On the need of improved gravity data to compute the next generation of quasigeoid models in Sweden

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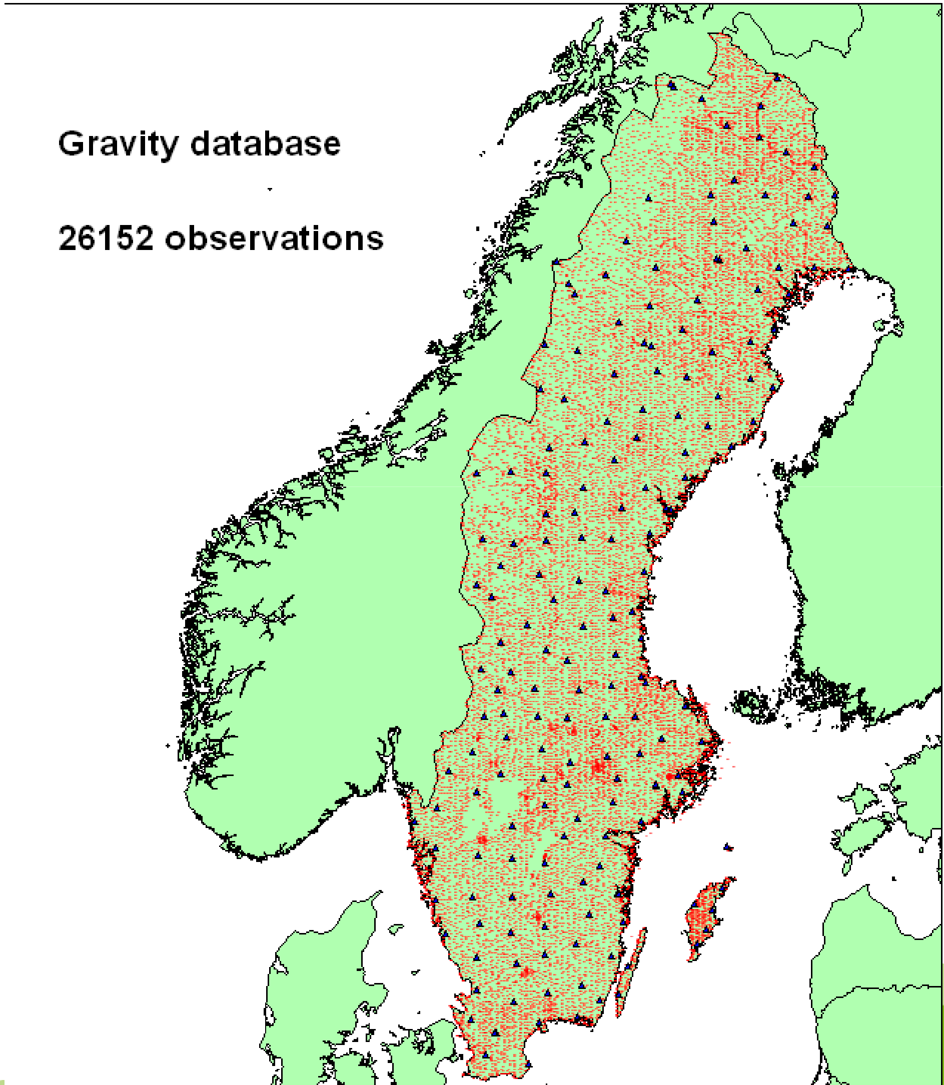
Background

- SWEREF 99 and RH 2000 have now been established on a national level and in many municipalities
- A new quasigeoid model SWEN08_RH2000 has been introduced, based on the gravimetric model KTH08. Standard error 10-15 mm on the main land.
- GNSS height accuracy will increase in the future. It is not unlikely that a ellipsoidal height standard error as low as 5 mm will be obtained fast and easily.
- What is required to obtain a comparable standard error for the geoid model?
- We here use the term “geoid model” for what is strictly a quasigeoid model. We also include different types of corrections, etc.

Purpose

- The main purpose of this talk is to find out in what way the *Swedish gravity data need to be improved* to make the computation of a gravimetric geoid model with 5 mm standard uncertainty *possible* in the future.
- Particularly whether 5 km resolution is sufficient.
- This is investigated by error propagations assuming a GOCE EGM with M = 200 and commission RMS-error of 1 cm.
- Please note that it will not automatically be possible to compute a 5 mm geoid model once the gravity data have been improved. Significant methodological improvements are also needed.
- Gravity improvements are also required on the Nordic level, but here we concentrate on Sweden.

The Swedish gravity database



- Most of the Swedish gravity data has 5 km resolution.
- Two different gravity systems are in use, RG 62 and RG 82. Complex situation.
- The gravity standard errors varies from around 0.1 up to a few mGal. Most below 0.5 mGal.
- The accuracy of the heights varies considerably
- The situation is worse in the big lakes and at Sea. Ship and air data with some gaps and a courser resolution
- The situation is also little worse in the highest mountains to the north west.
- Agreement at the borders?

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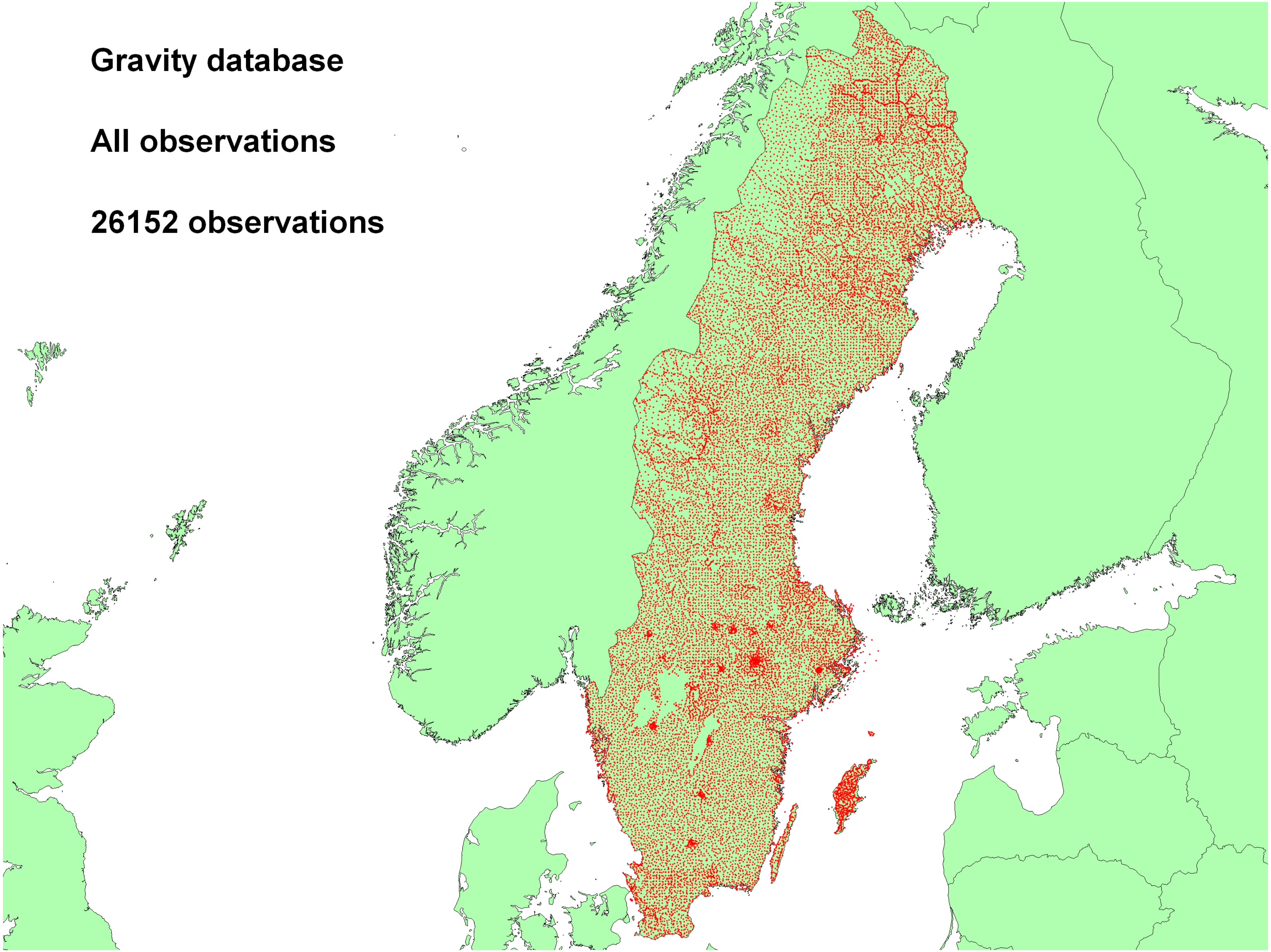
Gravity database

All observations

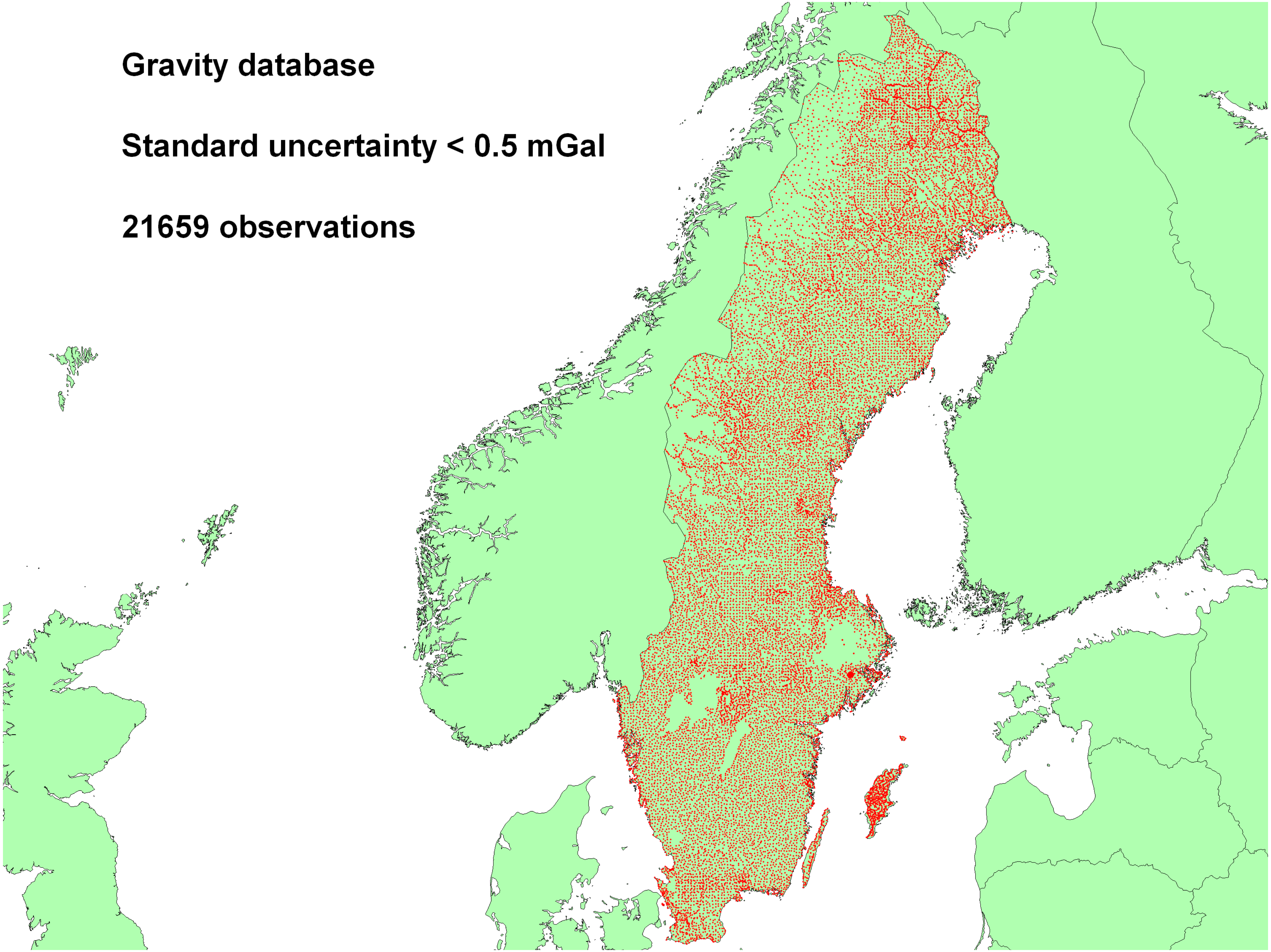
26152 observations



Gravity database

Standard uncertainty < 0.5 mGal

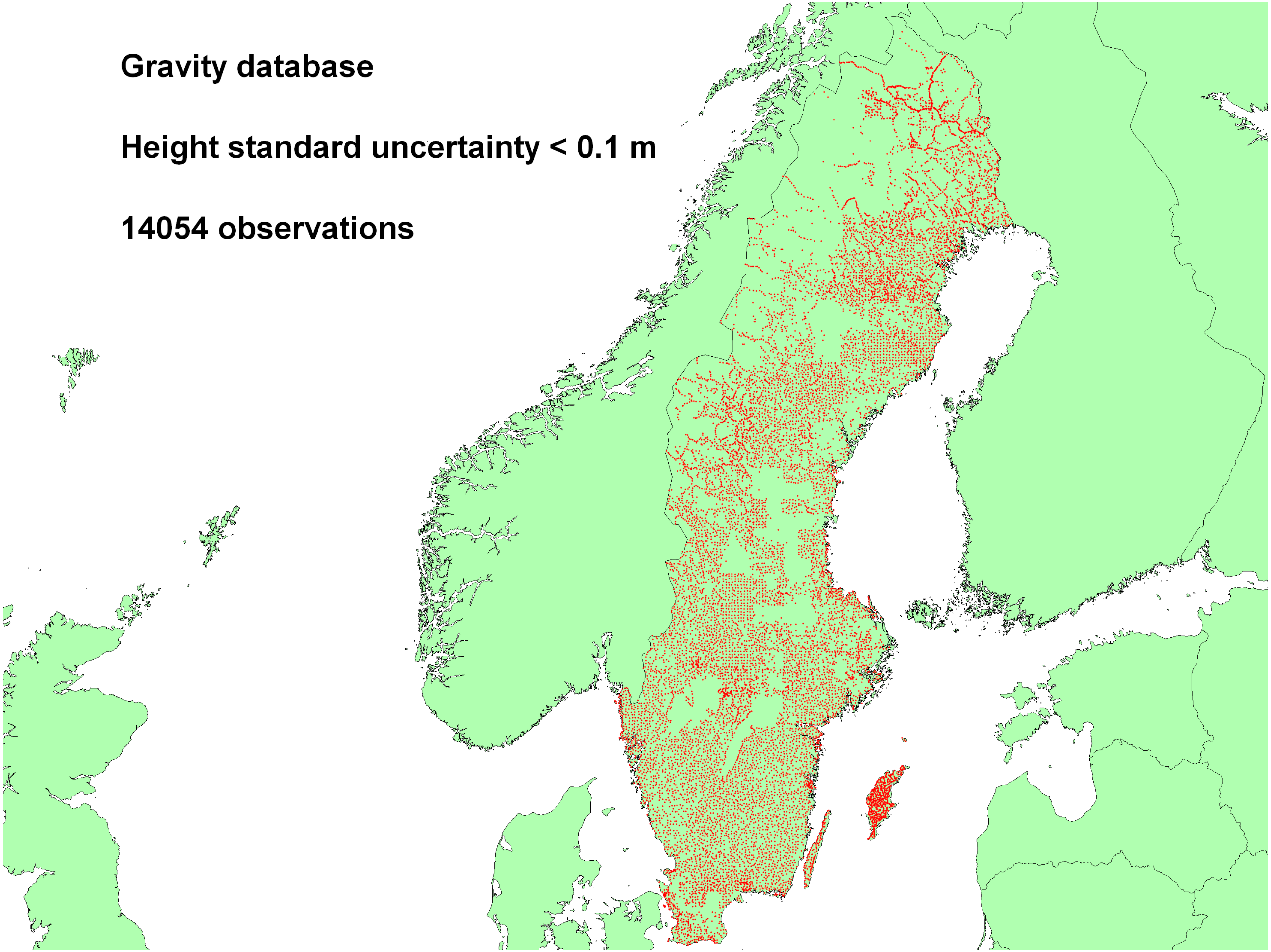
21659 observations



Gravity database

Height standard uncertainty < 0.1 m

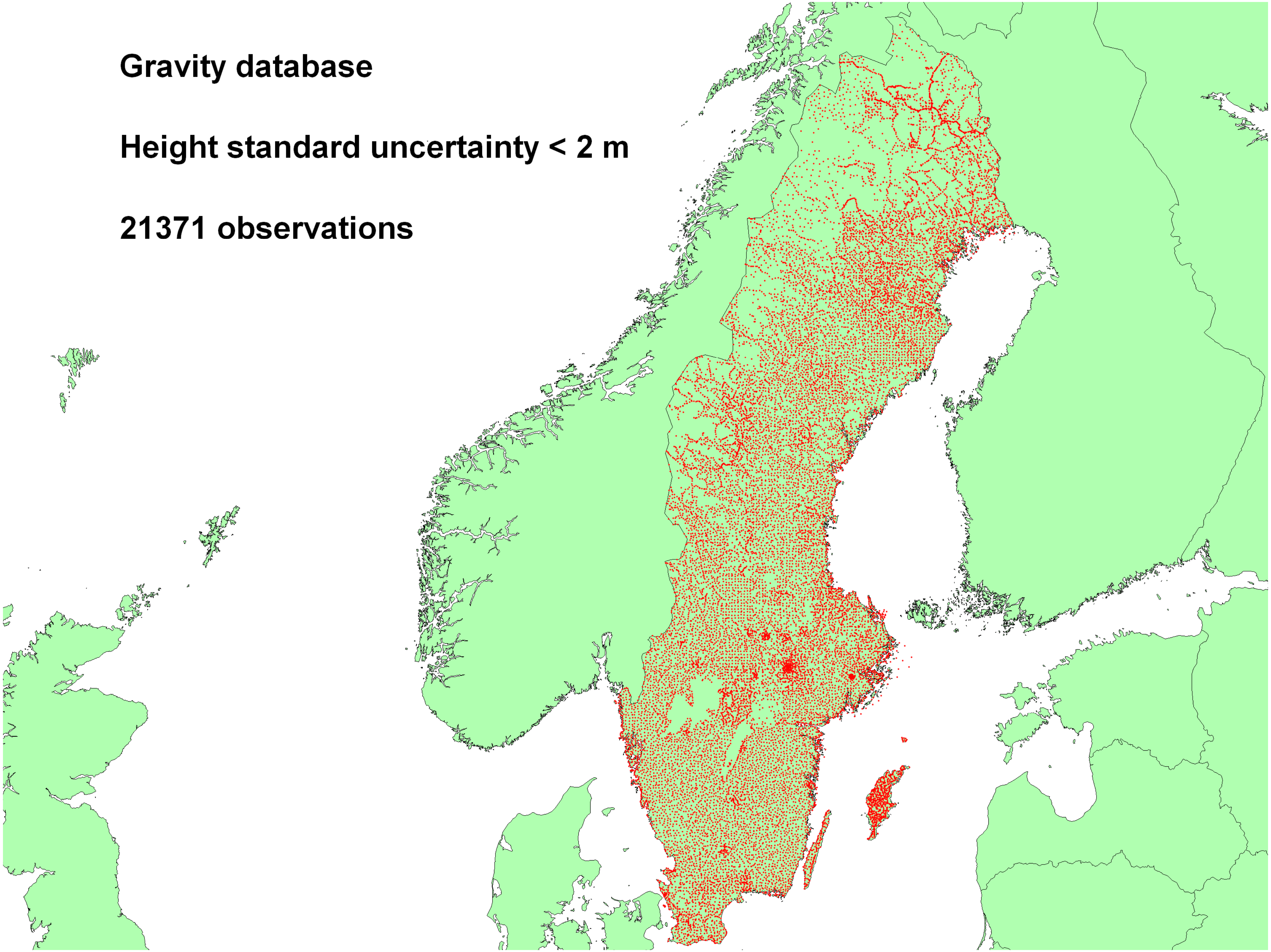
14054 observations



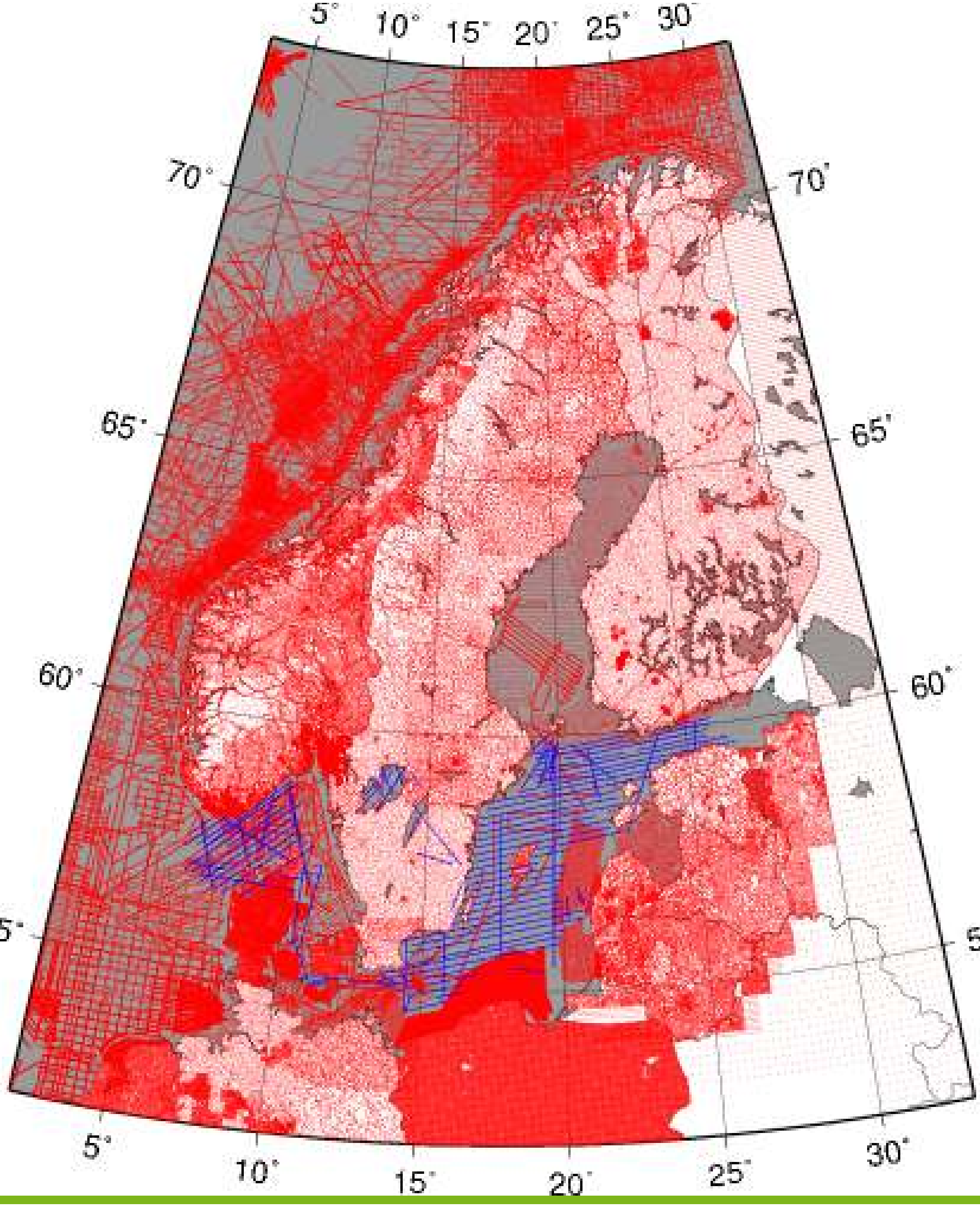
Gravity database

Height standard uncertainty < 2 m

21371 observations



The NKG database (Swedish copy 2010)



Error propagation

Remove-compute-restore estimator:

$$\tilde{N} = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\left(\Delta\tilde{g} - \Delta g^{DIR} - \sum_{n=2}^{M}\left(\frac{R}{r_p}\right)^{n+2}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right)\right)^{*} d\sigma$$

$$+\frac{R}{2\gamma}\sum_{n=2}^{M}\frac{2}{n-1}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right) + \delta N_I$$

In spectral form:

$$\tilde{N} = c\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^* - Q_n^M\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{\Delta g}\right)$$

$$+c\sum_{n=2}^{M}\left(Q_n^M + s_n\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{EGM}\right) + \delta N_I$$

The expected global mean square error:

$$\delta\bar{N}^2 = E\left[\frac{1}{4\pi}\iint_{\sigma}\left(\tilde{N} - N\right)^2 d\sigma\right] = c^2\sum_{n=2}^{M}\left[\left(\frac{2}{n-1} - s_n - Q_n^M\right)^2 \sigma_{n,\Delta g}^2 + \left(s_n + Q_n^M\right)^2 \sigma_{n,EGM}^2\right]$$

$$+c^2\sum_{n=M+1}^{\infty}\left[\left(\frac{2}{n-1} - Q_n^M\right)^2 \sigma_{n,\Delta g}^2 + \left(Q_n^M\right)^2 c_n\right]$$

We assume that

- a GOCE EGM with M = 200 and 1 cm commission RMS-error is used; see Ågren (2004) p. 31.
- either the high or the low reduced signal degree variance models are correct.
- the resolution 5 km is assumed for the gravity anomalies.
- the error degree variances for the gravity anomalies are
  - bandlimited white noise and/or
  - following the reciprocal distance model (Moritz 1980) with the correlation length 0.25 degrees.
  - Above the Nyquist degree, the signal degree variances are utilised.
  - Remember that the error covariance function is assumed to be **homogeneous** and **isotropic**.

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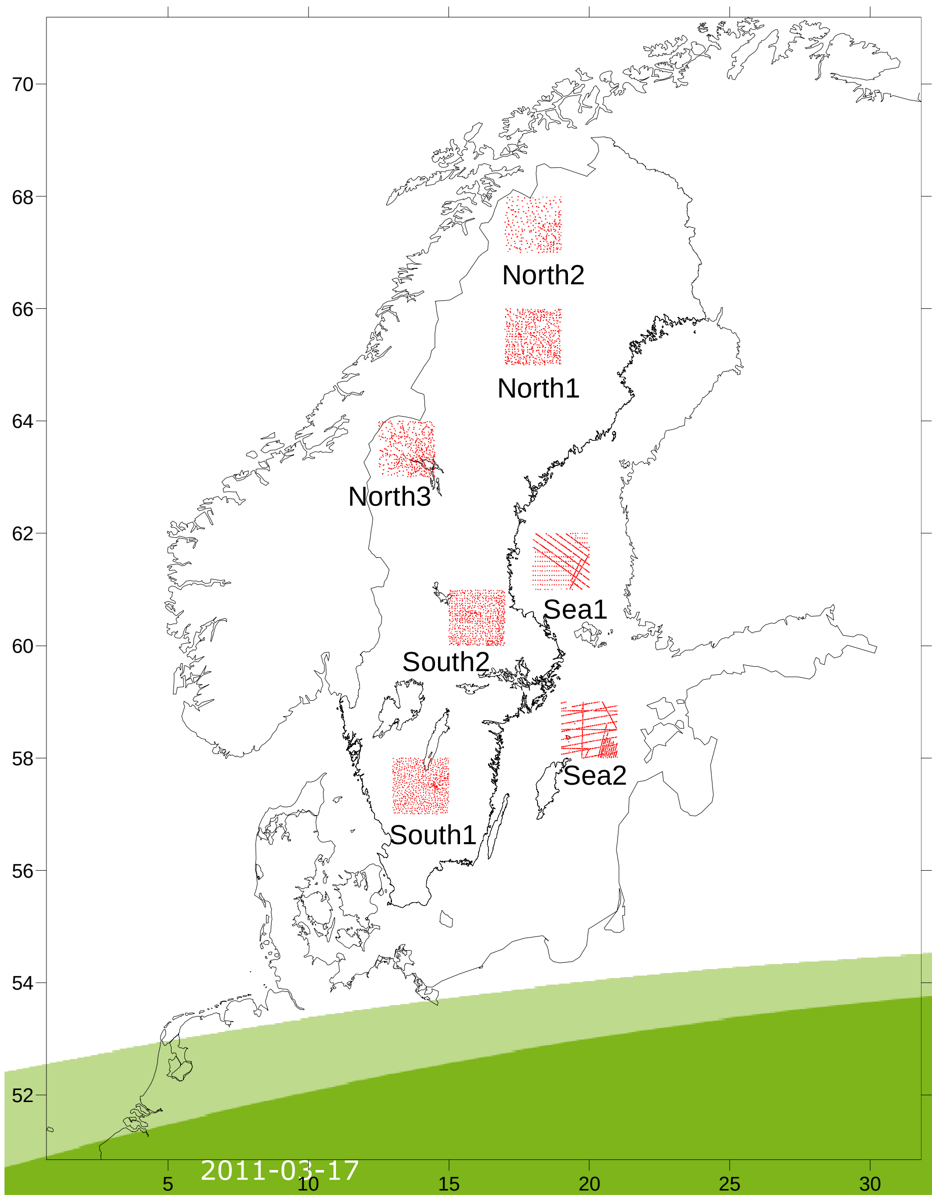
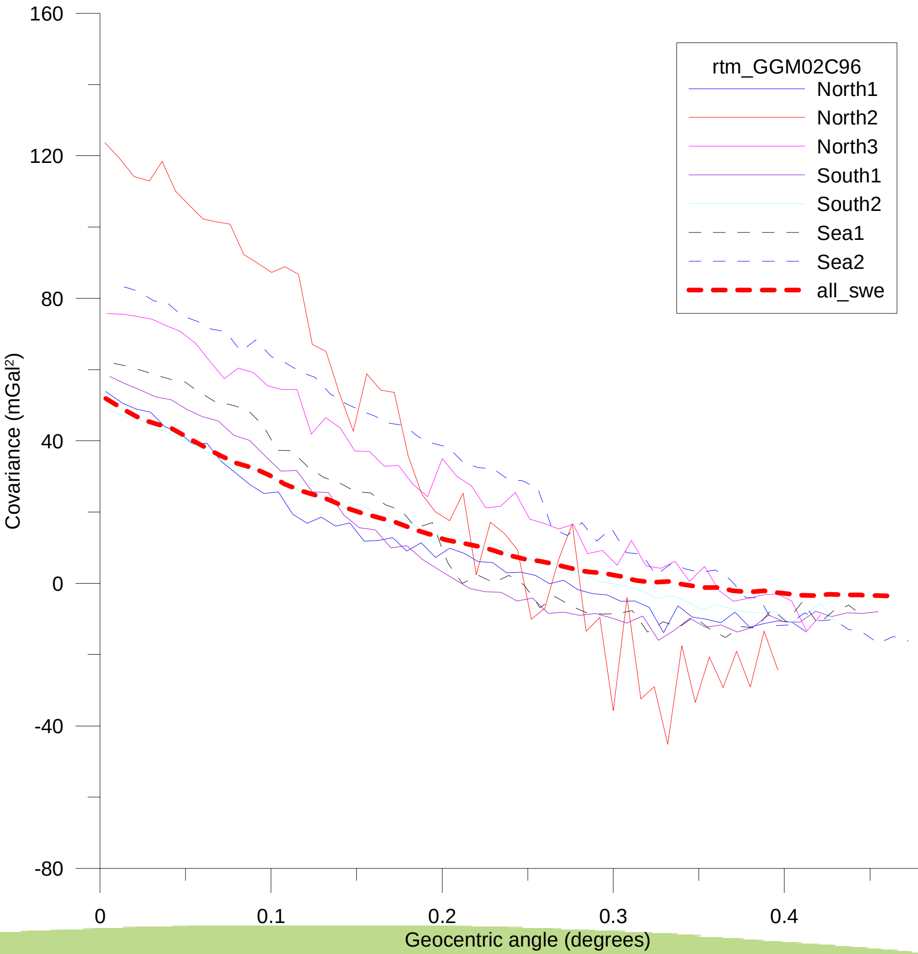
Spectral analysis of the gravity field in Sweden

- Cf. Forsberg (1986). The spectral analysis below is more or less only a confirmation of his results with updated data.
- We assume a remove-compute-restore (r-c-r) estimator
  - Sjöberg's LSMSA method needs to take advantage of the r-c-r strategy in the gridding phase to be efficient. The results are therefore relevant also for his case.
- Study Swedish gravity anomalies from the KTH08 computation reduced for
  - The EGM effect with M = 360 (GGM02C to 200 + EGM 96 above that)
  - The Residual Terrain Model (RTM) effect computed using GEOGRID (rect. prisms) with a 100 m x 100 m DEM (averaged from the Swedish photogrammetric DEM, now old).

Computed covariance functions





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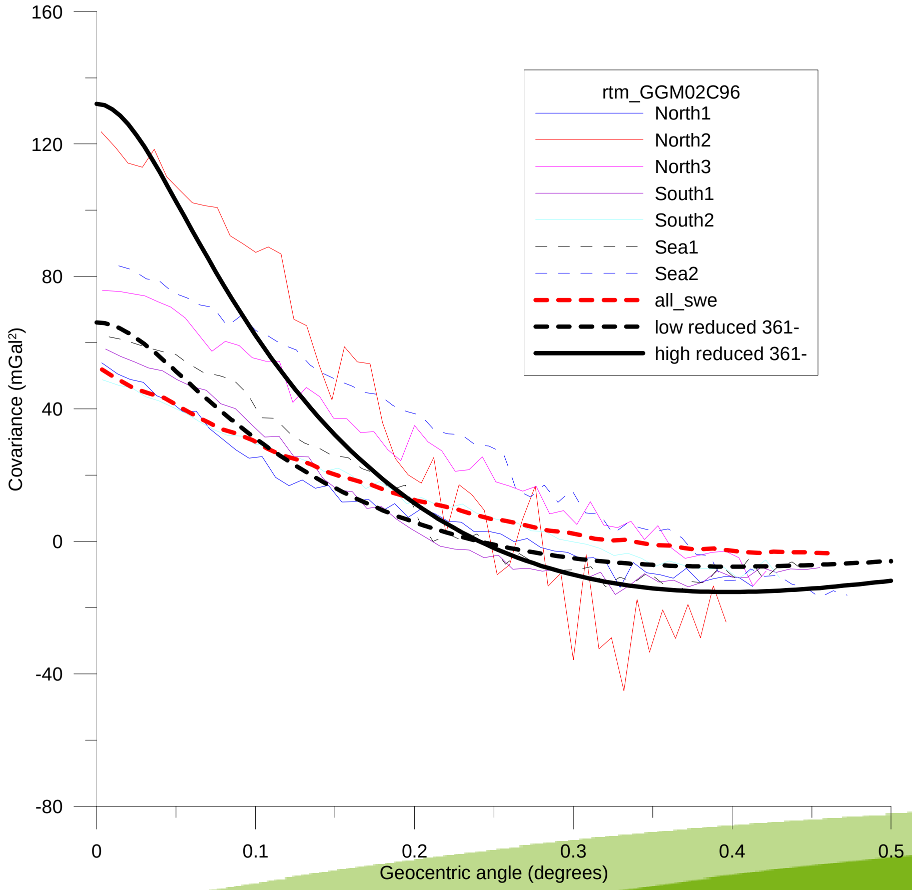
Degree variance models

Covariance function



Tscherning and Rapp (1974) type of model:

$$c_n = \alpha \frac{(n-1)}{(n-2)(n+4)} \left(\frac{(R-D)^2}{R^2} \right)^{n+2} \qquad n > 360$$

High reduced model:

α = 140 mGal^2 D = 2.5 km

Low reduced model:

α = 70 mGal^2 D = 2.5 km

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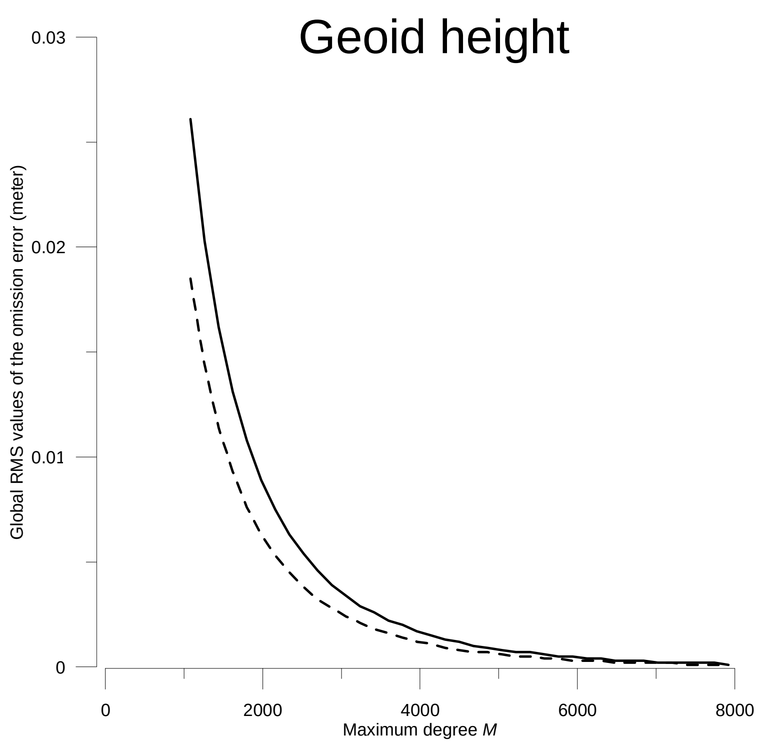
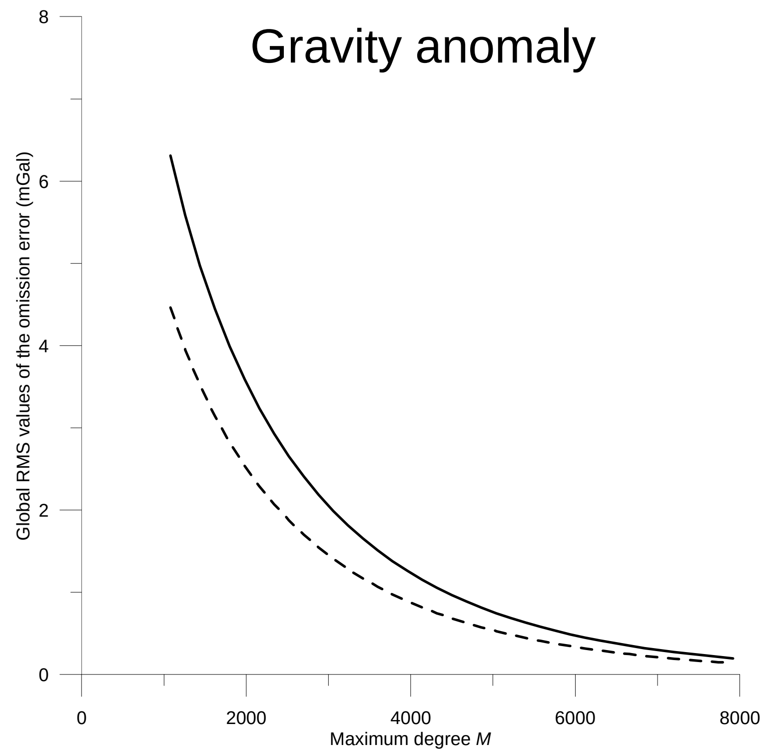
Global RMS omission errors





- 5 km resolution (corresponding to the Nyquist degree 3960) seems sufficient for all of Sweden.

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Sensitivity to correlated and uncorrelated gravity anomaly errors

- The high and low reduced signal degree variance models give very similar results
- The LSM method of Sjöberg (1991) with correct weighting. ψ_0 = 3 degrees.

Gravity anomaly noise model	Standard error (mGal)	Correlation length (deg)	Nyquist degree	Expected global RMS error (mm)
White	2	-	3960	9
White	1	-	3960	5
White	0.5	-	3960	3
White	0.2	-	3960	2
Reciprocal dist.	0.5	0.25	3960	9
Reciprocal dist.	0.2	0.25	3960	4
Reciprocal dist.	0.1	0.25	3960	3
Reciprocal dist.	0.05	0.25	3960	2

Comments

- It is important that the gravity data are as uncorrelated as possible. Systematic errors magnify the standard uncertainty very significantly.
- It is thus important to minimise errors in the higher order networks and in the connection upwards.
- White noise, generated for instance by bad heights, is not as dangerous.

Propagated geoid height RMS errors for a more realistic error model

- A combination of white and correlated noise is more realistic.

Gravity anomaly noise model	Standard error (mGal)	Correlation length (deg)	Nyquist degree	Expected global RMS error (mm)
White	2	-	3960	12
Reciprocal dist.	0.5	0.25		
White	1	-	3960	6
Reciprocal dist.	0.2	0.25		
White	0.5	-	3960	5
Reciprocal dist.	0.2	0.25		
White	0.1	-	3960	3
Reciprocal dist.	0.1	0.25		

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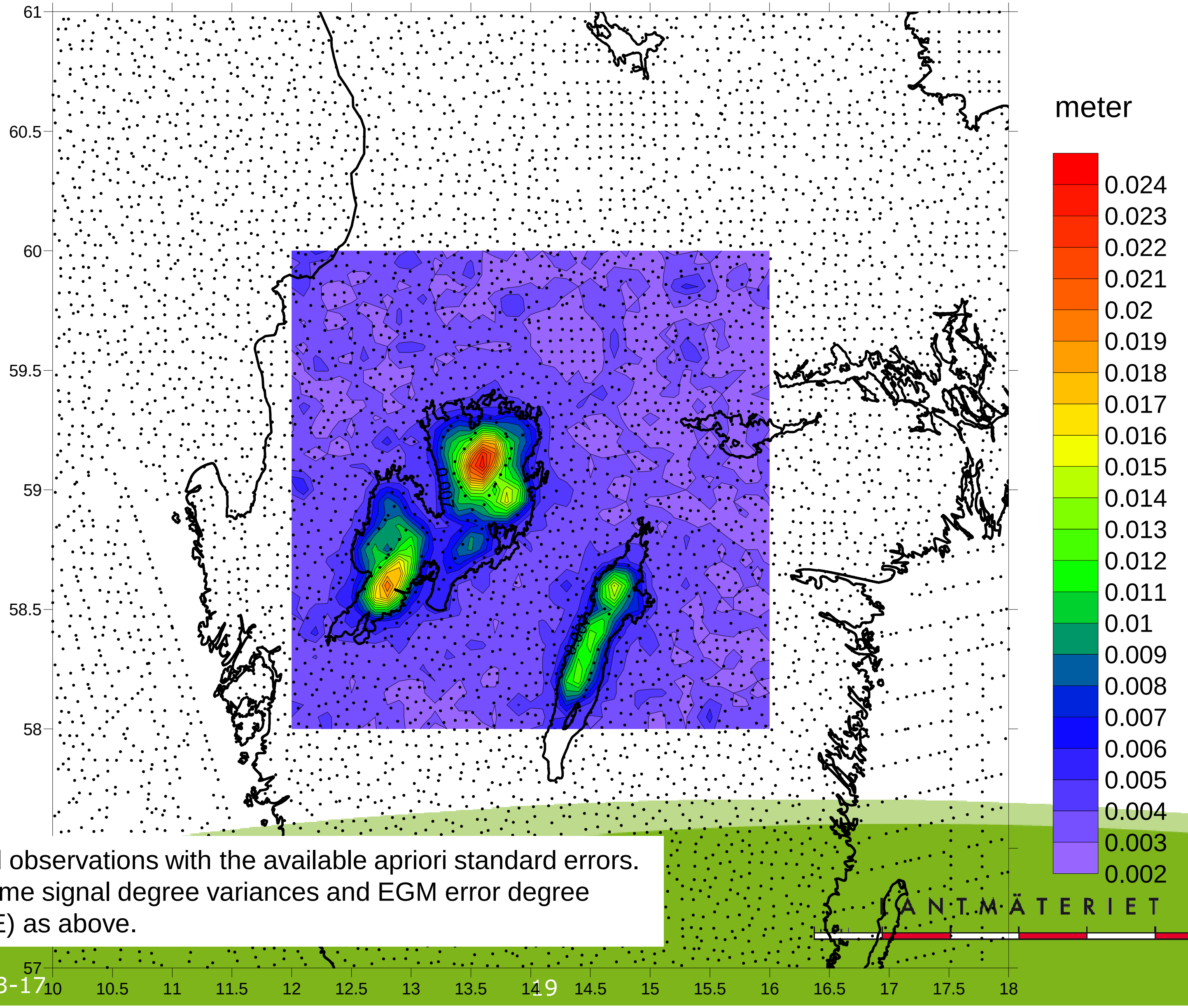
Limitations of the error propagations

- Note that the gravity data have been assumed to be homogeneous. How does non-homogeneous gravity data affect the situation? (Realistic sampling, data gaps, varying standard errors.)
- 5 km resolution seems sufficient to obtain a quasigeoid with 5 mm standard error over Sweden. One limitation here is that we used 5 km data in the spectral analysis.
- These questions are treated below.

Error propagation by Least Squares Collocation



Uncorrelated real observations with the available apriori standard errors. Otherwise the same signal degree variances and EGM error degree variances (GOCE) as above.

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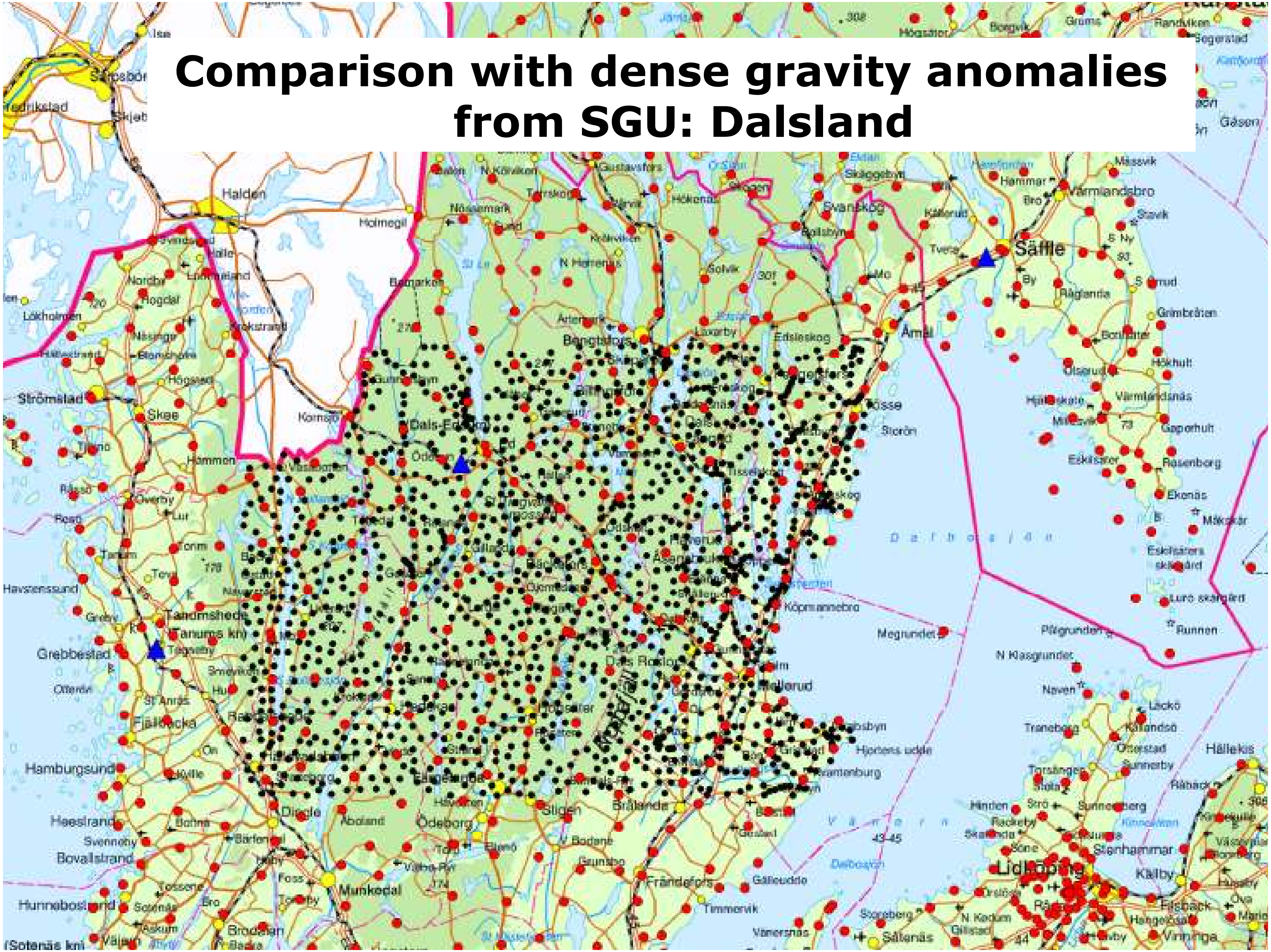
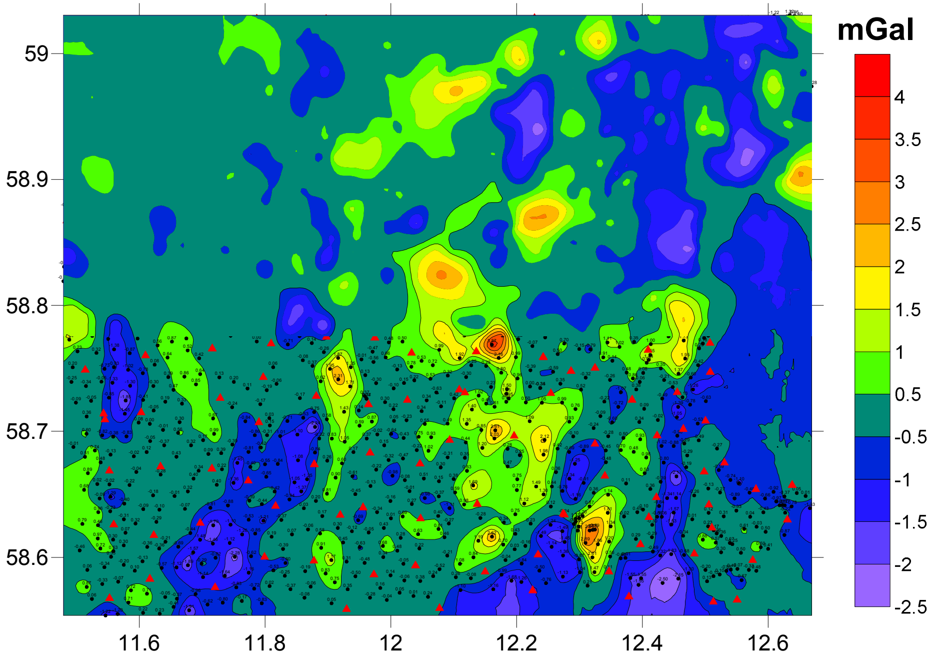
Comparison with dense gravity anomalies from SGU: Dalsland



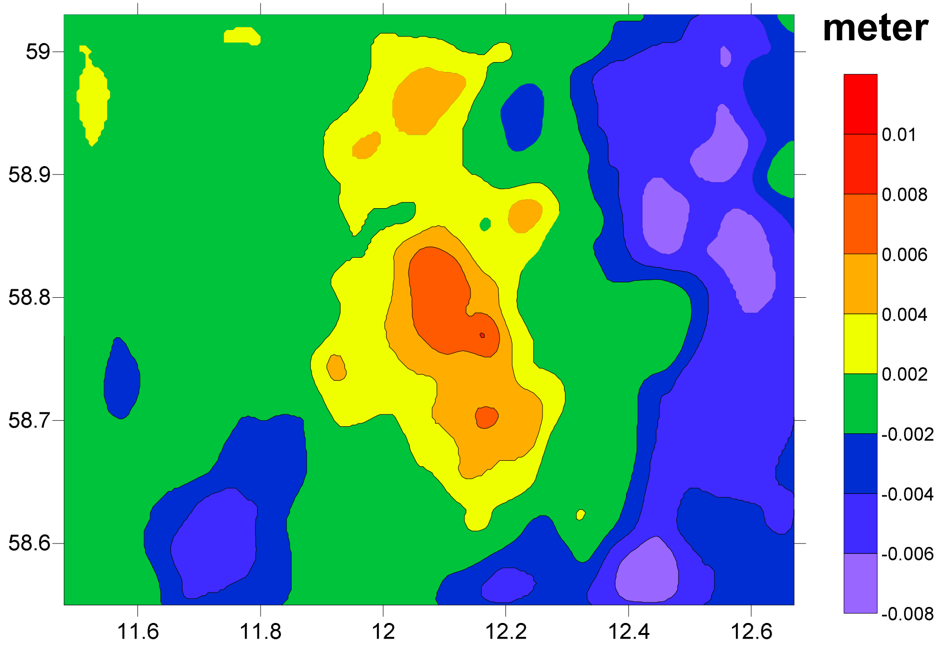
Comparison with dense gravity anomalies from SGU: Dalsland

Differences from the dense SGU gravity anomalies (mGal):



Min	Max	Mean	StdDev
-2.5	3.9	0.0	0.84

Propagated through the modified Stokes' operator (m):



Min	Max	Mean	StdDev
-0.008	0.008	-0.001	0.003

Refined Bouguer anomalies (100 m DEM), exact interpolation (Kriging Surfer 9)

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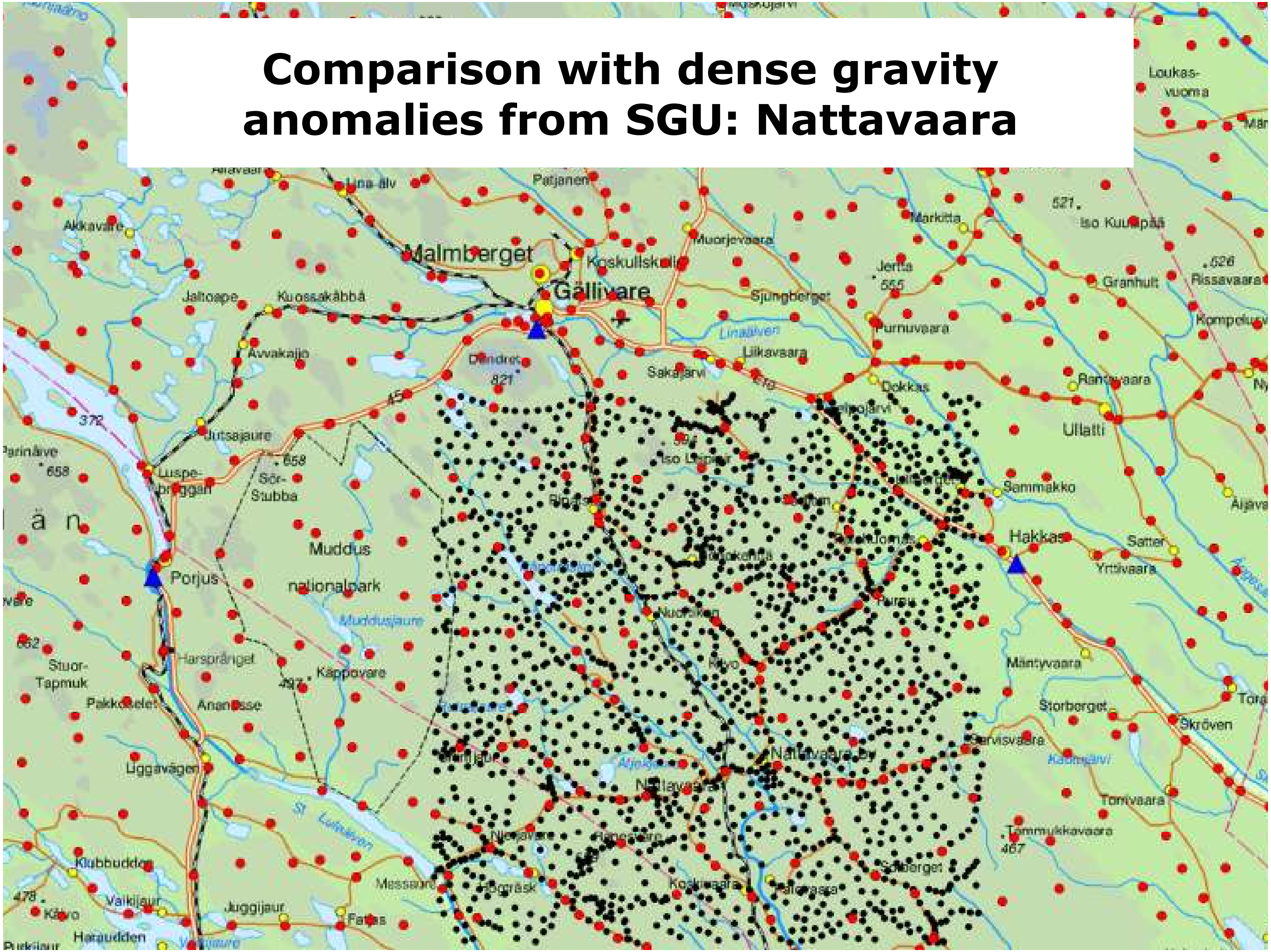
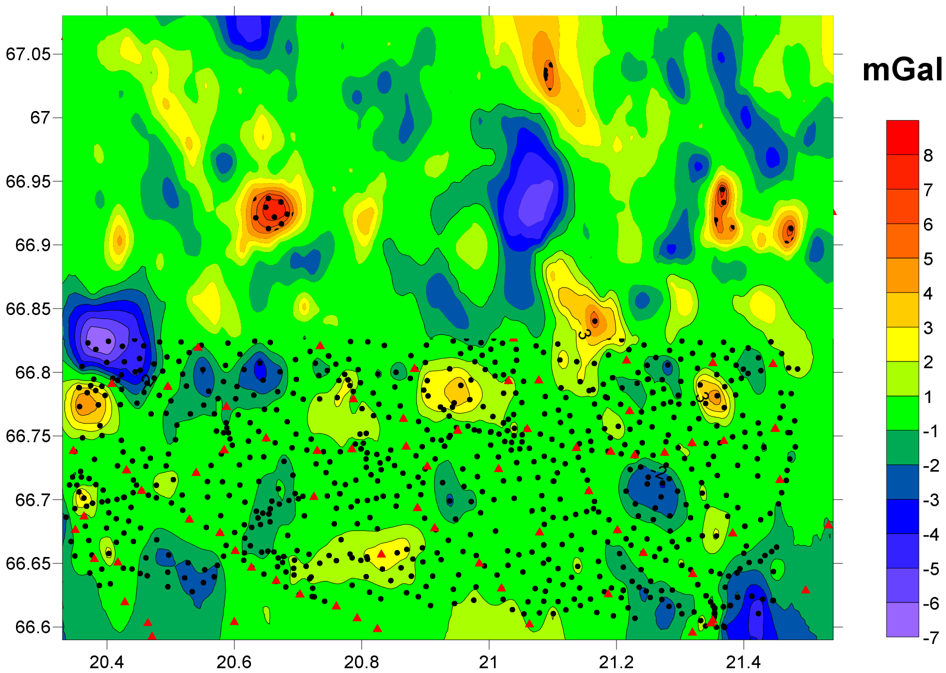
Comparison with dense gravity anomalies from SGU: Nattavaara



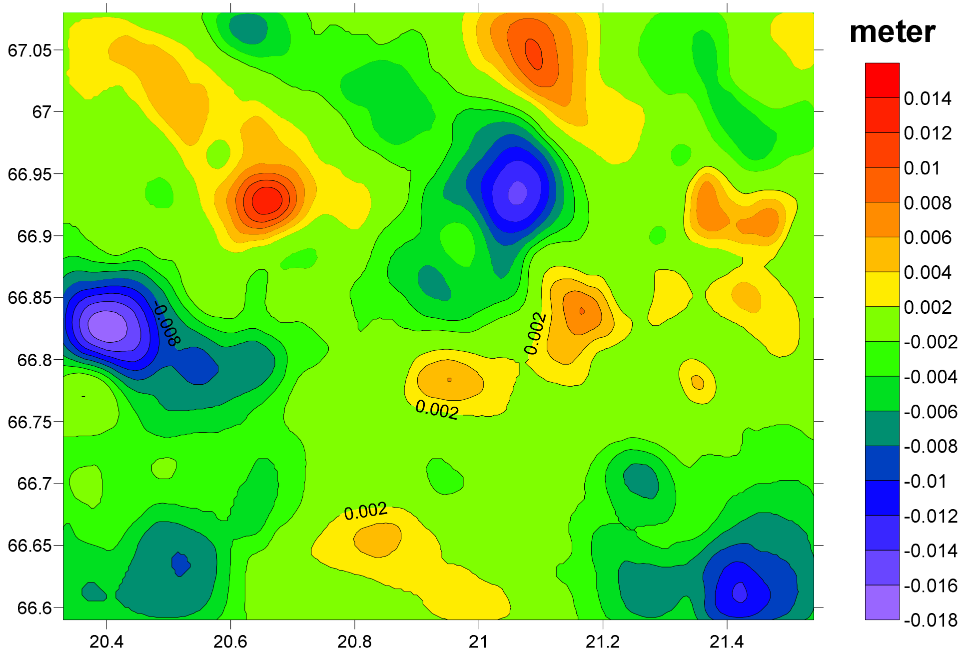
Comparison with dense gravity anomalies from SGU: Nattavaara

Differences from the dense SGU gravity anomalies (mGal)



Min	Max	Mean	StdDev
-6.6	7.7	0.0	1.5

Propagated through the modified Stokes' operator (m)



Min	Max	Mean	StdDev
-0.017	0.014	0.001	0.004

Refined Bouguer anomalies (100 m DEM), exact interpolation (Kriging Surfer 9)

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Conclusions

- This work indicates that it will be possible to compute a 5 mm geoid model for Sweden provided the gravity data are sufficiently updated.
- 5 km resolution is sufficient.
- Actions are needed to make the gravity data as uncorrelated as possible,
  - a new gravity system should be introduced (RG 2000), based on the Swedish stations in Nordic Absolute Gravity Project.
  - the old gravity networks and data should then be properly connected. This will require much work and many new measurements.
  - control measurements should be made with A10.
- All heights need to be determined with a standard uncertainty better than 1 - 2 meter.
- In those areas where the data are not accurate or dense enough, new measurements are needed. Data gaps in the big lakes and at sea should be filled (air borne, ship borne and/or ice measurements).
- Note again that significant methodological improvements are required to reach 5 mm, but this was not the issue here.