \frac{1}{32\pi R}\iint_\sigma \left(\frac{(T_1{}^2)_Q - (T_1{}^2)_P}{\sin^3 \;\; \psi/2}\right) d\sigma$$

$$T \approx T_1 = F(P) = \frac{1}{4\pi}[2f(P) - \sum_{n=0}^{\infty}\frac{1}{n+1}\sum_{m=-n}^{n} f_{nm}Y_{nm}(\theta,\lambda)]$$

$$= \frac{1}{4\pi}[\sum_{n=0}^{\infty}\left(2 - \frac{1}{n+1}\right)\sum_{m=-n}^{n} f_{nm}Y_{nm}(\theta,\lambda)].$$

$$f_{nm} = \begin{cases} [2\pi \;\; \mu H \;\; {}_{00} - A_{C0}]/(4\pi k) & ,if \qquad n=0 \\ \left[2\pi \;\; \mu H \;\; {}_{nm} - G_{nm}\right]/(4\pi k) & otherwise \end{cases}$$

b) Moho density contrast

$$\Delta\rho \;\; P \;\; \approx \frac{b \;\; P}{2\pi T \;\; P} - \frac{1}{(4\pi)^2 T \;\; P}\iint_\sigma H(\psi) b \;\; Q \;\; d\sigma + \frac{T\,\Delta\rho \;\; {}_P}{R}$$

$$-\frac{1}{32\pi R T \;\; P}\iint_\sigma \frac{\Delta\rho T^{\,2} \;\; {}_Q - \;\; \Delta\rho T^{\,2} \;\; {}_P}{\sin^3 \;\; \psi/2} d\sigma,$$

b-1) Moho density contrast by inversion method

$$b^j = R\sum_{i=1}^{n} K^{ij} \;\; \psi, s \;\; \Delta\rho_i \Delta\sigma_i \qquad \mathbf{b} = \mathbf{K}.\Delta\boldsymbol{\rho}$$

$$\|\mathbf{Ax} - \mathbf{L}\|^2 + \alpha\|\mathbf{x}\|^2 \rightarrow \min \qquad \mathbf{x}_{\mathrm{reg}} = \;\; \mathbf{A}^T\mathbf{PA} + \alpha\mathbf{I} \;\; ^{-1}\mathbf{A}^T\mathbf{PL} = \mathbf{N}^{-1}\mathbf{A}^T\mathbf{PL}$$

c) Recovery of the gravity anomaly from SGG data by VMM model

$$T_i \;\; P \;\; - V_i^{ti} \;\; P \;\; = \frac{R}{4\pi}\iint_\sigma S_i \;\; r,\psi \;\; \Delta g^* \;\; Q \;\; d\sigma \qquad i = zz, xx, yy, xy, xz \text{ and } yz$$

d) Recover of Moho from GOCE mission

$$T_j^* \;\; P \;\; - V_j^t \;\; P \;\; + \left[V_{C_0} \;\; P \;\;\right]_j + \left[dV_C \;\; P \;\;\right]_j \;\; \cong 0$$

$$\iint_\sigma \frac{\partial \bar{K}_{rr} \;\; r_P, \psi, s}{\partial s}\Delta s \;\; Q \;\; d\sigma = \bar{f} \;\; P$$

$$\bar{f}(P) = - \;\; T_{rr}^* \;\; P \;\; - V_{rr}^t \;\; P \;\; + \left[V_{C_0} \;\; P \;\;\right]_{rr}\Big|_{s=s_1} + \left[dV_C \;\; P \;\;\right]_{rr}\Big|_{s=s_1} \Big/ \; G\Delta\rho$$

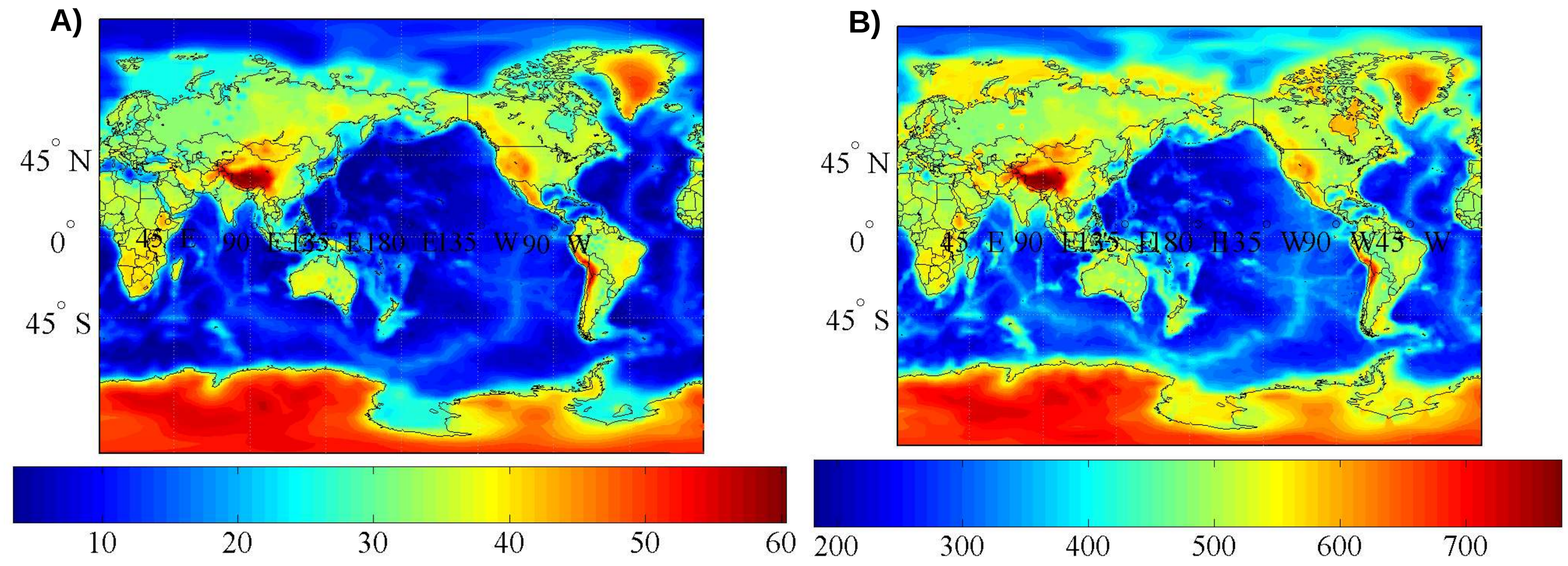

Fig.1: A) Moho depth, [Unit: km] and **B)** Moho density contrast by the VMM model [Unit: kg/m³] , with resolution 2 2 .

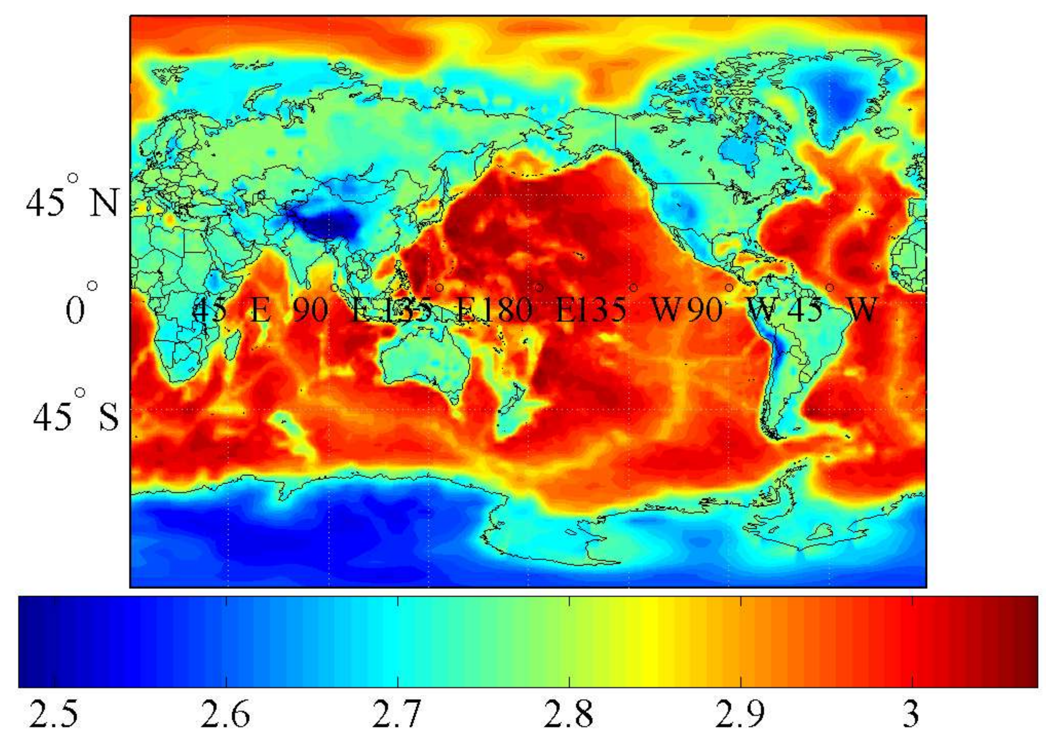

Fig.2: Crsutal densityby the VMM model with resolution 2 2 .
[Unit: g/cm³]

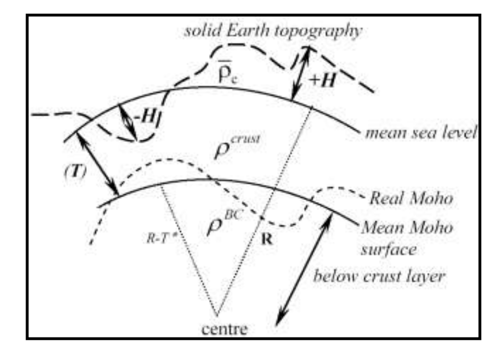

Fig.3: General schematic structure of Earth' topography and crust.

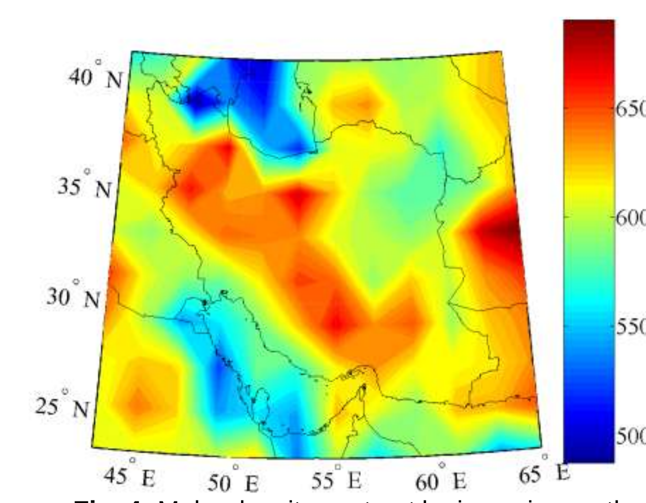

Fig. 4: Moho density contrast by inversion method with resolution 2 2 in Persia .
[Unit: kg/m³]

Table 1. Statistics of error of recovered gravity anomalies based on RCR and DC techniques. [Unit: mGal]

Noise		10 mE					1 mE				
	Method	Max	Mean	Min	Std	rms	Max	Mean	Min	Std	rms
T_{xx}	RCR-AH	44.9	-7.0	-62.5	17.7	12.2	32.0	-6.5	-46.7	12.7	9.1
	RCR-VMM	19.3	-2.2	-31.9	7.8	5.2	17.3	-2.0	-30.1	7.3	4.9
	DC	62.3	1.3	48.8	18.3	11.8	57.2	0.7	-71.8	18.7	12.0
T_{yy}	RCR-AH	39.2	2.1	-49.0	14.8	9.6	59.6	1.8	-38.5	13.0	8.4
	RCR-VMM	23.3	-3.7	-41.0	11.4	7.7	31.9	-3.8	-64.3	14.8	9.8
	DC	50.7	-5.5	-69.5	19.1	12.8	58.8	-5.6	-59.2	17.4	11.7
T_{zz}	RCR-AH	29.5	-4.1	-33.1	10.4	**7.1**	32.1	-3.9	-32.4	9.4	**6.5**
	RCR-VMM	17.2	1.1	-19.5	6.5	**4.3**	18.1	1.2	-20.2	5.9	**3.9**
	DC	37.9	-0.3	-43.9	13.7	8.8	42.1	-0.3	-.34.4	13.0	8.4
T_{xy}	RCR-AH	268.3	13.5	-206.1	85.1	55.3	148.8	20.1	-111.1	48.6	33.8
	RCR-VMM	154.1	20.6	-114.2	49.0	34.1	153.7	20.61	-115.7	48.9	34.0
	DC	418.1	-10.5	-573.9	117.9	114.3	420.7	-11.1	-567.5	180.5	116.0
T_{xz}	RCR-AH	24.3	-0.2	-35.5	10.5	6.7	25.2	-0.1	-32.6	9.9	6.3
	RCR-VMM	24.7	5.1	-19.9	7.2	5.6	24.0	5.0	-17.5	6.6	5.3
	DC	59.1	7.6	-40.8	17.3	12.1	56.4	7.4	-46.6	15.6	11.1
T_{yz}	RCR-AH	96.6	6.7	-63.7	25.4	16.9	96.9	6.9	-64.3	25.3	16.3
	RCR-VMM	90.1	14.5	-27.0	18.0	14.8	89.0	14.8	-26.1	18.0	14.0
	DC	122.1	-5.0	-109.2	37.9	25.7	124.3	-5.1	-109.0	38.9	25.1

RCR = Remvoe-Compute-Restore Technique ; **VMM** = Vening Meinesz-Moritz Model
AH = Airy-Heiskanen Model ; **DC** = Direct Downward Continuation

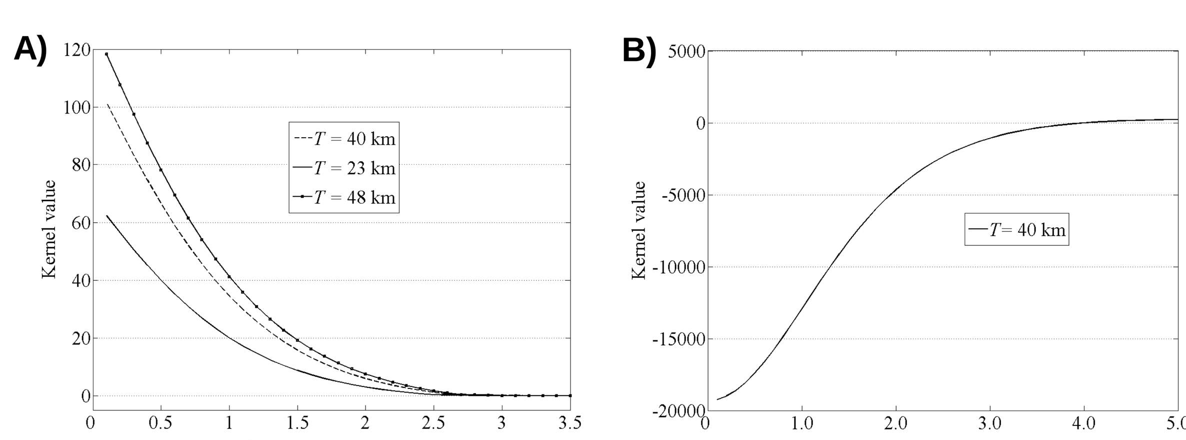

Fig. 5. A) $\bar{K}_{rr}(r_P,\psi,s)$ and **B)** $\partial\bar{K}_{rr}(r_P,\psi,s)/\partial s$ in different Moho depths.

Table 2. Statistics of **A)** Moho depths and **B)** Moho depths differences for Sjöberg's direct solution (T_{sj}), the nonlinear inversion approach ($T_{\Delta g}$), the Moho recovered from SGG data (T_{SGG}) and CRUST2.0 (T_{SM}). [Unit: km]

A)

Method	Max	Mean	Min	Std
T_{SGG}	45.0	36.4	20.9	4.2
$T_{\Delta g}$	45.3	36.7	22.0	4.5
T_{Sj}	44.8	35.3	23.0	5.0
T_{SM}	48.3	39.8	23.9	4.2

B)

	$T_{\mathrm{Sj}}-T_{\mathrm{SGG}}$	$T_{\mathrm{Sj}}-T_{\Delta g}$	$T_{\mathrm{SM}}-T_{\mathrm{Sj}}$	$T_{\mathrm{SM}}-T_{\mathrm{SGG}}$	$T_{\mathrm{SM}}-T_{\Delta g}$
Max	3.7	3.1	10.4	17.3	15.7
Mean	-0.4	-0.7	-0.9	3.4	3.1
Min	-4.3	-6.0	-11.9	-6.4	-6.8
Std	2.0	1.8	3.6	4.0	3.9
rms	2.0	1.9	3.8	5.2	5.0

Conclusions:

The main objective of this study is to investigate an isostatic Moho model in some essential aspects of geodetic, geophysical and geodynamical applications. The study shows that the VMM model is close to the seismic Moho model (CRUST2.0). In addition, It is important to mention that the VMM model is a flexible model and it can be altered for estimating the Moho density contrast, too.
The effects of the topographic masses on the satellite gradiometric data are large and in order to reduce the magnitude of these effects, some compensation mechanisms should be considered. We suggest to use the isostatic hypotheses for compensating the topographic effects on satellite gradiometric data to smooth data prior to their downward continuation to the gravity anomaly. Numerical results show that the topographic-isostatic effect based on the VMM's hypothesis smoothes the data better than that based on Airy-Heiskanen's hypothesis. Also the quality of inversions of the smoothed data by this mechanism is twice better than that of the non-smoothed ones.
In this study, the VMM model has been developed so that the satellite gravity gradiometry (SGG) data are used for recovering the Moho depth through a nonlinear inversion procedure. It is presented that the Moho depth recovered from the SGG data will be more or less the same as the one obtained from the terrestrial gravimetric data with a root mean square error of 2 km.
It is suggested to use sedimentary layers to define realistic density of the Earth's crust. The numerical results showed that the density data have significant effect for estimating the Moho depth. The significant corrections to computation of Moho are expected in forthcoming complementary studies to improve the results, where the isostatic gravity anomaly will be reduced for several disturbing signals. Also an additional compensation masses should be used, as thermal compensation.