COMBINING MARINE GEOID MODELS WITH THE NATIONAL VERTICAL DATUMS IN COASTAL REGIONS

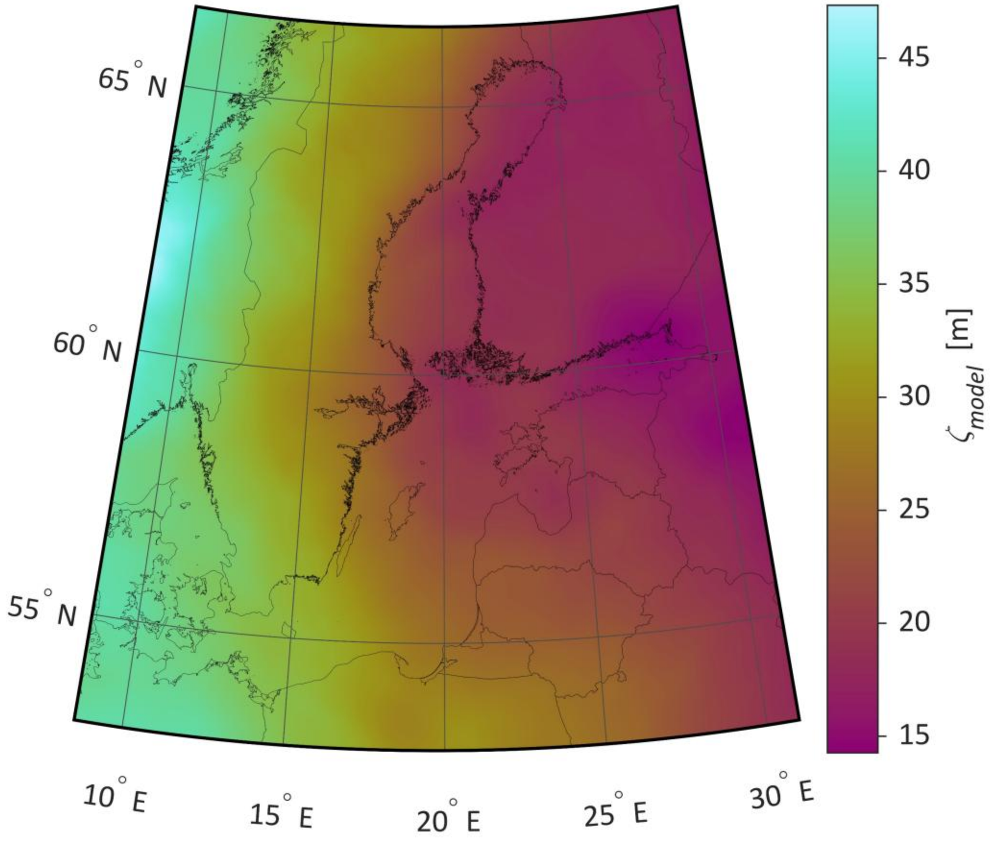
AND SOME MARINE GEOID MODELLING PROBLEMS IN GENERAL

Sander Varbla

26.08.2025

WHAT IS GEOID?

- An equipotential surface of the Earth's gravity field that coincides with the undisturbed (neglecting the influence of, e.g., wind and currents) sea level
- Represented either by:
  - Geoid model (described by geoid undulations N)
  - Quasigeoid model (described by height anomalies ζ)
  - N and ζ are given relative to a geodetic reference ellipsoid
- Offshore differences between geoid and quasigeoid are negligible ($N \equiv \zeta$) and have no practical implications



Min = 14.262 m   Max = 47.349 m   Mean = 26.498 m   SD = 8.130 m

Referred to as the GQM2022 quasigeoid model in the following

WHY BOTHER?

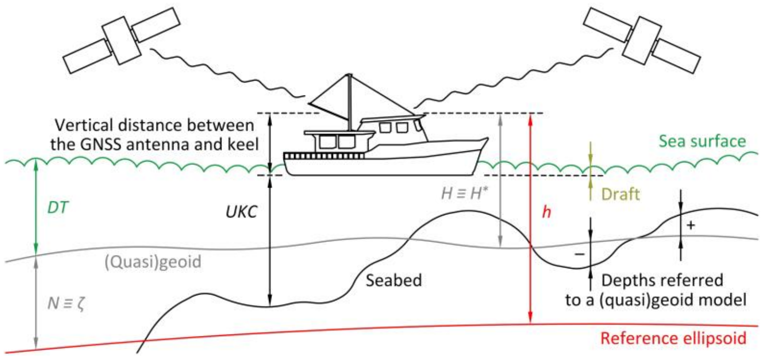
Practical applications require orthometric/normal heights

- (Quasi)geoid models are needed for height determination using modern satellite (GNSS) techniques (e.g., construction and maintenance of offshore structures)

GNSS-determined height relative to a reference ellipsoid → $h = N + H = \zeta + H^*$ ← normal height

geoid undulation (N), orthometric height (H), height anomaly (ζ)

- Marine (quasi)geoid models allow real-time GNSS-based monitoring of vessels' under-keel clearance



- Marine (quasi)geoid models are needed to determine the mean dynamic topography from temporally averaged sea surface height measurements to study marine processes/dynamics

TAL TECH

(CONVENTIONAL) GRAVIMETRIC QUASIGEOID MODELLING

- A gravimetric quasigeoid model can be determined via gravity integration, for instance, using the unbiased least-squares modified Stokes's formula with additive corrections (LSMSA)

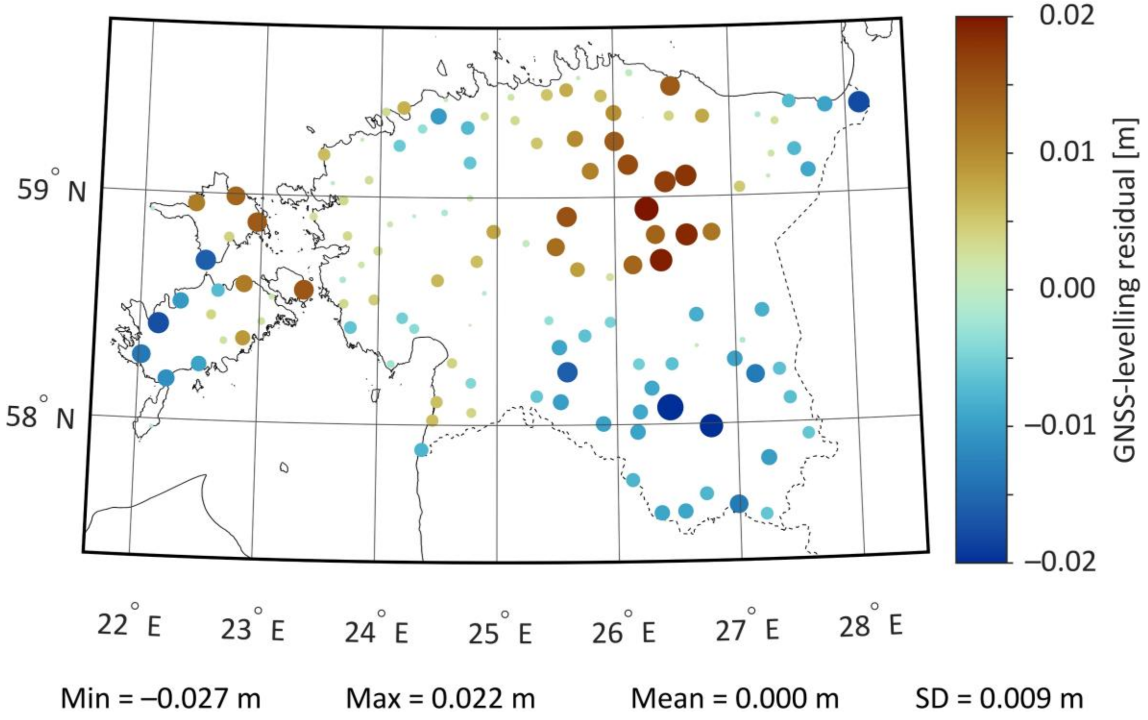
$$\zeta_{model} = \frac{R}{4\pi\gamma_0} \iint_{\sigma_0} S^L(\psi) \Delta g_G^{FAA} \, d\sigma + \frac{R}{2\gamma_0} \sum_{n=2}^{M} (s_n + Q_n^L) \, \Delta g_n^{GGM} + \delta\zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL}$$

- Examples are:
  - NKG2015 quasigeoid model
  - (Gravimetric) BSCD2000 quasigeoid model

VALIDATION USING GNSS-LEVELLING CONTROL POINTS

- Quasigeoid modelling solutions are conventionally validated using GNSS-levelling control points
- No information is retrieved regarding marine quasigeoid modelling accuracy

GNSS-levelling residuals of the GQM2022 quasigeoid model in Estonia



THE PROBLEM

- The errors of marine (quasi)geoid models are expected to reach up to a few decimetres due to gravity data void areas and/or inaccurate data
  - Since the conventional GNSS-levelling control points used for validating (quasi)geoid modelling solutions cannot be established offshore, the marine (quasi)geoid modelling accuracy estimates are primarily conjecture
- Already, the currently available (quasi)geoid models may not satisfy the industry and scientific needs, whereas the demand for improved accuracy increases ever further with technological and methodological advancements

OFFSHORE VALIDATION PRINCIPLE

- Offshore geometric height anomalies can be determined using sea surface height (SSH) measurements and dynamic topography (DT) estimates

$$\zeta_{geom} = SSH - DT$$

- Sea surface heights (relative to a reference ellipsoid) can be determined using various methods, for example:
  - Shipborne GNSS measurements and airborne laser scanning surveys (discussed first)
  - Satellite altimetry (discussed later)
- Dynamic topography information can be obtained from tide gauge stations and hydrodynamic models

TAL TECH

DERIVATION OF DYNAMIC TOPOGRAPHY

- The distribution of tide gauge stations is generally sparse and restricted to land-bound coastal locations (i.e., tide gauges represent a limited spatial domain)
- Hydrodynamic models contain spatiotemporal biases relative to the used vertical datums
- A combination of the two datasets can provide the best solution
  - Tide gauge readings (given relative to a vertical datum **at a defining epoch t_0**) allow determining the spatiotemporal bias (DB) of a hydrodynamic model

The bias consists of a datum shift and hydrodynamic modelling errors

$$DB = DT_{HDM} - DT_{TG}^{ASL} = DT_{HDM} - \left[DT_{TG}^{RSL} + VLM_{leveled}(t - t_0)\right]$$

DERIVATION OF DYNAMIC TOPOGRAPHY

- The resulting spatiotemporal bias estimates are available only at the locations of tide gauge stations
- There are two primary approaches to estimate the offshore spatiotemporal bias:
  1. Use of gridding approaches (e.g., inverse distance weighted interpolation or use of the least-squares collocation principle)

Ocean Modelling 180 (2022) 102124

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

ELSEVIER

Towards realistic dynamic topography from coast to offshore by incorporating hydrodynamic and geoid models

Vahidreza Jahanmard [a,*], Nicole Delpeche-Ellmann [b], Artu Ellmann [a]

Journal of Geodesy (2023) 97:24

https://doi.org/10.1007/s00190-023-01711-7

ORIGINAL ARTICLE

Iterative data assimilation approach for the refinement of marine geoid models using sea surface height and dynamic topography datasets

Sander Varbla[1] · Artu Ellmann[1]

The approach shown on slides 10 and 11

  2. Use of the machine learning strategies

Ocean Modelling 186 (2023) 102286

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

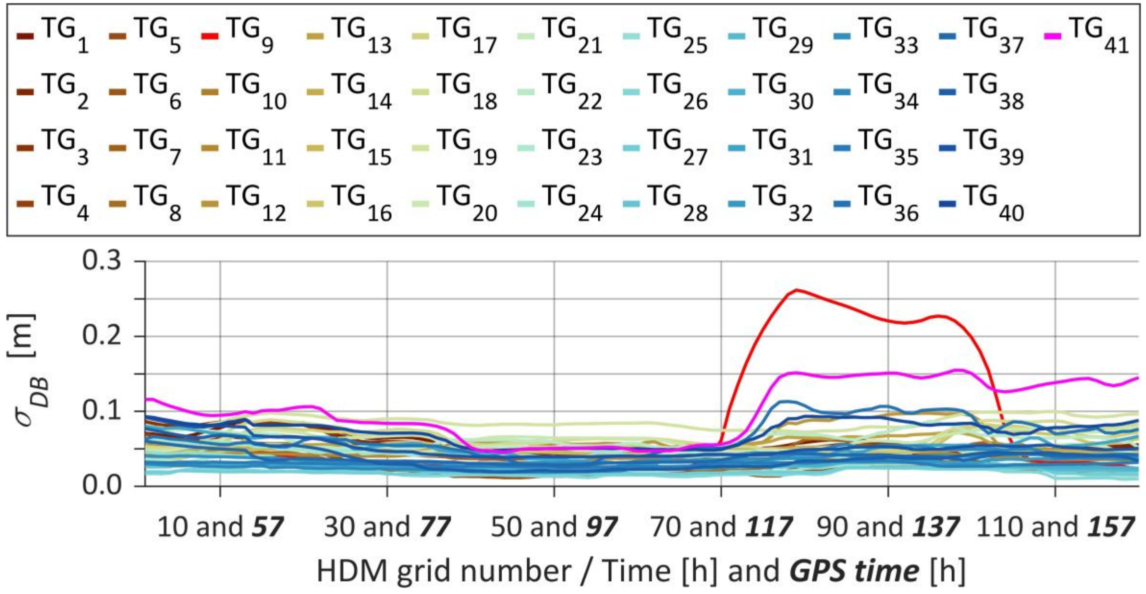
ELSEVIER

Quantification of hydrodynamic model sea level bias utilizing deep learning and synergistic integration of data sources

Vahidreza Jahanmard [a,*], Robinson Hordoir [b,c], Nicole Delpeche-Ellmann [d], Artu Ellmann [a]

UNCERTAINTY OF SPATIOTEMPORAL BIAS

- Two assumptions are posed:
  1. Over short time frames, spatiotemporal bias remains relatively stable
  2. A long-wavelength signal represents the spatiotemporal bias

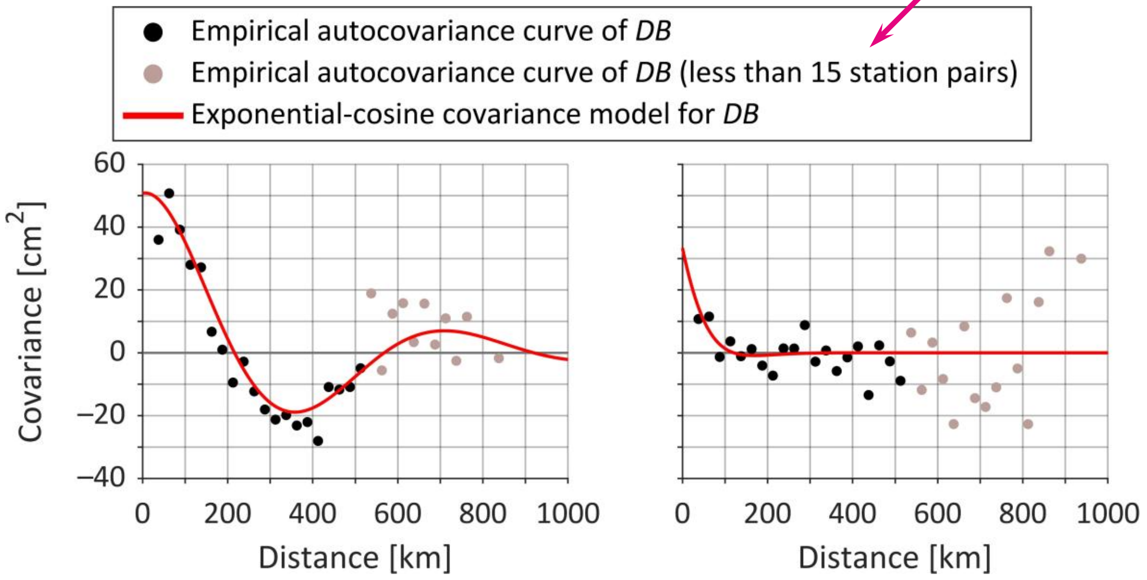


Combined spatiotemporal bias uncertainty representing stormy conditions – notice the sharp increase in uncertainty values at around hour 70/117, which coincides with the strongest storm winds

OFFSHORE ESTIMATION OF SPATIOTEMPORAL BIAS AND DYNAMIC TOPOGRAPHY

Not considered in model fitting

- Spatial dependency of spatiotemporal bias is modelled empirically
  - Notice variations at two arbitrary time instances
- Considering empirically modelled covariances and estimated uncertainties, offshore spatiotemporal bias can be estimated using the least-squares collocation principle
- The final dynamic topography is obtained by removing the estimated bias from the hydrodynamic model data



Constructed using combined spatiotemporal bias uncertainty estimates

$$\widehat{\mathbf{DB}} = \mathbf{C_{st}}(\mathbf{C_{tt}} + \mathbf{C_{nn}})^{-1}\mathbf{DB}$$

Constructed using covariance modelling results

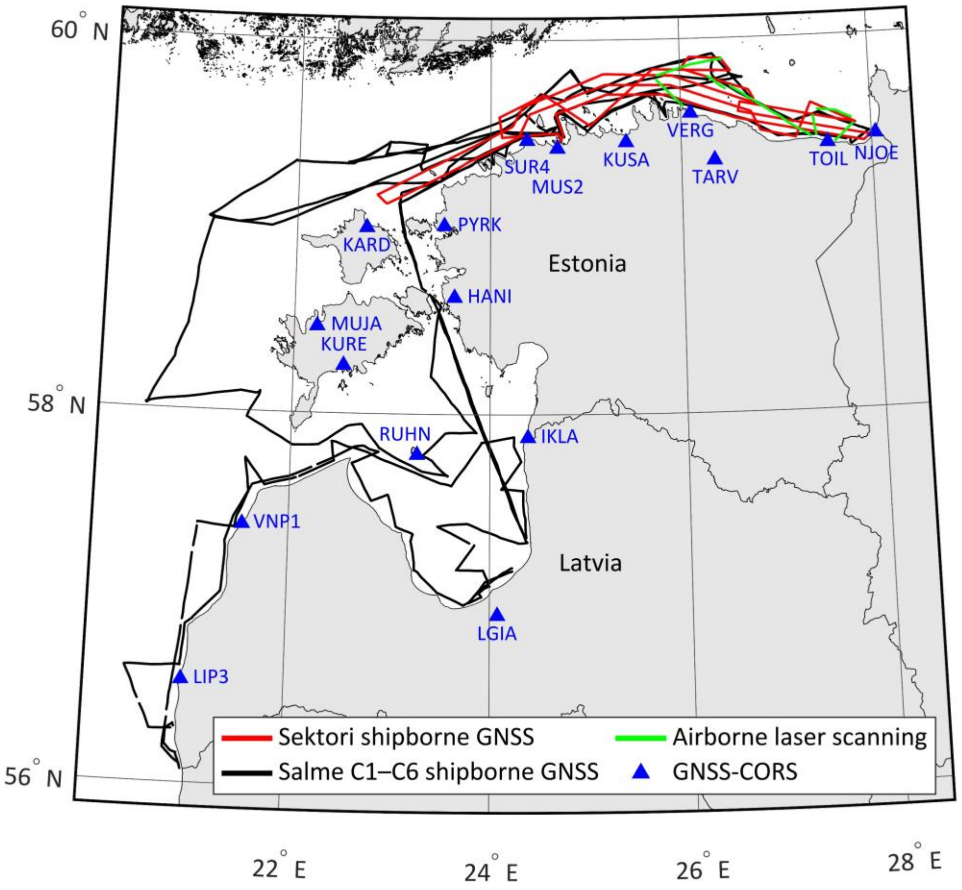
$$DT = DT_{HDM} - \widehat{DB}$$

$$\mathbf{E_{ss}} = \mathbf{C_{ss}} - \mathbf{C_{st}}(\mathbf{C_{tt}} + \mathbf{C_{nn}})^{-1}\mathbf{C_{st}}^{T}$$

SOURCES OF SEA SURFACE HEIGHT DATA

1. A dedicated shipborne gravity and GNSS campaign (Sektori) conducted in July 2017

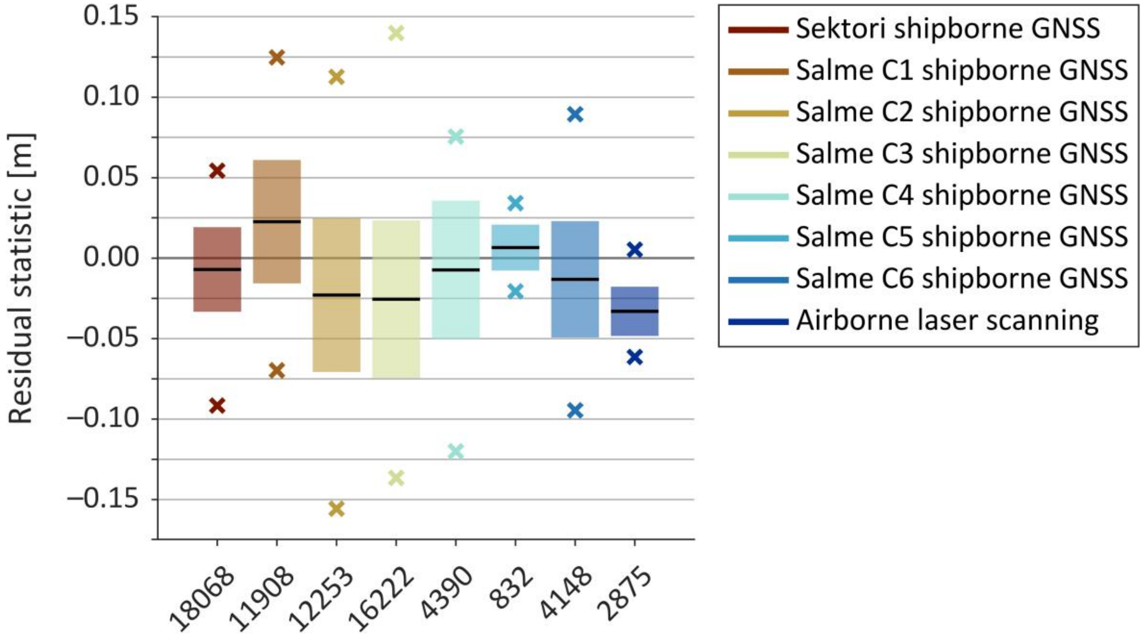
2. Six shipborne GNSS campaigns (Salme) conducted in the spring and summer of 2021 (data collection was autonomous)



3. An airborne laser scanning survey performed within the frames of routine mapping of offshore islands in May 2018

VALIDATION OF THE GQM2022 QUASIGEOID MODEL
DATA CONSISTENCY

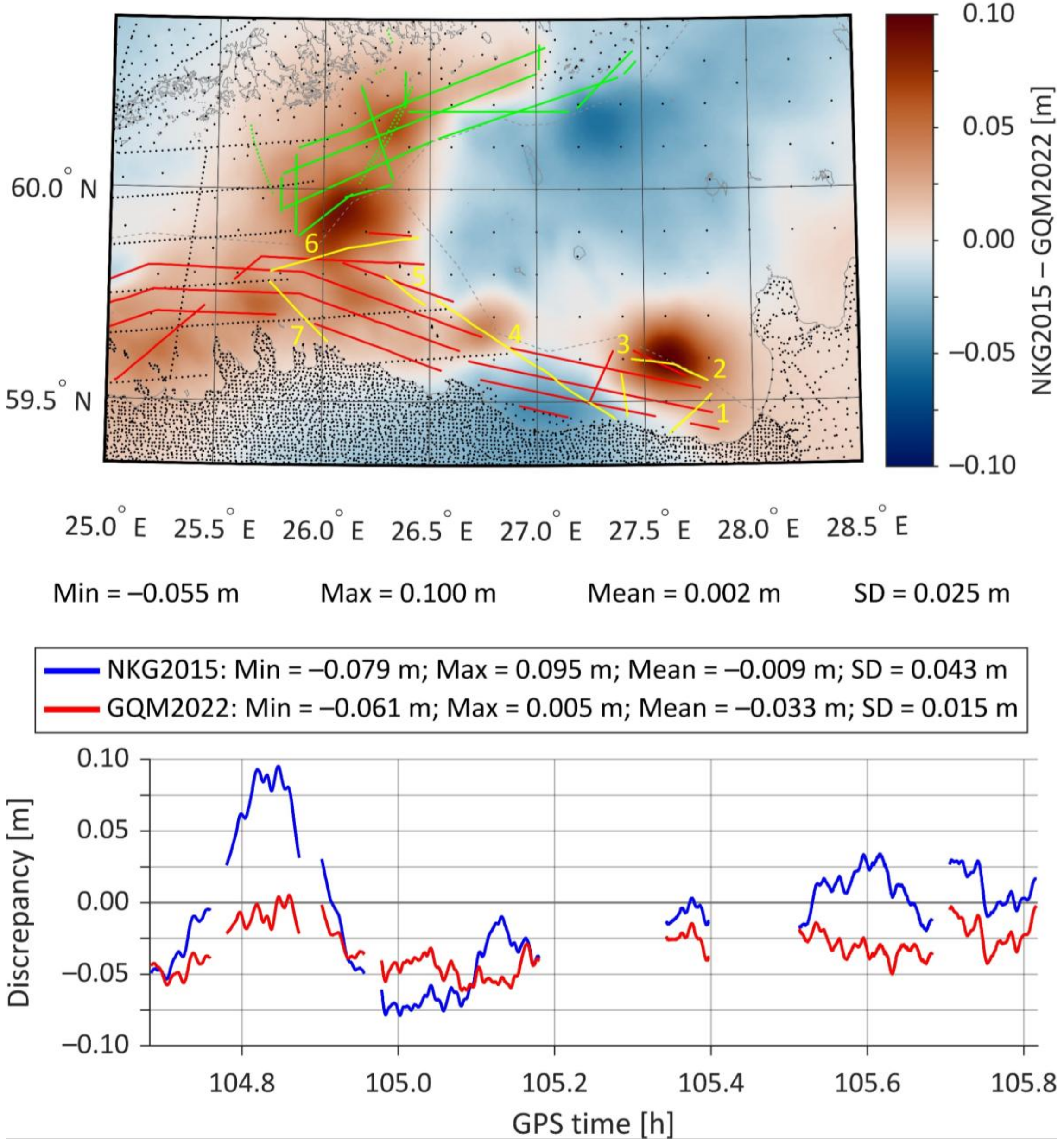
- After processing the raw observational data, dynamic topography estimates can be removed from the obtained sea surface heights, yielding geometric height anomalies
  - Geometric height anomalies provide a means for the validation
- It can be safely assumed that marine quasigeoid modelling is possible with an accuracy better than 5 cm (in the examined study area)
  - Considering also that the quasigeoid model is just one component contributing to the presented residuals



Black lines denote **mean values**, **coloured bars standard deviation estimates**, and coloured crosses minimum and maximum residuals. The X-axis shows the number of corresponding data points used in calculating the estimates.

IMPACT OF NEW GRAVITY DATA

- Comparison with the official NKG2015 quasigeoid model
  - Development of the NKG2015 model did not employ gravity data (denoted as dots) shown with red and green colour
- Airborne laser scanning reveals that quasigeoid modelling has improved up to 9 cm in previously existing gravity data void areas
  - Notice also only a 1.5 cm standard deviation from a comparison with the GQM2022 quasigeoid model

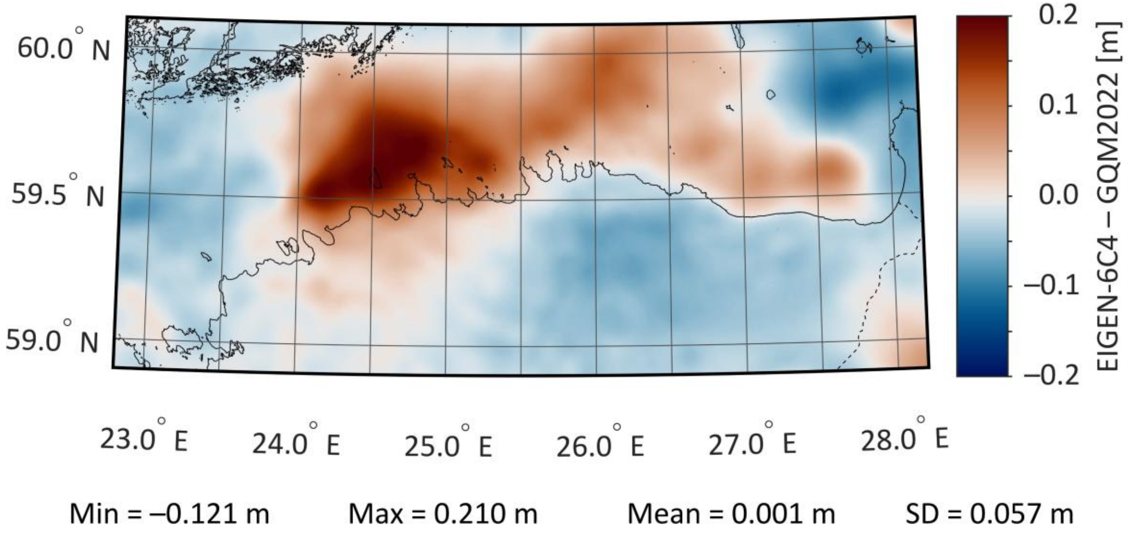


IMPLICATION

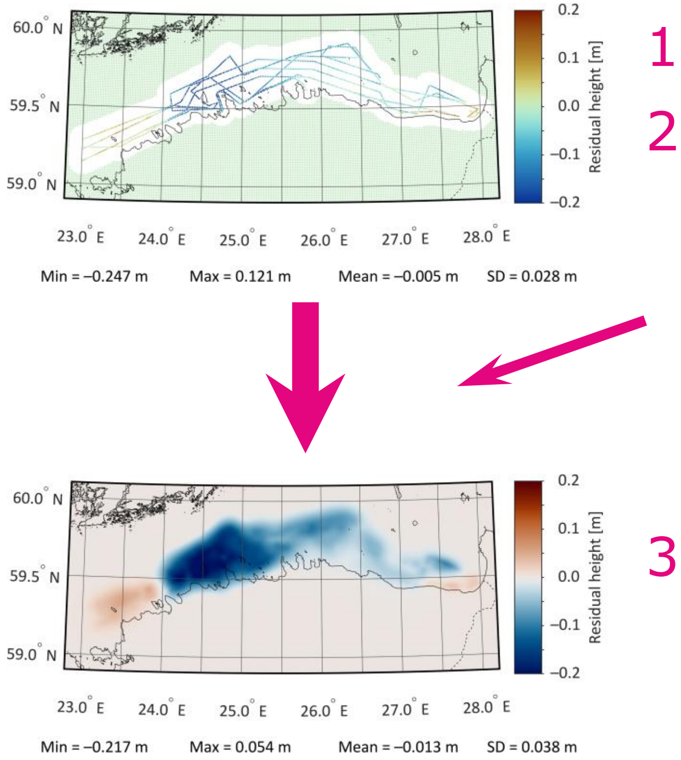
- Geometric height anomalies, determined using sea surface heights measured by shipborne GNSS or airborne laser scanning, are suitable for quantifying quasigeoid modelling errors offshore
- There is potential to use geometric height anomalies for refining quasigeoid modelling solutions, especially in gravity data void areas and regions where data quality is poor (i.e., where significant modelling errors could be expected)
  - A straightforward iterative data assimilation scheme, as shown in the following case study, can be employed

CASE STUDY

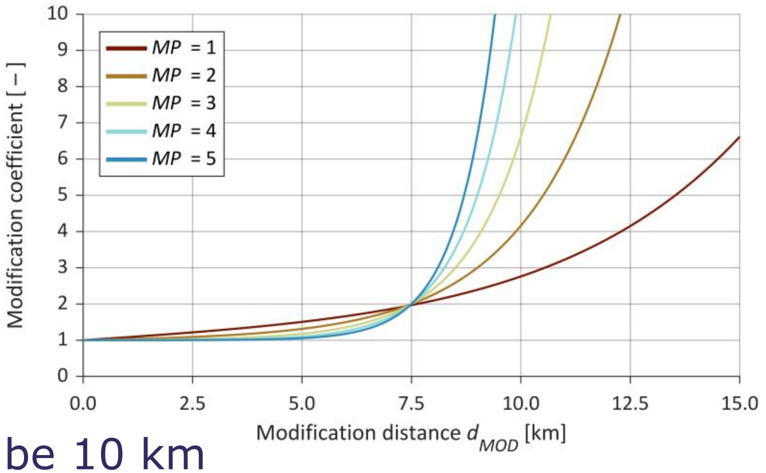
- Height anomalies synthesised from the EIGEN-6C4 global geopotential model were employed for refinement due to significant errors in the Gulf of Finland
- GQM2022 was used for validation as its good accuracy has been proven through validations (GNSS-levelling and offshore geometric height anomalies)
- Geometric height anomalies were separated between two assimilation iterations



ITERATIVE DATA ASSIMILATION SCHEME



1. Uncertainty estimation for geometric height anomalies
2. Gridding of geometric height anomalies
   - Reduction to residual heights
   - Zero-padding of data void regions
   - Gridding using the least-squares collocation principle; covariances are modelled empirically, and location dependently
   - Restoration to geometric height anomalies
3. Modification of geometric height anomalies' uncertainty to limit assimilation to the sea surface height data region



Modification coefficient values if minimum zero padding distance is assumed to be 10 km

ITERATIVE DATA ASSIMILATION SCHEME

4. Combination of modelled and geometric height anomalies

Weight (0, 1] determined from the agreement between modelled and geometric height anomalies

$$\hat{\zeta}^{k+1}_{model} = \frac{P^k\left(\sigma^k_{model}\right)^2 \hat{\zeta}^k_{geom} + \left(2 - P^k\right)\left(\sigma^k_{MOD}\right)^2 \zeta^k_{model}}{P^k\left(\sigma^k_{model}\right)^2 + \left(2 - P^k\right)\left(\sigma^k_{MOD}\right)^2}$$

$$\sigma^{k+1}_{\widehat{model}} < \sigma^k_{model}$$

and

$$\sigma^{k+1}_{\widehat{model}} < \sigma^k_{MOD}\sqrt{F_{scale}}$$

Scale factor ≥ 1 for scaling up the uncertainty of modified geometric height anomalies

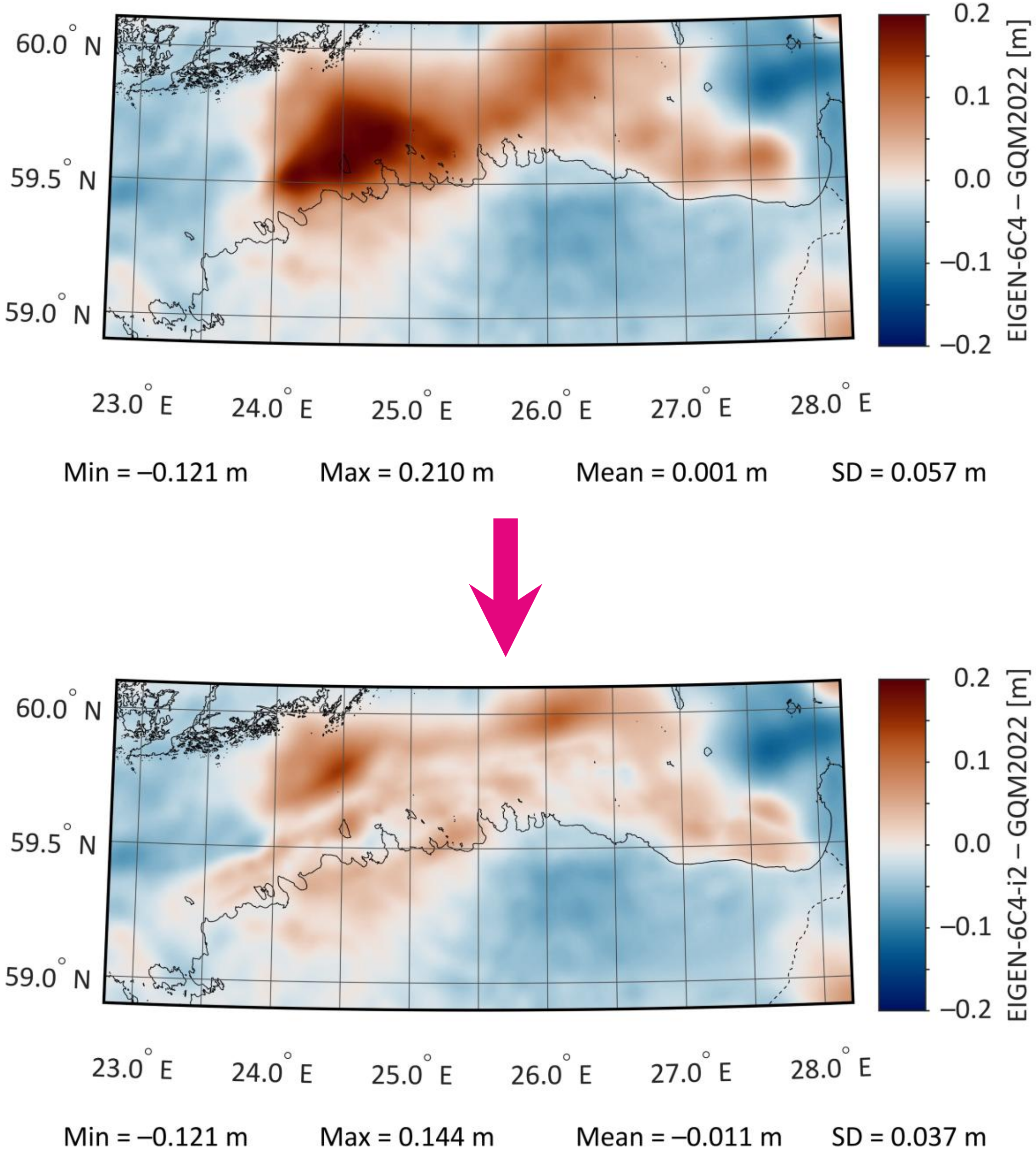
$$\sigma^{k+1}_{\widehat{model}} = \sqrt{\frac{F_{scale}\left(2 - P^k\right)\left(\sigma^k_{model}\right)^2\left(\sigma^k_{MOD}\right)^2}{P^k\left(\sigma^k_{model}\right)^2 + F_{scale}\left(2 - P^k\right)\left(\sigma^k_{MOD}\right)^2}}$$

If $\sigma^k_{MOD} \gg \sigma^k_{model}$, then $\sigma^{k+1}_{\widehat{model}} \approx \sigma^k_{model}$ and $\hat{\zeta}^{k+1}_{model} \approx \zeta^k_{model}$

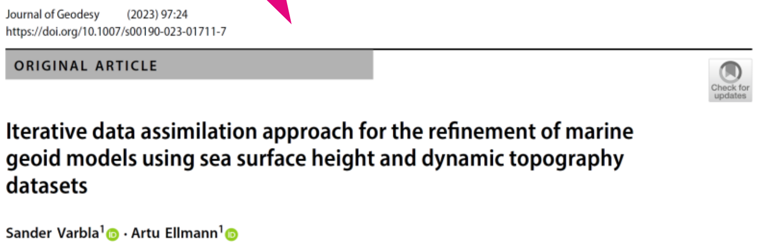
The reason for uncertainty modification

ASSIMILATION RESULTS

- Along survey routes, the results suggest a reduction in root mean square error from 10.1 cm to 2.2 cm – a five-fold improvement



More details about the approach can be found in


Journal of Geodesy (2023) 97:24
https://doi.org/10.1007/s00190-023-01711-7

ORIGINAL ARTICLE

Check for updates

Iterative data assimilation approach for the refinement of marine geoid models using sea surface height and dynamic topography datasets

Sander Varbla[1] · Artu Ellmann[1]

LIMITATIONS OF THE APPROACH

- Although shipborne GNSS measurements and airborne laser scanning surveys can provide high-accuracy data, some limitations need to be addressed:
  - Data collection is expensive and time-consuming
  - It is challenging to achieve repeated measurements
  - Data distribution is generally very limited

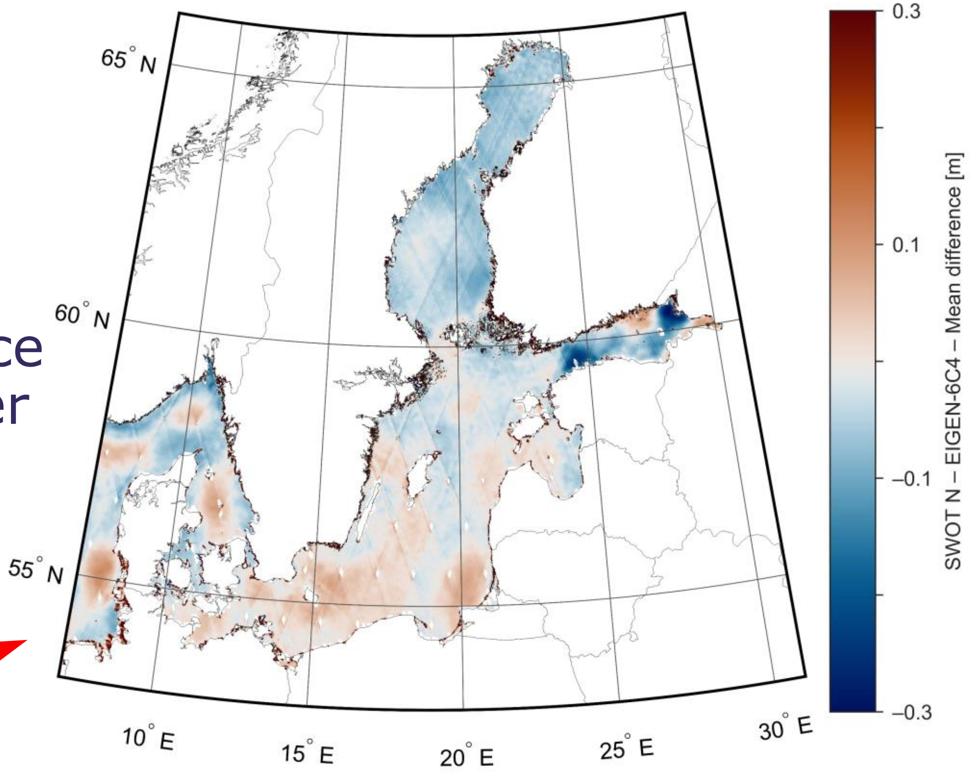
- The above problems can be solved using satellite altimetry measurements, and more specifically, SWOT altimetry due to its dense data coverage of the sea surface
  - In the following conceptual experiment, SWOT data are used to determine the Baltic Sea region marine portion of a quasigeoid model

GENERAL NOTES REGARDING THE DATA

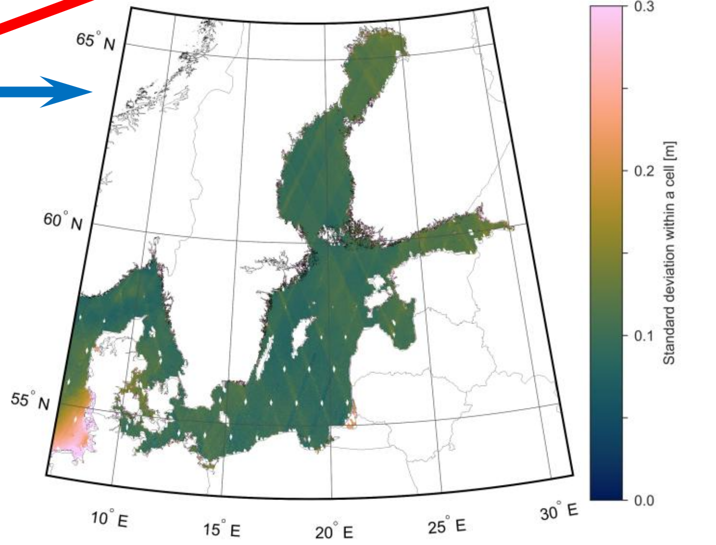
- The employed data are version C SWOT Level 2 KaRIn low-rate sea surface heights
  - 2 × 2 km resolution expert dataset
  - Data coverage spans from July 26, 2023, to May 3, 2025
- The 0.083° × 0.083° resolution Global Ocean Physics Analysis and Forecast model is used for dynamic topography information
  - Tide gauge data is neglected
  - Instantaneous dynamic topography estimates are determined at a mean time stamp (considering only the Baltic Sea marine measurements) of a satellite pass

DETERMINATION OF MEAN RESIDUAL HEIGHTS

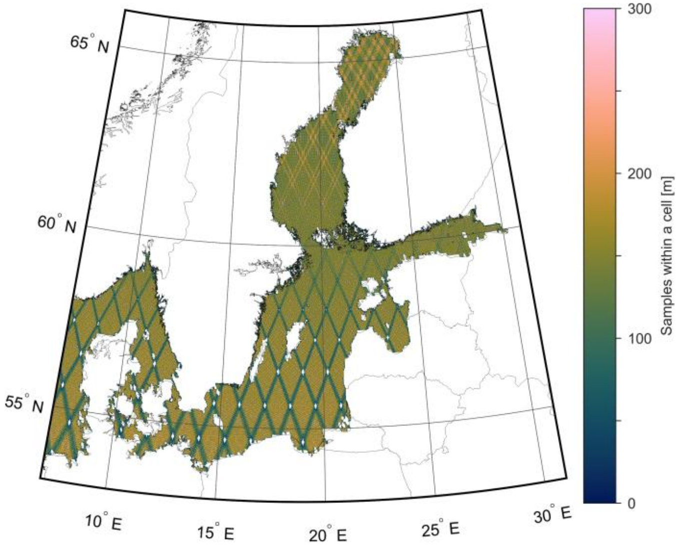
- After dynamic topography removal from sea surface heights, residual heights are determined for further processing using EIGEN-6C4
    - All residual heights over 3 m (relative to the mean residual height) are considered as obvious outliers
    - The remaining data points are analysed in 0.02° × 0.04° cells
        - Residual heights > 3×SD (relative to a cell's mean value) are removed
        - New mean residual height and standard deviation is determined for each cell



Min = −2.691 m Max = 2.991 m Mean = 0.000 m SD = 0.146 m



Min = 0.001 m Max = 2.594 m Mean = 0.133 m SD = 0.130 m



Min = 1 m Max = 322 m Mean = 139 m SD = 43 m

TAL TECH

COVARIANCE MODELLING

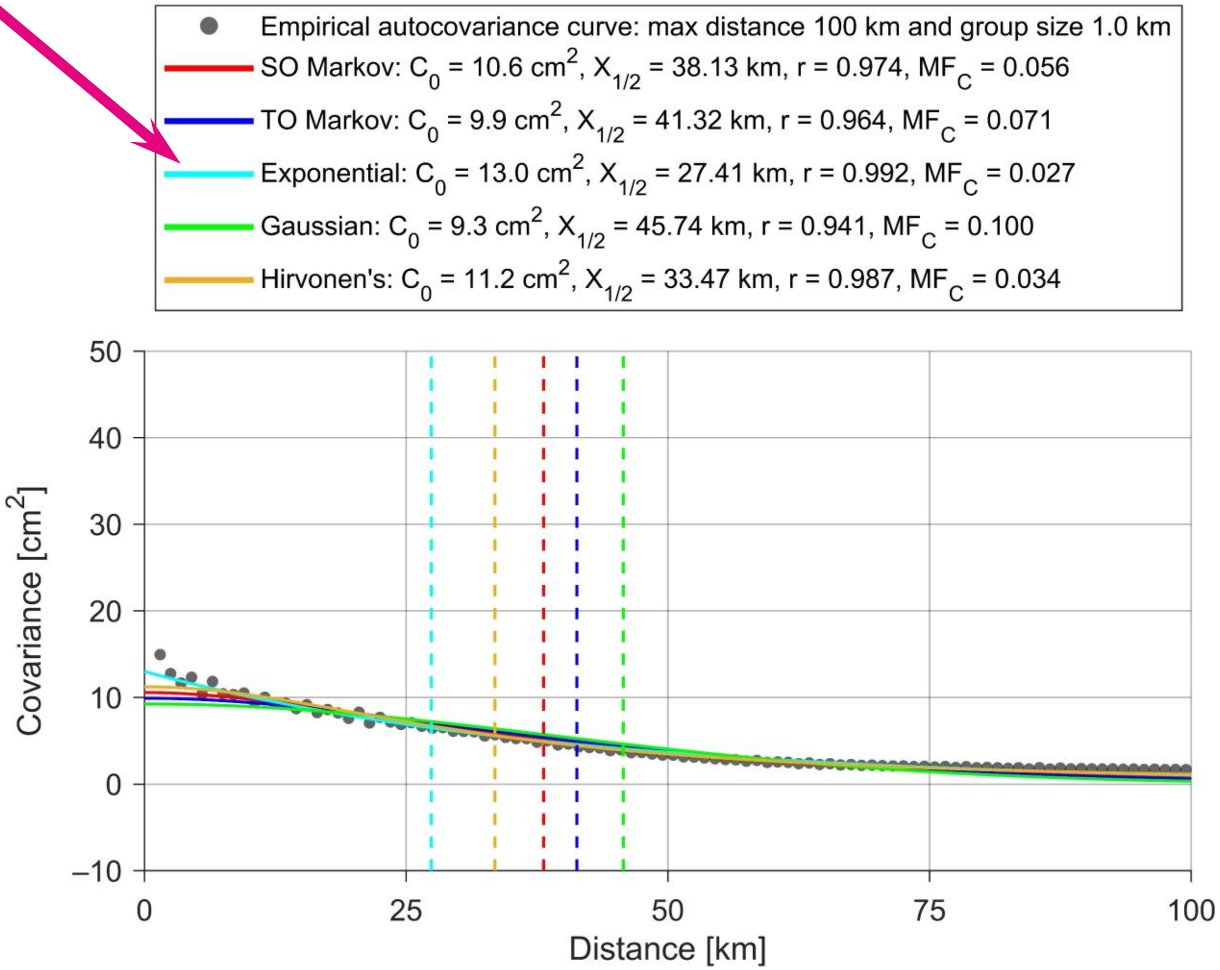
$$MF_C = \frac{1}{C_0}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[C_{model}(l_i) - C_{emp}(l_i)\right]^2}$$

The most optimal solution, as determined through experiments

- Empirical autocovariance curve computation is weighted using covariance uncertainties determined via uncertainty propagation
- Signal variance (C_0) and correlation length ($X_{1/2}$) are determined via weighted least-squares fitting
- Exponential covariance model appears to be most suitable

$$C_{model}(l) = C_0 e^{-l/\alpha}$$

$$\alpha = (-ln0.5)^{-1} X_{1/2}$$



DETERMINATION OF THE SURFACE MODEL

- Discrete mean residual heights can be gridded using the least-squares collocation principle

Constructed using cell-based standard deviation estimates

$$\hat{\zeta}_{\mathbf{residual}} = \mathbf{C}_{\mathbf{st}}(\mathbf{C}_{\mathbf{tt}} + \mathbf{C}_{\mathbf{nn}})^{-1}\zeta_{\mathbf{residual}}$$

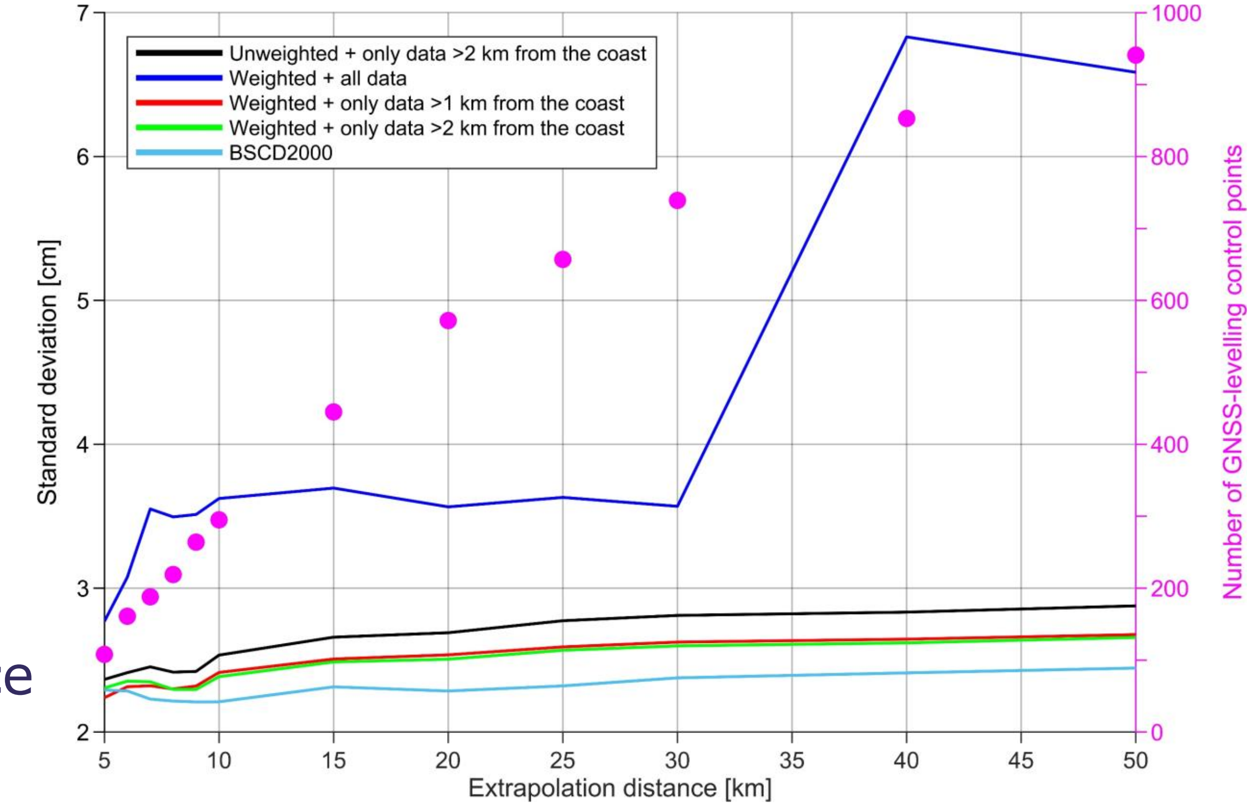
Constructed using covariance modelling results

$$\mathbf{E}_{\mathbf{ss}} = \mathbf{C}_{\mathbf{ss}} - \mathbf{C}_{\mathbf{st}}(\mathbf{C}_{\mathbf{tt}} + \mathbf{C}_{\mathbf{nn}})^{-1}\mathbf{C}_{\mathbf{st}}{}^{T}$$

- The initially removed signal of EIGEN-6C4 is finally added back

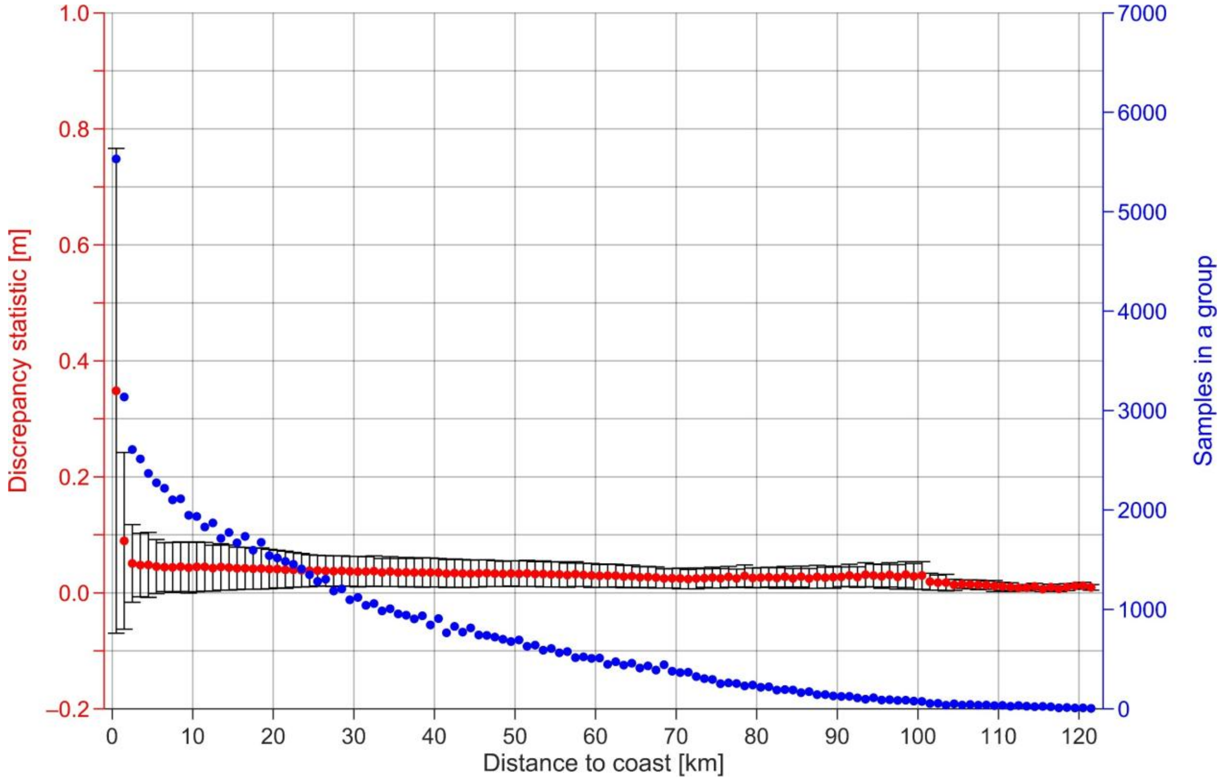
VALIDATION USING GNSS-LEVELLING CONTROL POINTS

- The Baltic Sea region GNSS-levelling control points used during the development of the BSCD2000 model are employed
- Extrapolation distance refers to the distance from available mean residual heights used during gridding
- All later results refer to the "weighted + only data >1 km from the coast" variant
  - Also, the previously shown covariance modelling results
- BSCD2000 quasigeoid model is the gravimetric variant



LAND SIGNAL

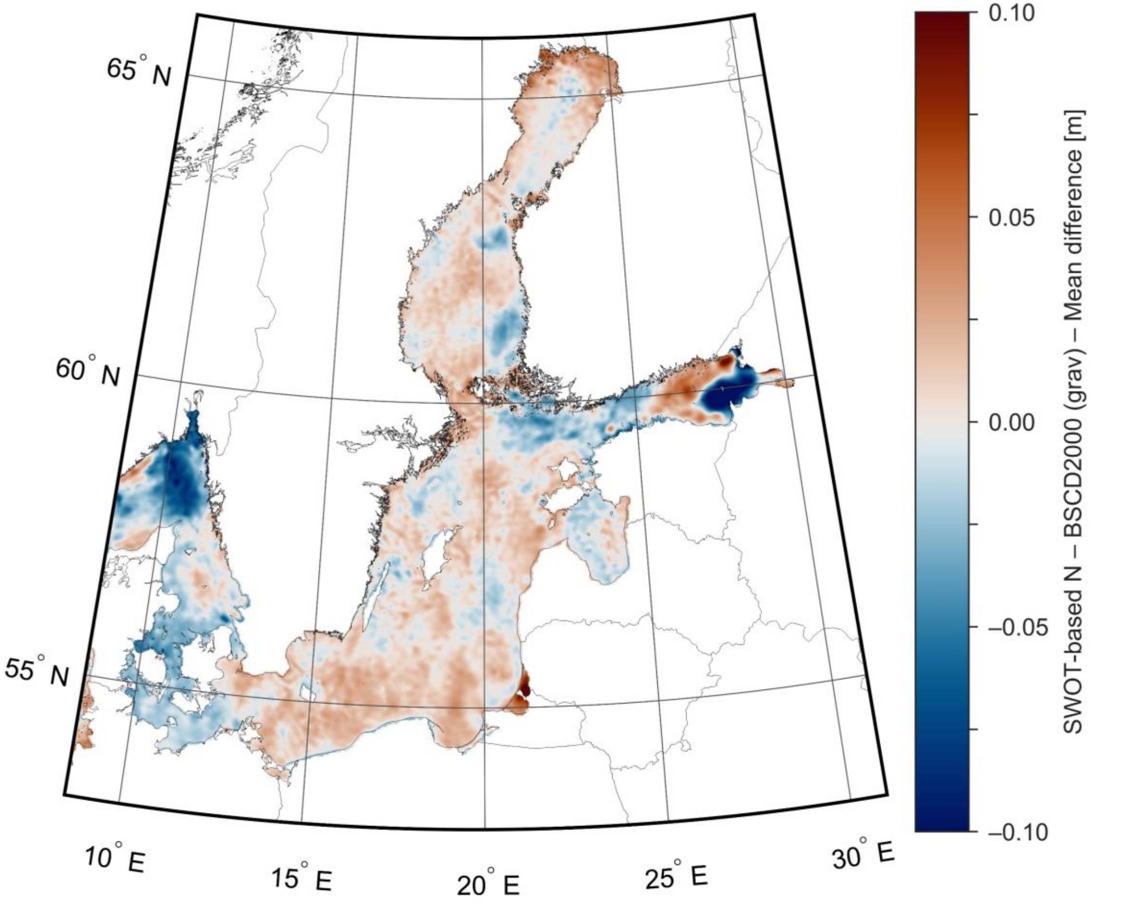
- Land signal effect in data (notice the first two groups, i.e., 0-1 km and 1-2 km groups)
  - Red dots represent averaged cell-based mean absolute residual heights within a distance group, and whiskers show the corresponding standard deviation estimates



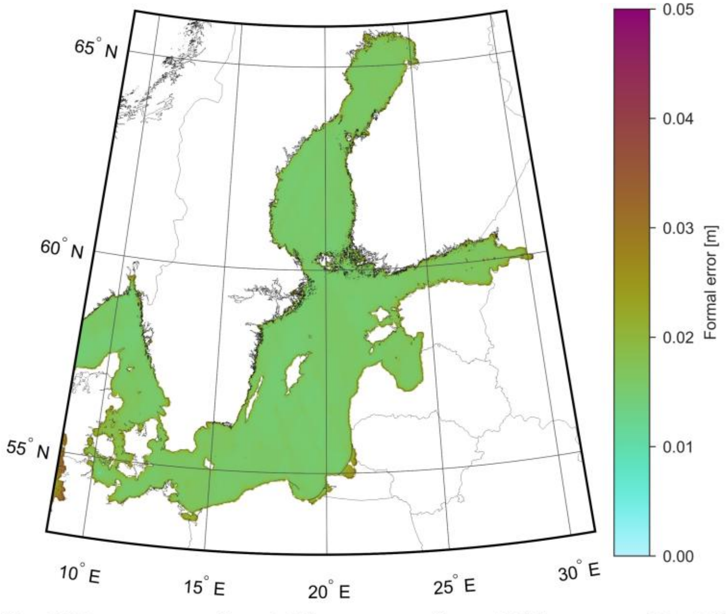
TALLINN UNIVERSITY OF TECHNOLOGY

COMPARISON WITH THE GRAVIMETRIC BSCD2000 MODEL

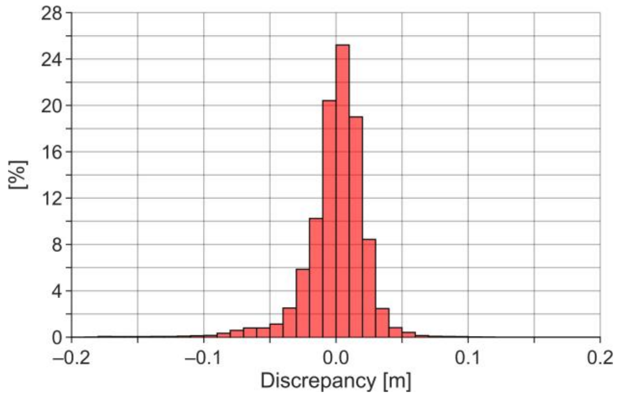
- Comparison between the SWOT-based model and BSCD2000 demonstrates excellent potential of SWOT data in determining the marine geoid
  - Discrepancies remain generally within 3-4 cm



Min = –0.185 m   Max = 0.116 m   Mean = –0.000 m   SD = 0.023 m



Min = 0.006 m   Max = 0.034 m   Mean = 0.017 m   SD = 0.002 m



TALTECH

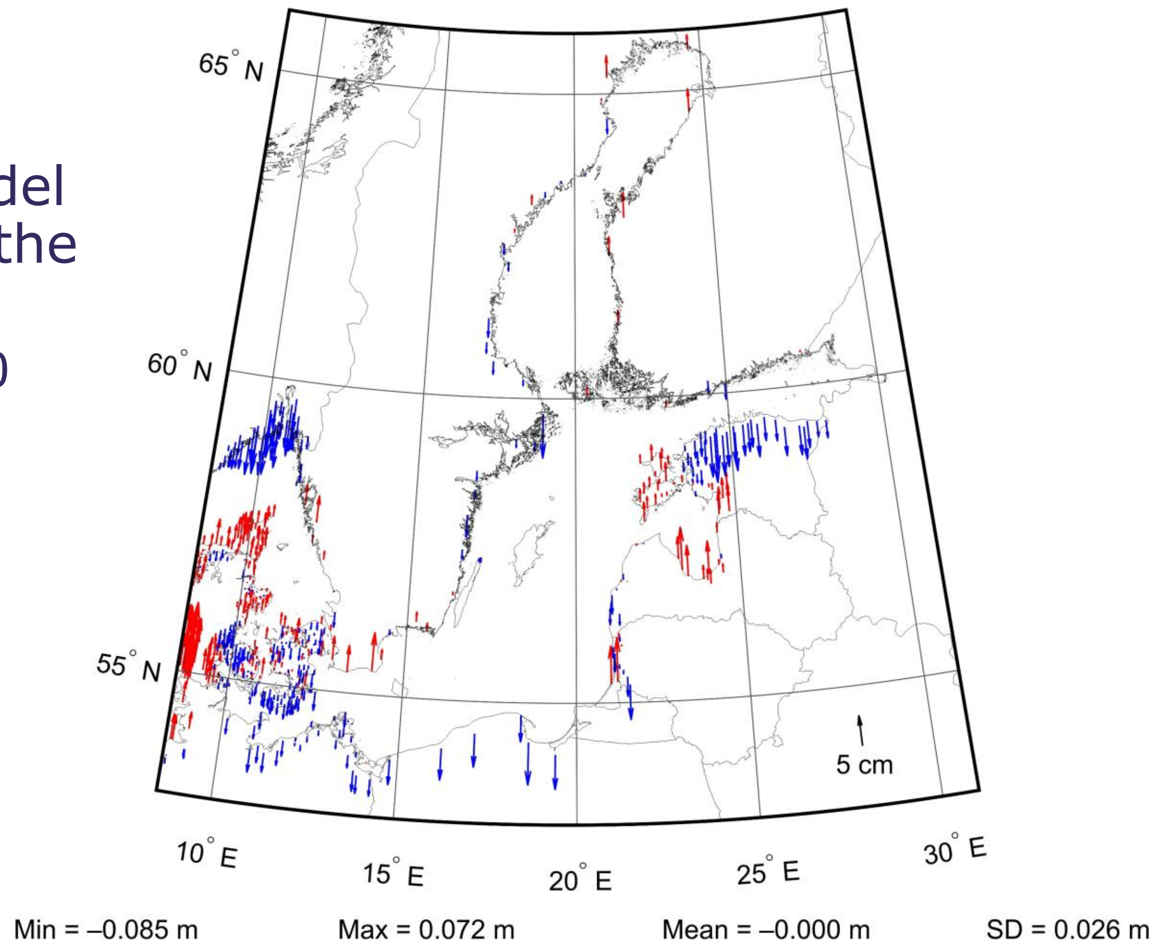
IMPLICATION

- In marine regions of unavailable, missing, or poor gravity data, SWOT-based geoid surface geometry models could potentially be considered as an alternative
- SWOT-based information can be used to:
  - refine gravimetric models, as in the earlier case study
  - replace the marine portion of a gravimetric model (i.e., blending of the models)
    - Principles of the BSCD2000 height transformation grid construction can be adapted

Shown in the following

BLENDING OF MODELS

- The SWOT-based model and gravimetric model should be given a one-dimensional fit using the same set of GNSS-levelling control points
  - In this conceptual experiment, the BSCD2000 gravimetric quasigeoid model is used
  - Extrapolation of the SWOT-based model is allowed up to 25 km
  - After fitting, the discrepancies between the two models have a mean value of 6 mm



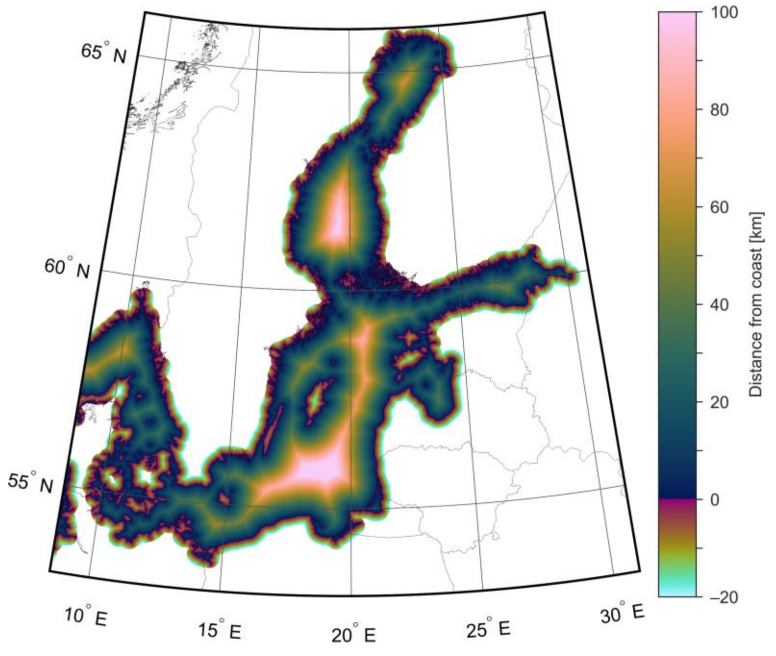
GNSS-levelling residuals considering 25 km extrapolation distance

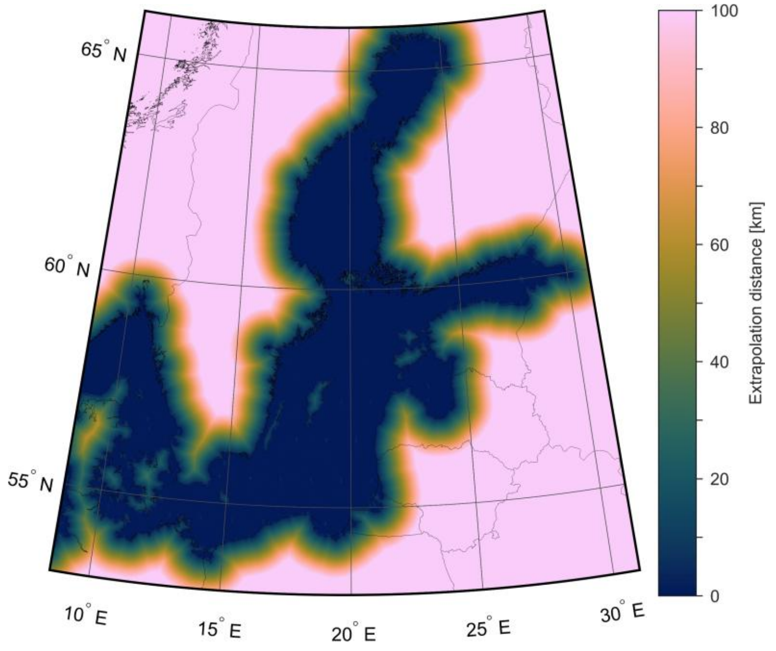
BLENDING OF MODELS

- The blending considers distances from the coast (D_C) and extrapolation distances (D_E)

$$
w = \begin{cases} 1, & D_C < 0 \; OR \; D_E > D_E^{max} \\ \frac{1}{2} + \frac{1}{2}\cos\left(\pi \frac{D_C}{T}\right), & 0 \leq D_C \leq T \; AND \; D_E \leq D_E^{max} \\ \frac{1}{2} + \frac{1}{2}\cos\left(\pi \frac{D_E - (D_E^{max} - T)}{T}\right), & D_C > T \; AND \; D_E^{max} - T \leq D_E \leq D_E^{max} \\ 0, & D_C > T \; AND \; D_E < D_E^{max} - T \end{cases}
$$

- The transition zone (T) is set to 10 km and the maximum allowable extrapolation distance (D_E^{max}) to 17 km

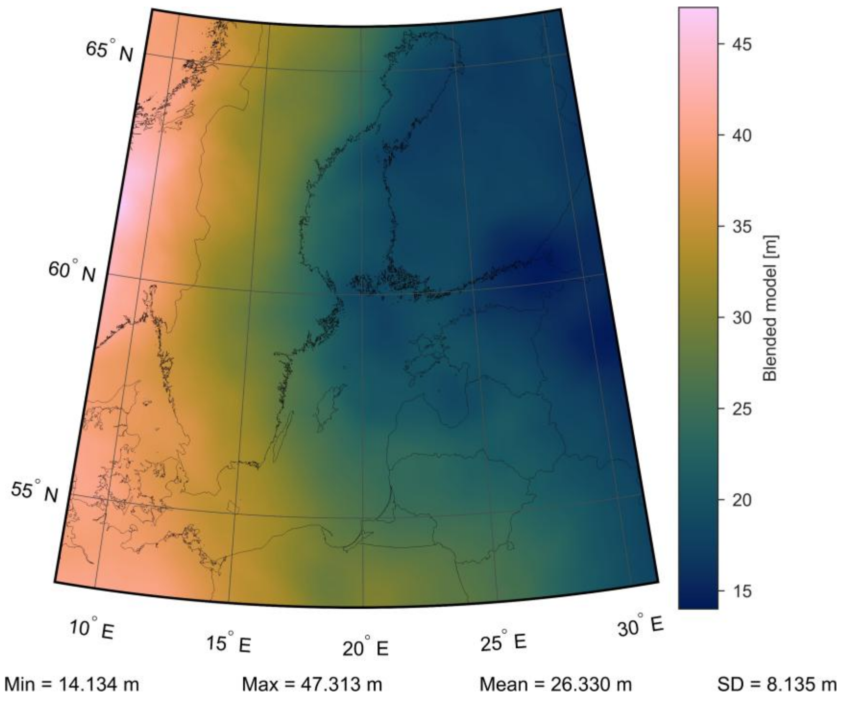


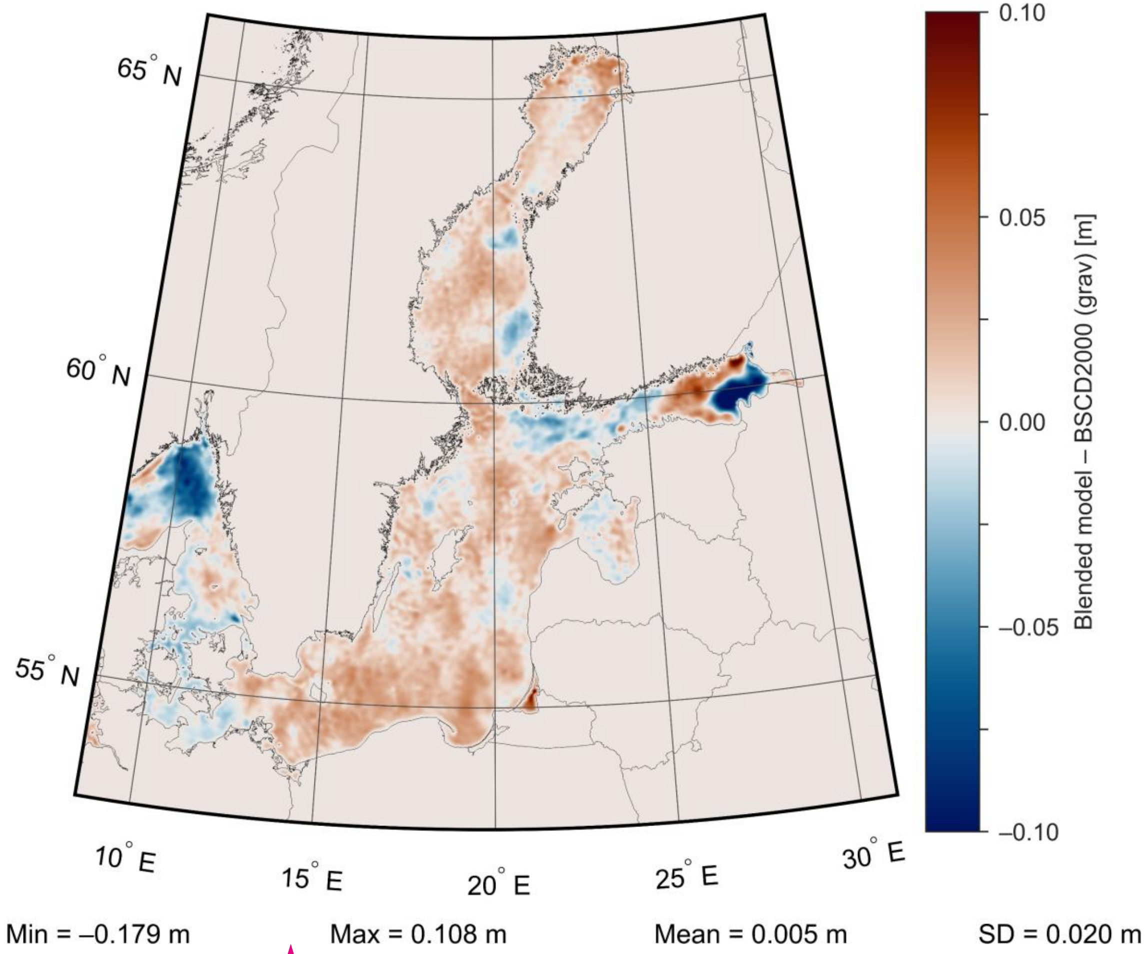


THE BLENDED MODEL

- The final blended model is determined

$$\zeta_{blend} = \zeta_{BSCD2000}^{fitted} \cdot w + \hat{\zeta}_{SWOT}^{fitted} \cdot (1 - w)$$





Statistics refer only to the marine portion of discrepancies

THANK YOU FOR YOUR ATTENTION!