FINALISING SURVEYS FOR THE BALTIC MOTORWAYS OF THE SEA

Federal Agency for Cartography and Geodesy

GEOID BASED CHART DATUMS

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NKG SUMMER SCHOOL "FROM STRUVE TO THE SPACE" IN TARTU, AUGUST 25-29, 2025

LANTMÄTERIET

*) *Part 2 of this presentation to a large extent based on Schwabe et al. (2023b) "Documentation of the BSCD2000 height reference grid"*

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INTRODUCTION – BALTIC SEA CHART DATUM 2000 (BSCD2000)

- The Baltic Sea is a shallow, non-tidal area with dense marine traffic
- The Baltic Sea Hydrographic Commission (BSHC) has recently introduced the Baltic Sea Chart Datum 2000 (BSCD2000) as new common chart datum for hydrographic surveying, hydrographic engineering, nautical charts, and publications in the Baltic Sea
- What is new here is that BSCD2000 is a geodetic height system that uses an equipotential surface (geoid) as zero level
- The purpose of this lecture is to
  - Explain how BSCD2000 is defined
  - Describe how the has been realised in practice, including the work with the BSCD2000 height transformation grid
  - Discuss how it relates to old chart datums and national height systems (on land)
- In order to do this we also need to explain some basics in physical geodesy related to the geoid and heights/depths

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  - National height systems and national geoid models (national height transformation grids) (Swedish example: RH 2000 and SWEN17_RH2000)
- Part 2 – Development and implementation of BSCD2000
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  - Definition and realisation of BSCD2000
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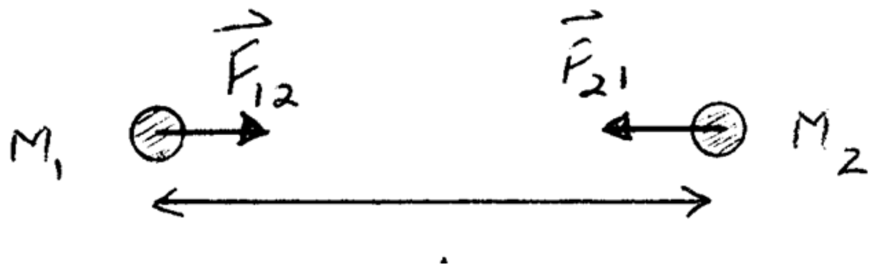
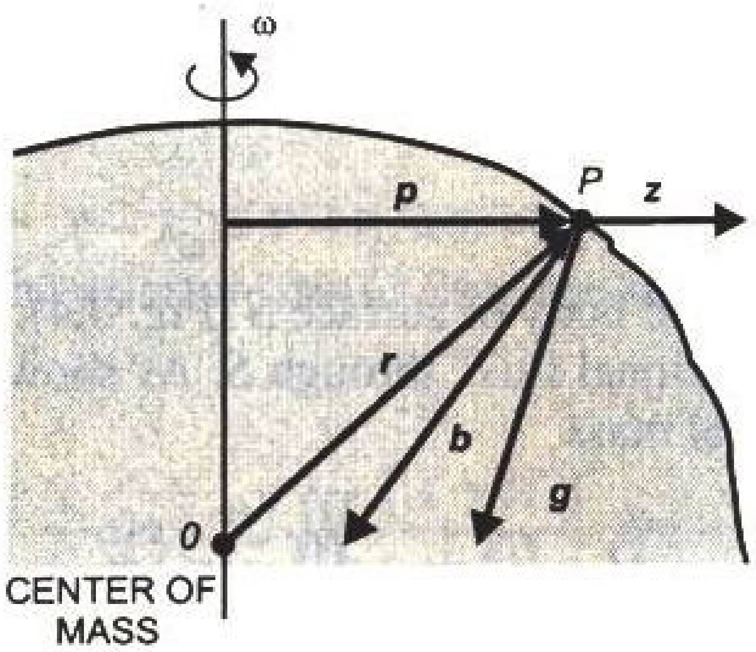
  - Computation of the final BSCD2000 height transformation grid
  - Final words

PART 1 – BASICS IN PHYSICAL GEODESY RELATED TO THE GEOID AND HEIGHTS

WHAT IS GRAVITY?





- Newton's law of gravitation:

$$F_{12} = F_{21} = G\frac{M_1 M_2}{d^2}$$

Gravity vector = gravitational force vector + centrifugal force vector

$$\mathbf{g} = \mathbf{b} + \mathbf{z}$$

- In Geodesy we study the force affecting a unit mass, i.e. acceleration (cf. Newton's 2nd law, F = m ·a). This means that the magnitude of gravitation becomes

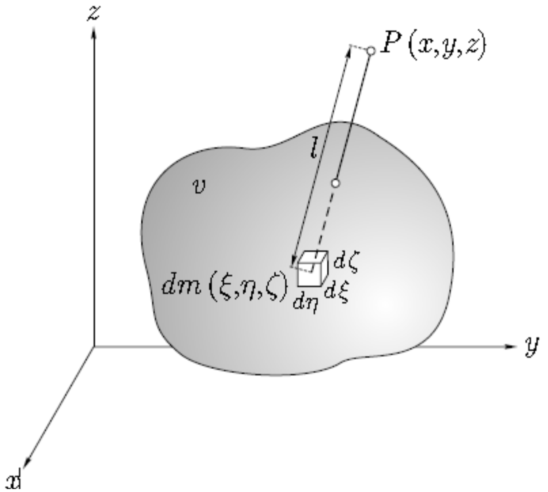
$$b = G\frac{m}{d^2}$$

- By summing (integrating) the vector contributions generated by all masses, we get the gravity vector **g**

THE GRAVITY POTENTIAL (NEWTON'S INTEGRAL)

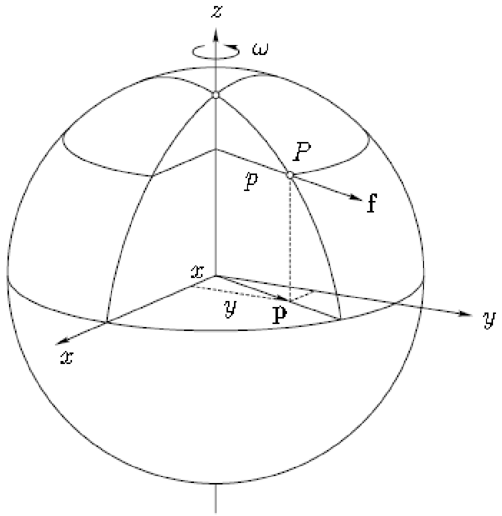
Gravitational, centrifugal and gravity potentials

$$W = V + \Phi = G\iiint_v \frac{\rho}{l}\,dv + \frac{1}{2}\omega^2\left(x^2 + y^2\right)$$



W	Gravity potental
V	Gravitational potential
G	Newton's gravitational constant
v	Density of the Earth's masses
l	Distance between the computation point and the mass element
Φ	Centrifugal potential
ω	Earth's rotational velocity

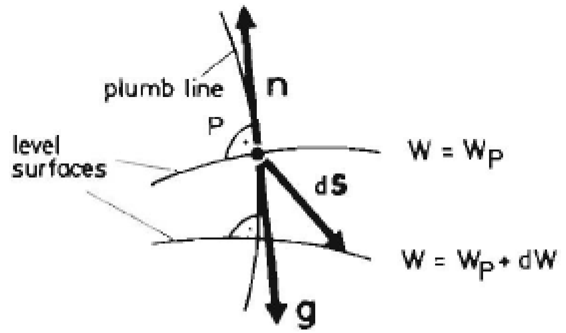
$p = \sqrt{x^2 + y^2}$ Distance from the rotation axis



GRAVITY VECTOR, GRAVITY, EQUIPOTENTIAL SURFACES AND PLUMB LINES

The gravity vector **g** (in Cartesian coordinates) and gravity g

$$\mathbf{g} = \left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}, \frac{\partial W}{\partial z} \right)^T \qquad g = |\mathbf{g}| = \left| \frac{\partial W}{\partial n} \right|$$



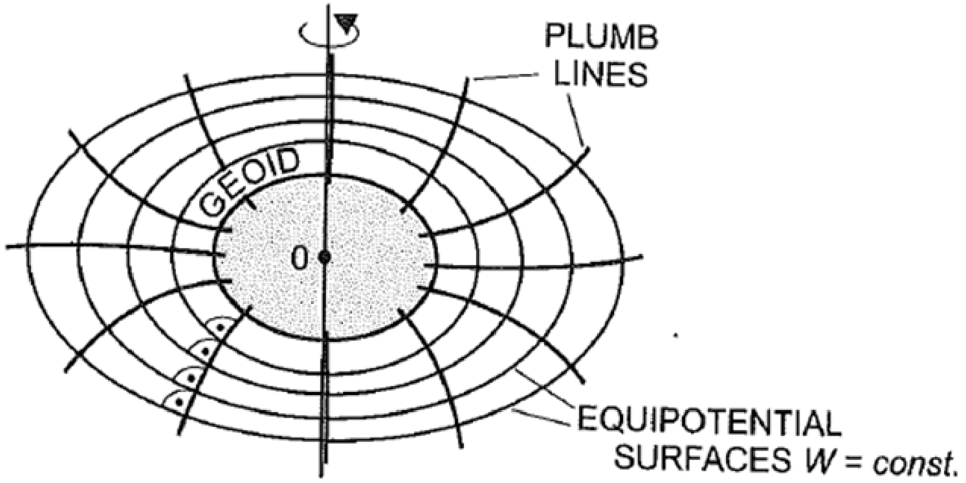
Equipotential surfaces (or level surfaces)

Surfaces where $W = const$

$$dW = \mathbf{g} \cdot d\mathbf{s} = g \cdot ds \cdot \cos(\mathbf{g}, d\mathbf{s}) = 0 \text{ for d}\mathbf{s} \text{ along a level surface}$$

Thus the gravity vector **g** is perpendicular to the equipotential surface through the point.

Plumb lines are lines to which **g** is tangent at every point. They are perpendicular to the equipotential surfaces.



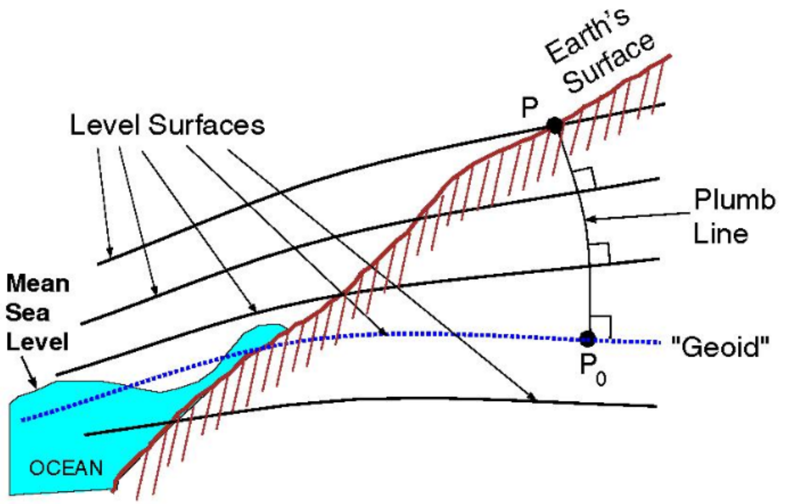
THE GEOID

- Very important reference surface used as zero level in geodetic height systems

- The geoid was first defined as the ”equipotential surface of the earth’s gravity field coinciding with the mean sea level of the oceans” by C. F. Gauss in 1828, the name geoid was suggested by Listing in 1873.

$$W = W(\mathbf{r}) = W_0$$

- Due to variations of MSL (Mean Sea Level) both in space and time, this has to be changed to “the equipotential surface which **best fits mean sea level at a certain epoch”** (e.g. Rapp 1995)



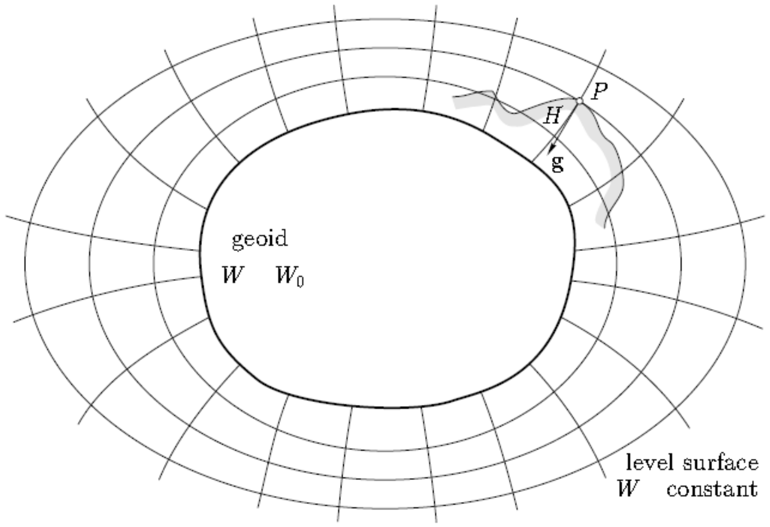
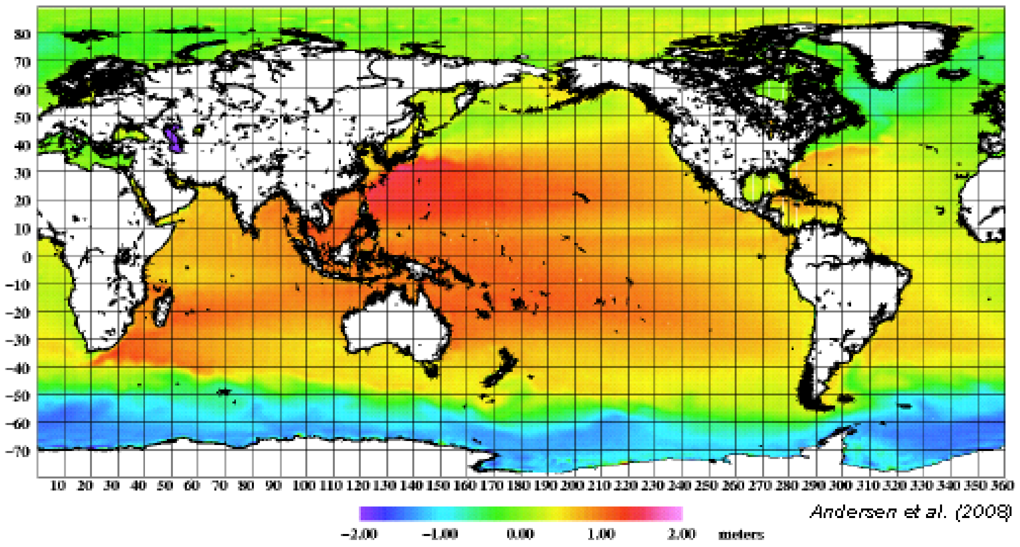


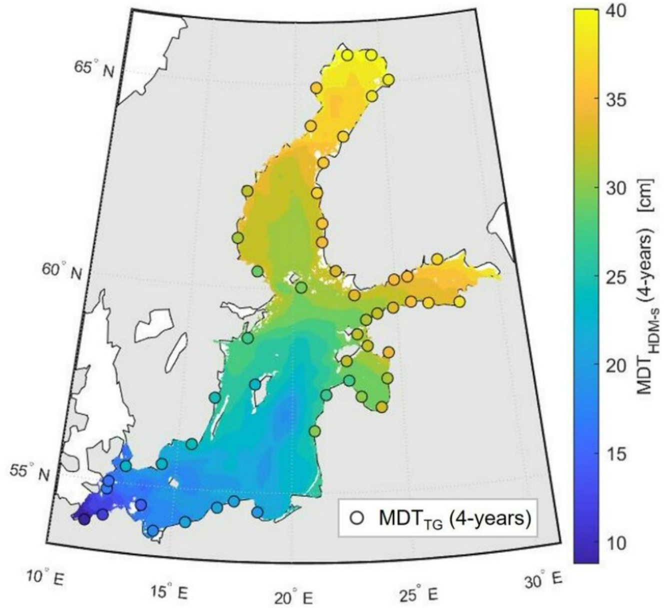
Fig. 2.2. Level surfaces and plumb lines

MEAN SEA LEVEL (MSL) AND MEAN DYNAMIC TOPOGRAPHY (MDT)

- Sea Surface Topography makes Mean Sea Level (MSL) deviate from the geoid
- The sea surface varies both in time and in space. Long term averaging gives MSL
- Mean Dynamic Topography (MDT) is MSL relative to the geoid
- The spatial variations of MSL (MDT) are mainly due to temperature variations (thermal expansion), permanent currents, and salinity differences (Baltic Sea, see below)

- EXAMPLE: DNSC07 MDT
  - DNSC08 MSS minus the EGM2008 global geoid model
  - 1 min x 1min resolution, global coverage



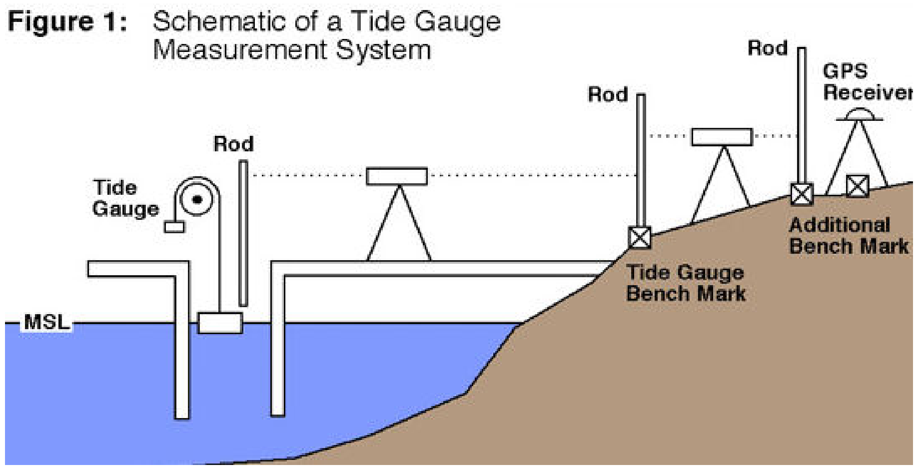


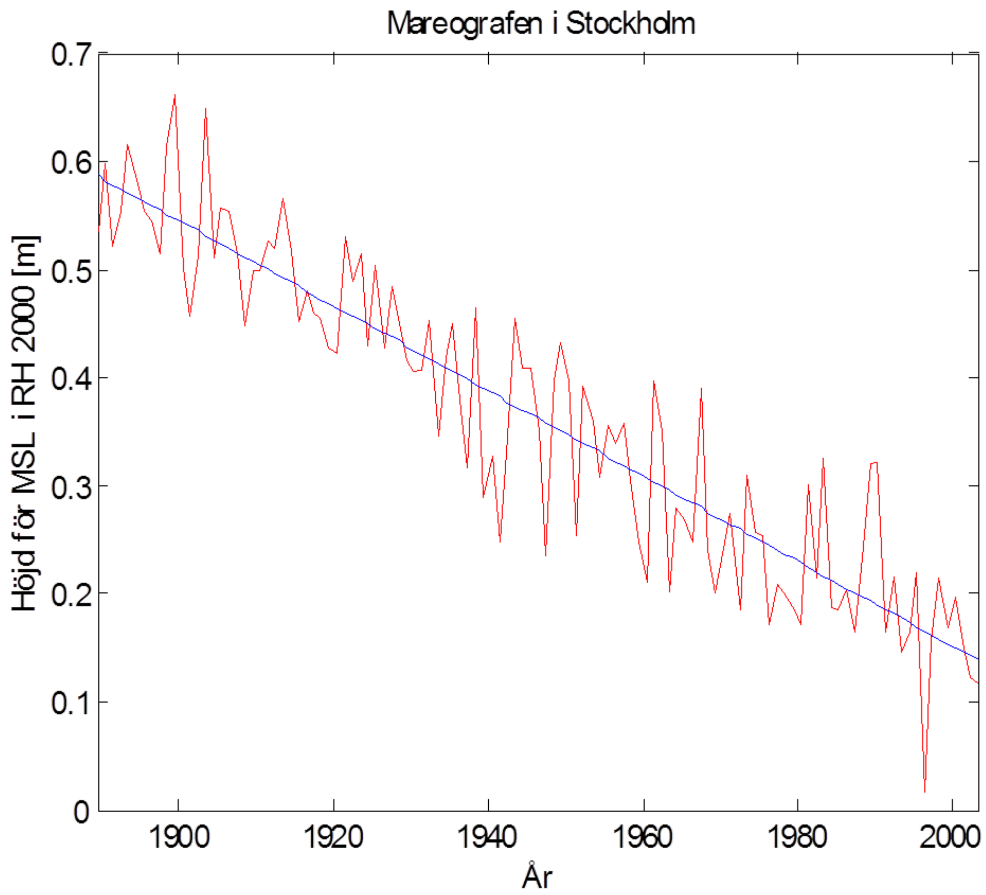
Jahanmard et al. (2022)

MSL was previously used as reference surface for the Baltic Sea charts (non-tidal area)

In tidal areas LAT (Lowest Astronomical Tide) is used

MSL IN TIDE GAUGES





- Tide gauges continuously record the height of the water level relative to a local benchmark
- Averaging over long time intervals eliminates most of the time variation. Often this is done over the period 18.6 years (the lunar nutation cycle), but also very long time spans are used in areas with postglacial land uplift

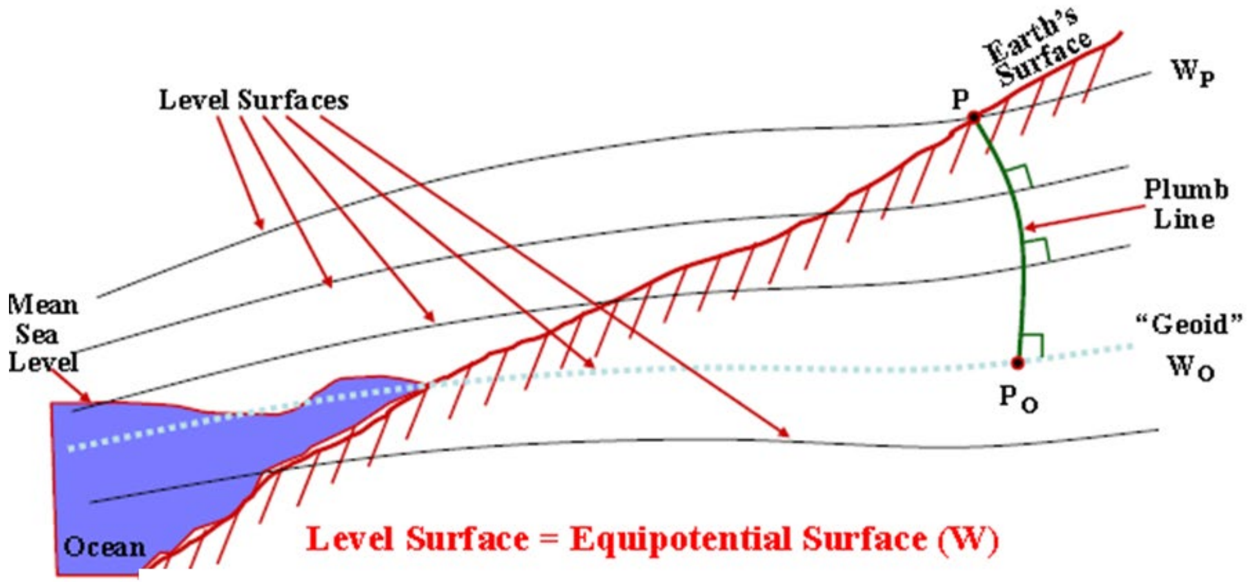
ORTHOMETRIC HEIGHTS

- The orthometric height is the distance between the surface point and the geoid along the (slightly) curved plumb line
- It can be computed from the geopotential number as follows:

$$H_P = \frac{C_P}{\bar{g}_P}$$

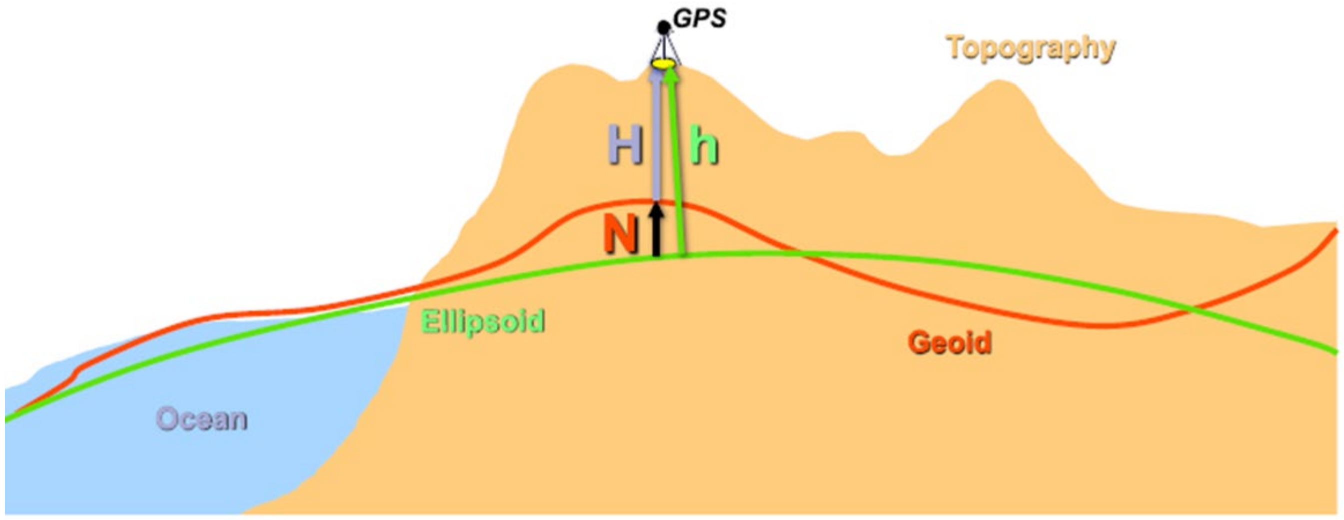
where $\bar{g}_P$ is the average value of gravity along the plumb line

- The computation of the orthometric height thus strictly requires that gravity is known inside the masses, along the plumb line
- In practice, one has to compute the average gravity from observed gravity at the surface, assuming a density model for the topographic masses.



THE GEOID AND GEOID HEIGHTS

- The <u>geoid</u> is defined as the equipotential surface of the Earth's gravity field that best coincides with the Mean Sea Level (MSL).



$$H_{(\text{from leveling})} = h_{(\text{from GPS})} - N_{(\text{height of the geoid})}$$

H = Orthometric height (height above the geoid). Can be directly measured using levelling

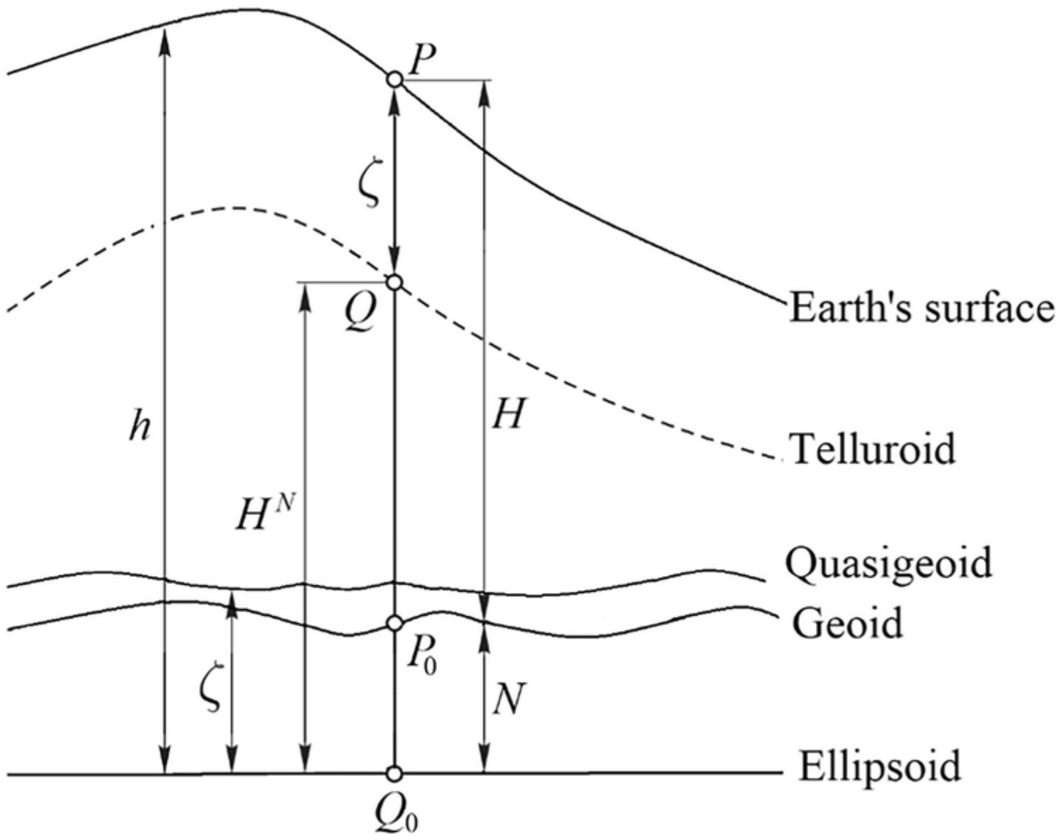
h = Ellipsoidal height in a 3-dim. reference system. Can be directly measured using GNSS

N = Geoid height. Modelled by a **geoid model**

NORMAL HEIGHTS AND THE QUASIGEOID

- The geometric distance between the ellipsoid and telluroid along the normal plumb line
- The telluroid is defined as the surface at which the normal potential U_Q is equal to real potential W_P at the Earth's surface
- The height anomaly ζ is the analogue of the geoid height N in Molodensky's Boundary Value Problem
- The normal height and height anomaly can be determined without any knowledge of the topographic density
- If the height anomaly is moved down to the ellipsoid one gets the quasigeoid; see the figure
- The quasigeoid is not an equipotential surface (this makes it a little less smooth than the geoid)
- We have the following relation to the ellipsoidal height determined by GNSS

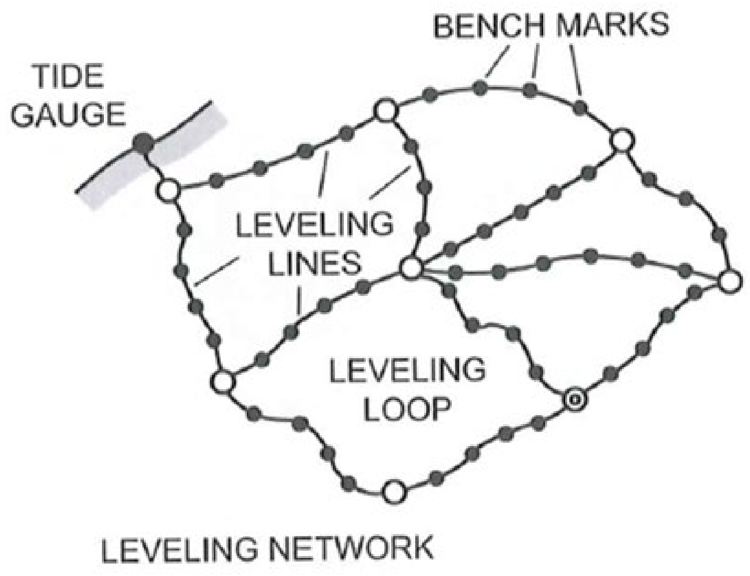
$$h_{GNSS} = \zeta + H^N$$

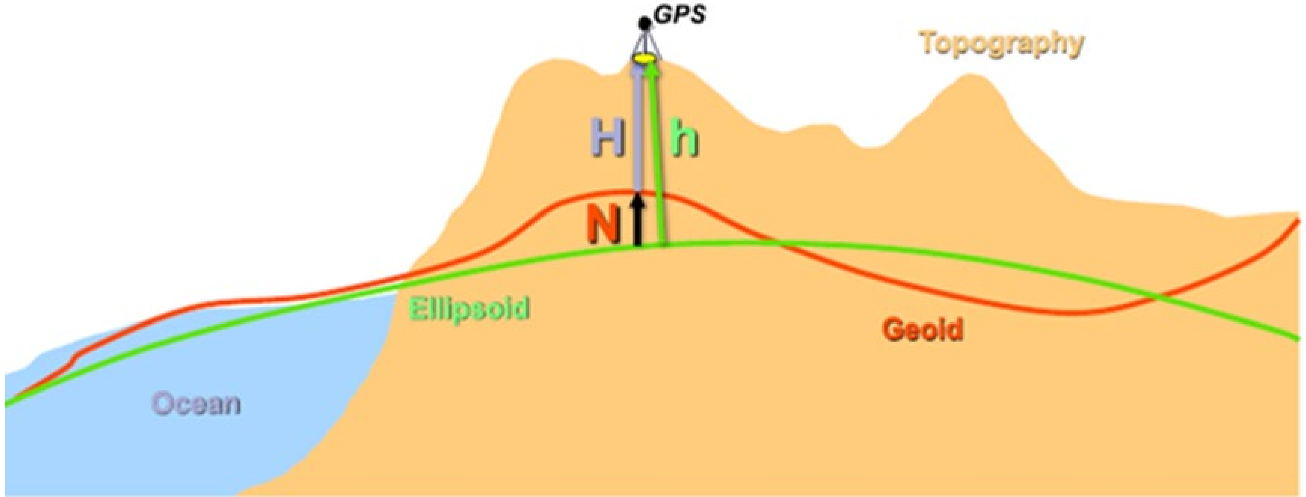


- At sea, normal height/depth is identical to orthometric height/depth

GEODETIC HEIGHT SYSTEMS

- The vertical coordinate is the orthometric height or the normal height
- Traditional levelling-based height systems are realized using precise levelling and MSL in one or more tide gauges
- Modern GNSS/geoid-based height systems can nowadays be realized using GNSS and a gravimetric geoid model





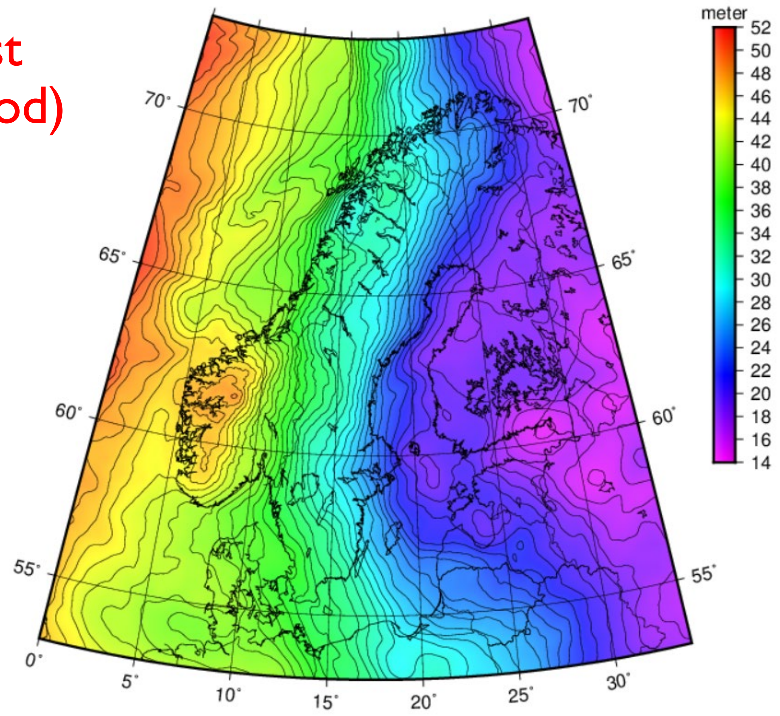
$$H_{\text{(from leveling)}} = h_{\text{(from GPS)}} - N_{\text{(height of the geoid)}}$$

GRAVIMETRIC GEOID MODELS (EXAMPLE: NKG2015)

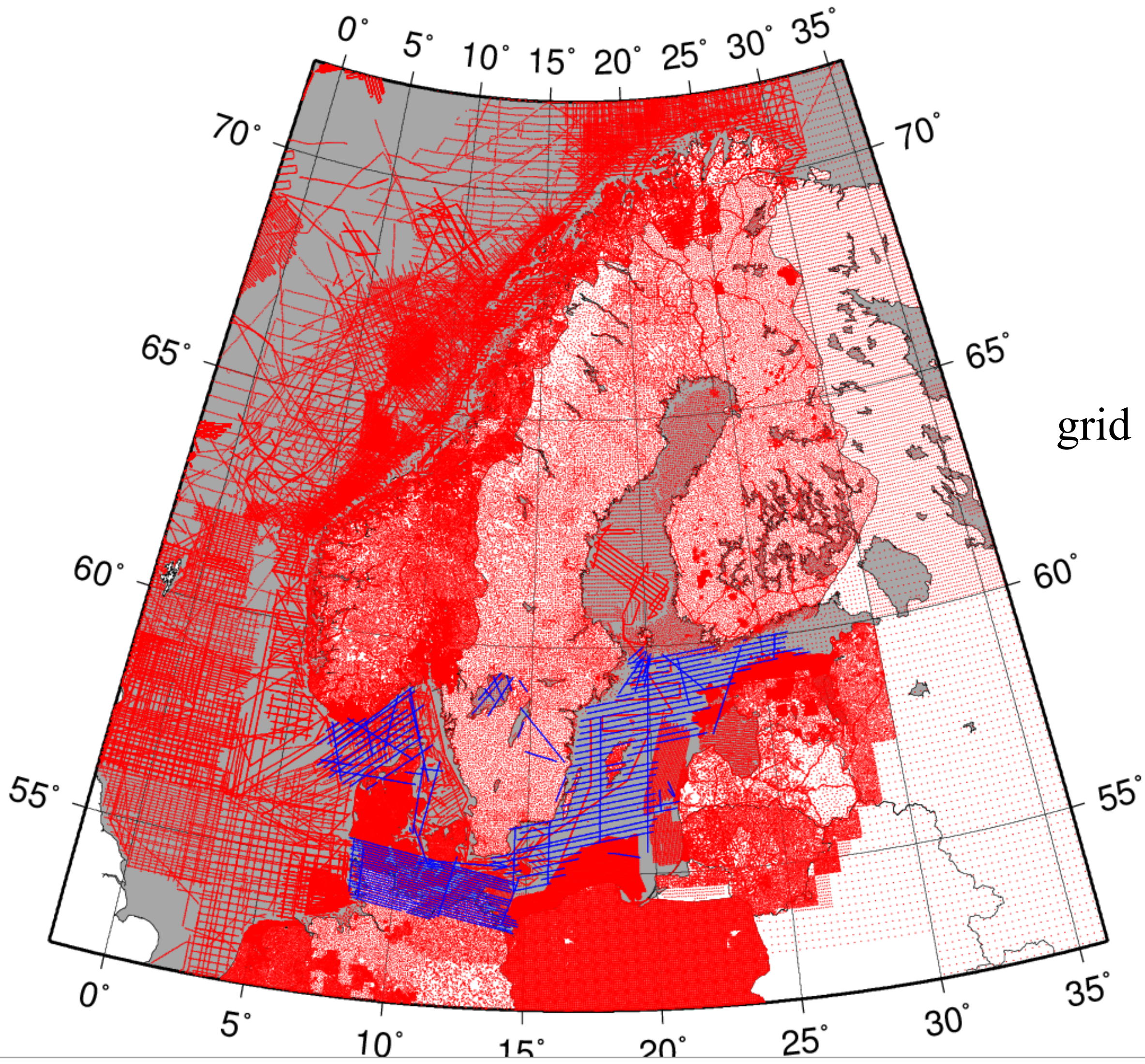
- The geoid is determined based on terrestrial gravity data in combination with a long wavelength satellite-only Earth Gravitational Model (EGM) (plus other data…)
- A global combined EGM with high resolution like EGM2008 is one type of gravimetric model
- A regional gravimetric geoid model with very high resolution is another. It can for instance be computed by modified Stokes' formula (combination of regional gravity with a satellite-only EGM) or other similar technique. Many corrections and modifications are needed
- The NKG2015 gravimetric quasigeoid model was computed by Least Squares Modification with Additive correction (LSMSA) (KTH-method) using updated gravity data from the NKG database

$$\zeta = \frac{(GM_{GGM} - GM_{\mathrm{GRS\,80}})}{r \cdot \gamma} - \frac{(W_{0,IHRS} - U_{\mathrm{GRS\,80}})}{\gamma} + \frac{R}{4\pi\gamma}\iint S^M(\psi)\Delta g\, d\sigma$$

$$+\frac{R}{2\gamma}\sum_{n=2}^{M}\left(s_n + Q_n^M\right)\Delta g_n^{EGM} + \zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL}$$



NKG GRAVITY DATABASE



- Gravity data are needed from a larger overlapping area than the gravimetric geoid model => We always need gravity data from our neighbours
- The Nordic and Baltic countries share their gravity data for NKG work and keeps the NKG gravity database
- The NKG database is managed by DTU Space in Lyngby outside Copenhagen
- It was created in the 1980-ies and was thoroughly updated and improved in the NKG2015 geoid project
- It is continuously updated in other projects (FAMOS/BSCD2000, NKG2026…)
- It is very important that the NKG database survives in the future! ☺

NATIONAL HEIGHT SYSTEMS AND NATIONAL GEOID MODELS (HEIGHT TRANSFORMATION GRIDS)

- A national (geodetic) height system is needed to standardise measurement and handling of heights in a country
- To make height determination by GNSS possible, a national (quasi) geoid model (or national height transformation grid) is required

$$H_{Nat.\,height\,system} = h_{GNSS} - N_{Nat.\,(quasi)geoid\,model}$$

- Height determination by GNSS and a national geoid model is a very efficient way to determine heights in a national height system
- For a traditional levelling-based height system, a national geoid model is typically determined by fitting a gravimetric geoid model to pointwise GNSS/levelling observations obtained as

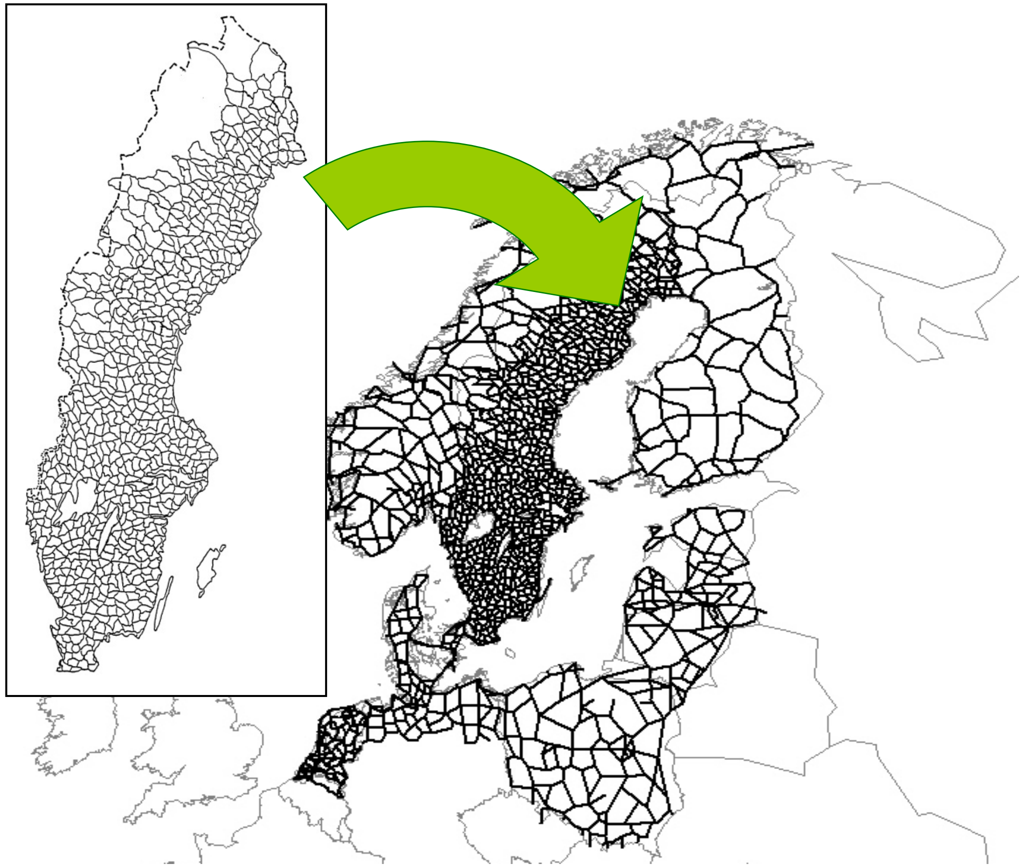
$$N_{GNSS/LEV} = h_{GNSS} - H_{Levelling}$$

- An interpolated residual surface is usually also needed to better adapt the gravimetric model to the national height system (realised by the heights of the benchmarks)

EXAMPLE: SWEDISH NATIONAL HEIGHT SYSTEM RH 2000

- The third precise levelling 1979–2003
- 100 km long loops
- About 1 km between the benchmarks
- Around 50 000 benchmarks
- Connection points for local networks planned from the beginning
- About 50 000 km double run levelling (totally 122 582 km levelling)

DEFINITION AND COMPUTATION OF RH 2000



Baltic Levelling Ring (BLR)

- Defined in accordance with the European Vertical Reference System (EVRS)
  - Zero level is the equipotential surface in the level of Normaal Amsterdams Peil (NAP)
  - Normal heights
  - Zero system for the permanent tide.
- Conversion to geopotential numbers using observed gravity
- Land uplift model: NKG2005LU
- Epoch: 2000.0
- Computation made by adjusting the whole Baltic Levelling Ring (BLR) in cooperation with the other Nordic countries (NKG) and EUREF
- Finished in 2005

EUROPEAN VERTICAL REFERENCE SYSTEM (EVRS)

(1) The vertical datum is defined as the equipotential surface for which the Earth gravity field potential is constant:

$$W_0 = W_{0E} = const.$$

and which is in the level of the Normaal Amsterdams Peil.

(2) The unit of length of the EVRS is the meter (SI). The unit of time is second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modelling.

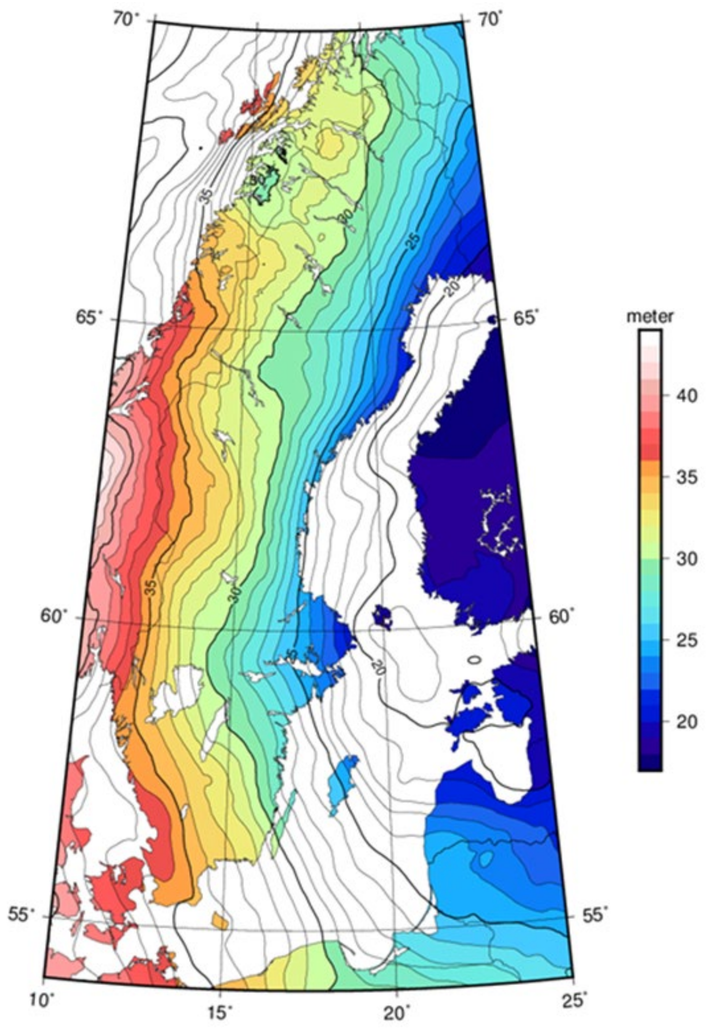
(3) The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points P, and the potential W_{0E} of the EVRS conventional zero level. The potential difference $-\Delta W_P$ is also designated as the geopotential number c_P

$$-\Delta W_P = c_P = W_{0E} - W_P$$

Normal heights are equivalent with geopotential numbers, provided that the reference gravity field is specified.

(4) The EVRS is a zero tidal system, in agreement with the IAG Resolutions No. 9 and 16 adopted in Hamburg in 1983 (Appendix 2).

SWEN17_RH2000, THE SWEDISH NATIONAL GEOID MODEL (HEIGHT TRANSFORMATION GRID)



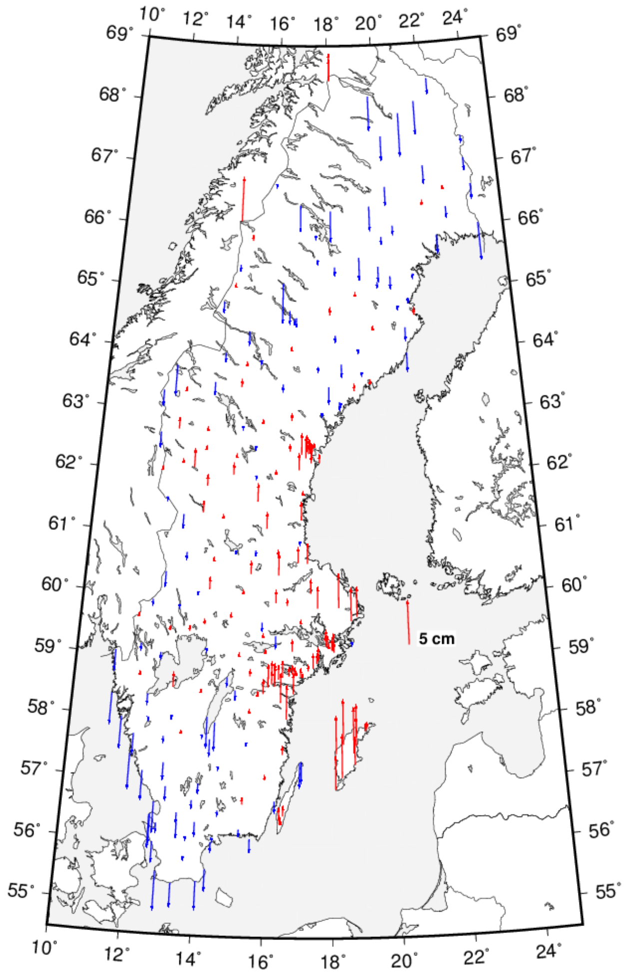
- Computed by fitting the gravimetric model NKG2015 to the Swedish reference systems SWEREF 99 and RH 2000

$$H_{\mathrm{RH\ 2000}} = h_{\mathrm{SWEREF\ 99}} - N_{\mathrm{SWEN17\_RH2000}}$$

- This is made by using a new dataset of 250 GNSS/levelling observations over Sweden to
  - Make a 1-parameter fit/transformation (i.e., a shift)
  - Add a smooth residual surface interpolated using least squares collocation
  - (A correction for different permanent tide systems is also needed)
- The standard uncertainty (1 sigma) of SWEN17_RH2000 has been estimated to approximately 8–10 mm on land
- SWEN17_RH2000 is strictly a quasigeoid model

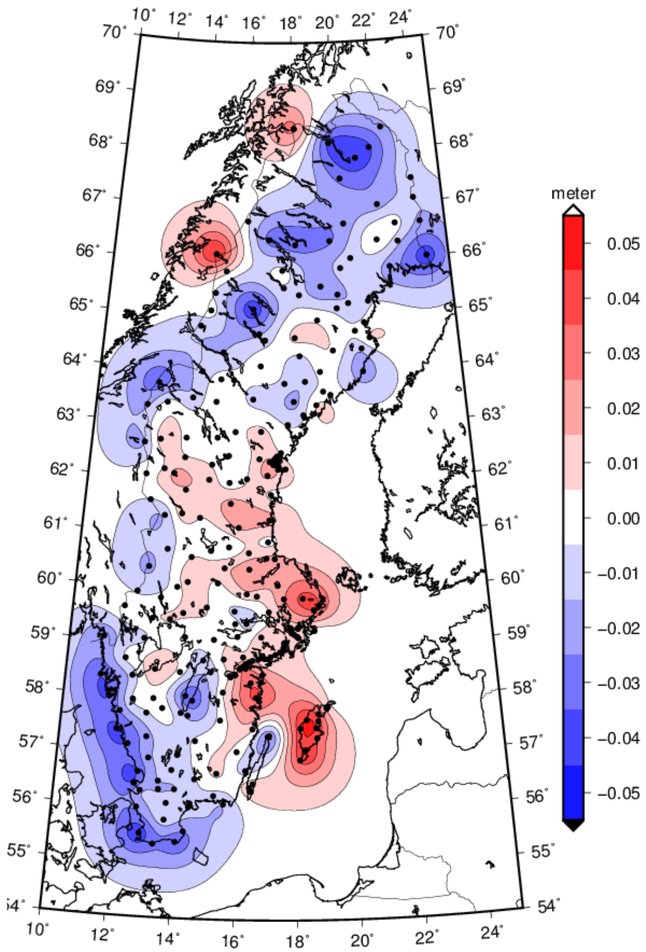
SWEN17_RH2000

GNSS/levelling residuals for NKG2015

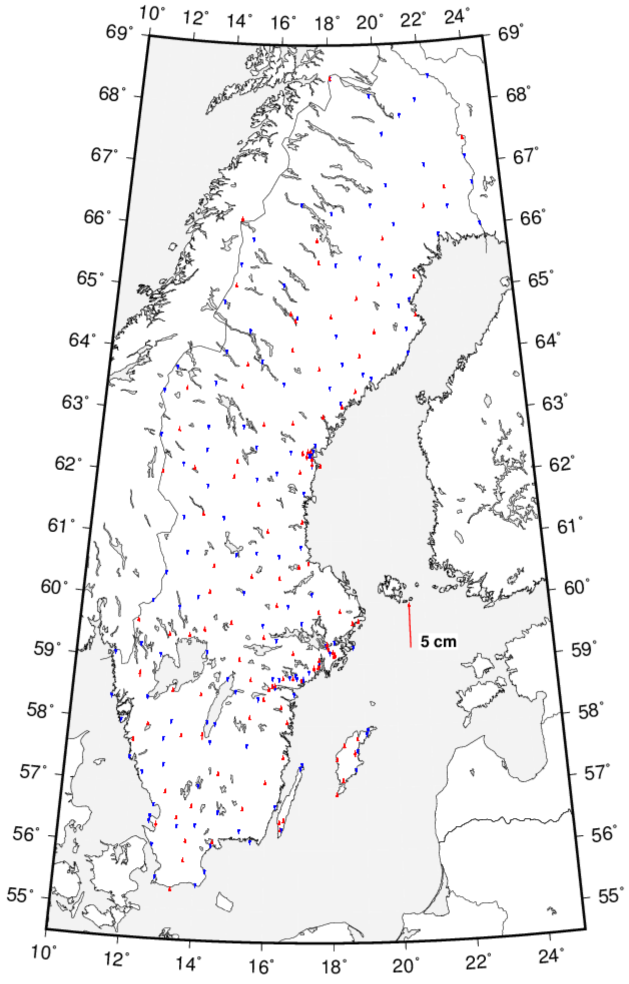


#	Min	Max	Mean	StdDev
250	-0.0487	0.0518	0.0001	0.0190

Smooth residual surface (Least Squares Collocation)

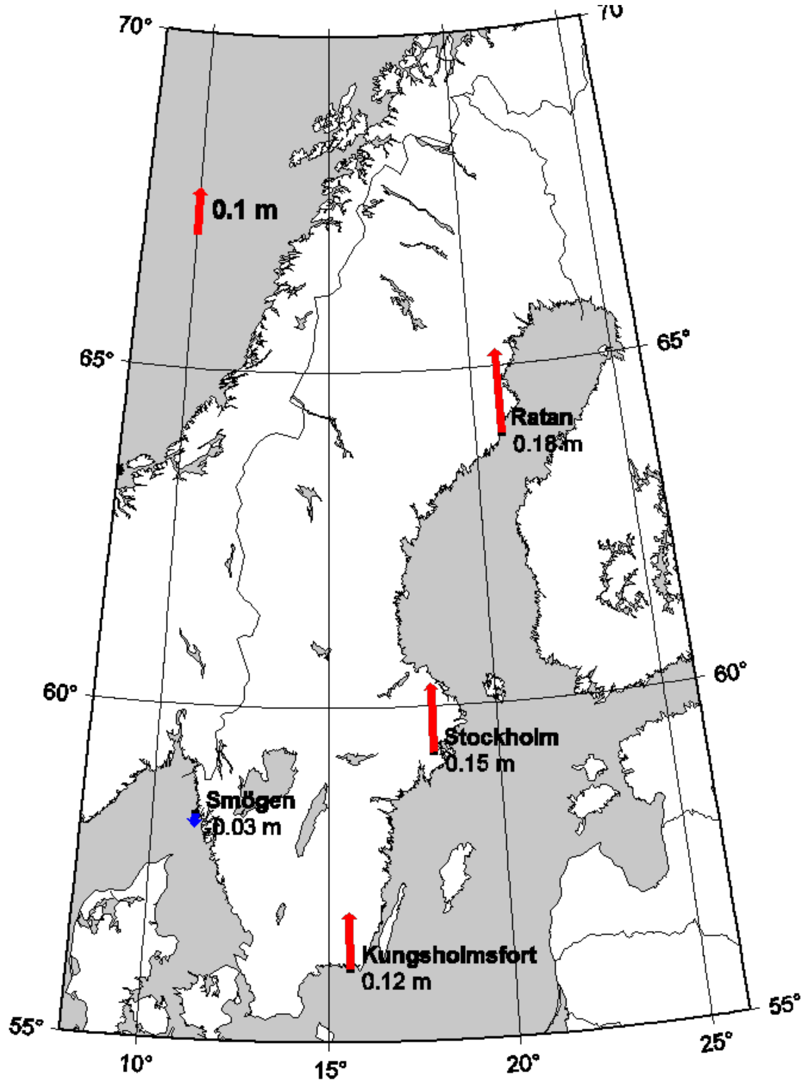


GNSS/levelling residuals for SWEN17_RH2000

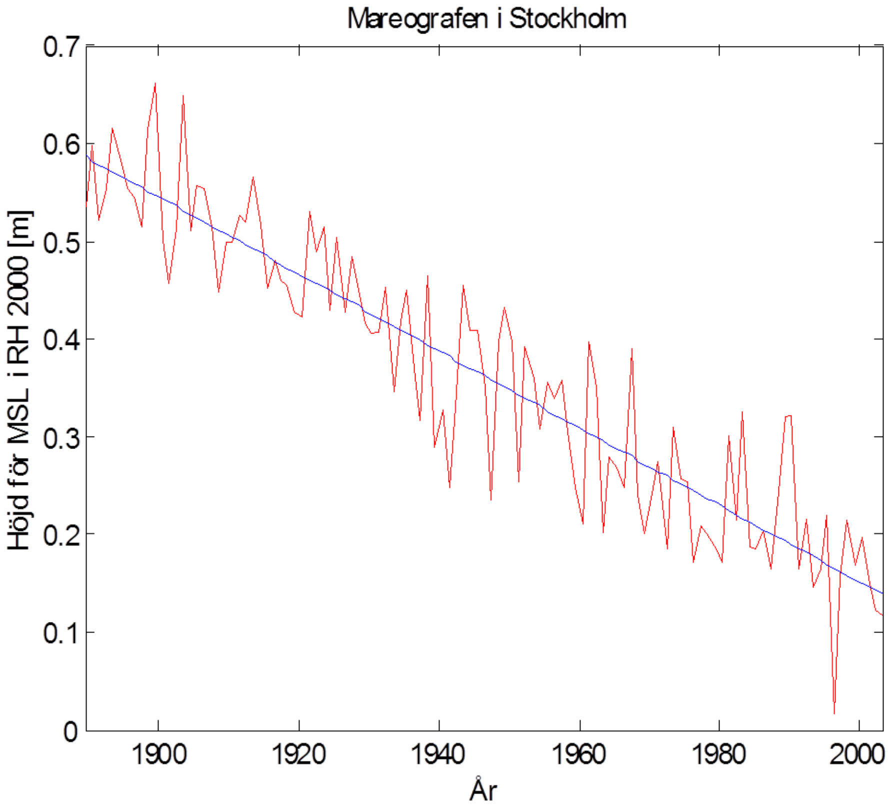


#	Min	Max	Mean	StdDev
250	-0.0075	0.0093	0.0000	0.0030

MSL IN RH 2000 FOR THE EPOCH 2000.0

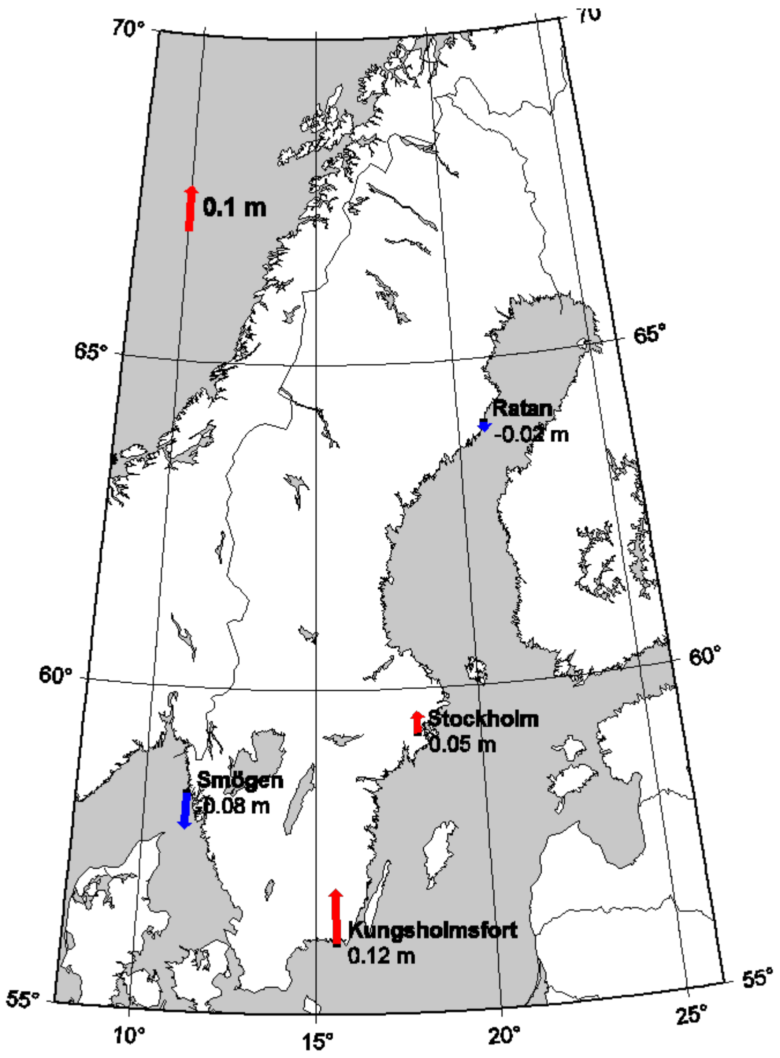


- Tide gauge data 1892–2003
- MSL computed using linear regression

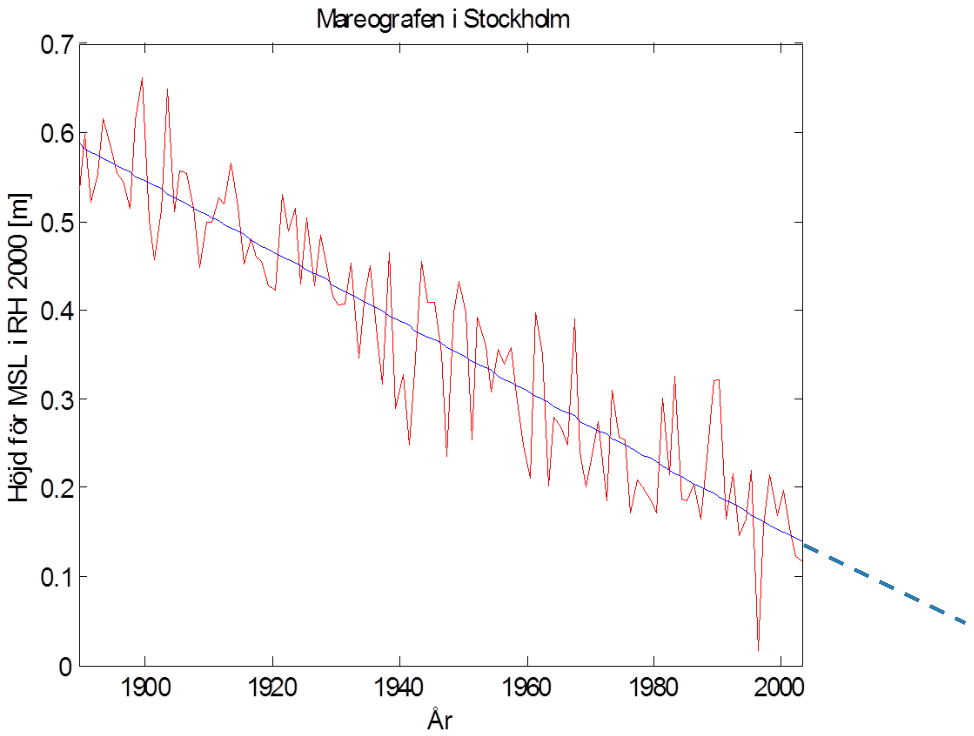


From Ågren and Svensson (2007)

MSL IN RH 2000 IN THE EPOCH 2026.0



- MSL predicted by continuing along the regression line
- This study was made when finalising RH 2000 back in 2005 (Ågren and Svensson, 2007)
- More recent figures are presented in Part 2 below



From Ågren and Svensson (2007)

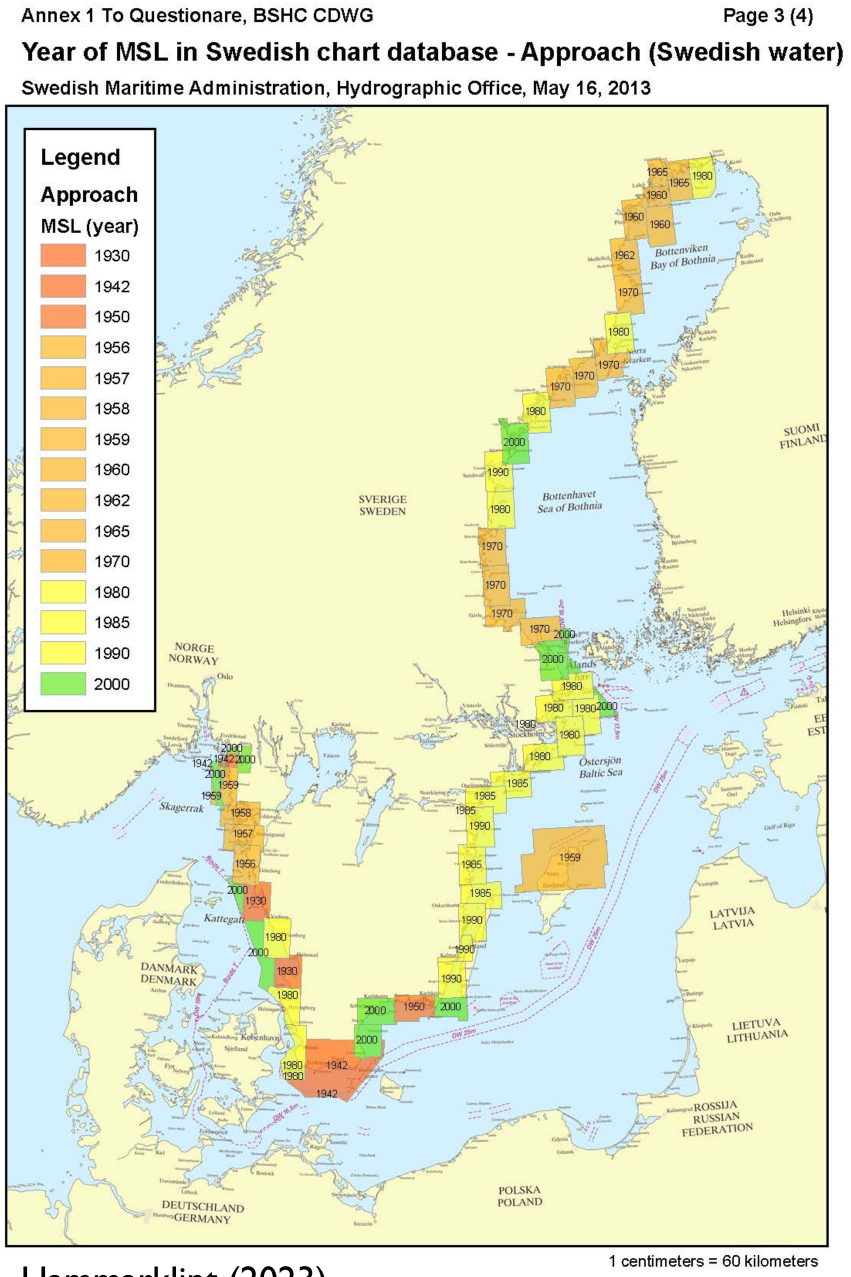
PART 2 – DEVELOPMENT AND IMPLEMENTATION OF BSCD2000

BALTIC SEA CHART DATUM 2000 (BSCD2000)

- The Baltic Sea is a shallow, non-tidal area with dense marine traffic
- The Baltic Sea Hydrographic Commission (BSHC) has recently introduced the Baltic Sea Chart Datum 2000 (BSCD2000) as new common chart datum for hydrographic surveying, hydrographic engineering, nautical charts, and publications in the Baltic Sea
- What is new here is that BSCD2000 is a geodetic height system that uses an equipotential surface (geoid) as zero level

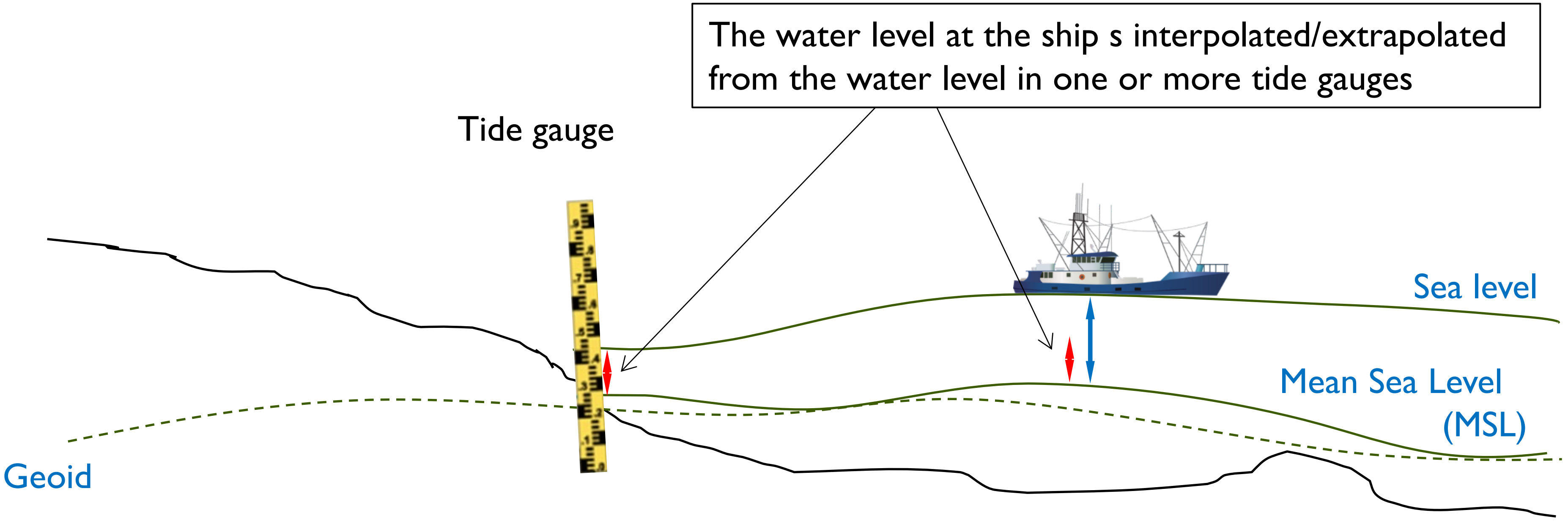
MOTIVATION BEHIND BSCD2000

- Previously, the nautical charts of the Baltic Sea were in most cases referenced to Mean Sea Level (MSL) as given by tide gauges along the coasts
- Many such tidal chart datums were introduced during the years, referring to for instance different reference epochs for the postglacial land uplift
- This multitude of tidal datums became very confusing to the mariners and resulted in a constant risk of intermixing datums, which led to unnecessarily high safety margins
- Another problem is that it is difficult to reliably determine MSL far away from the coasts with high accuracy, as required by the GNSS-based navigation and bathymetric surveying

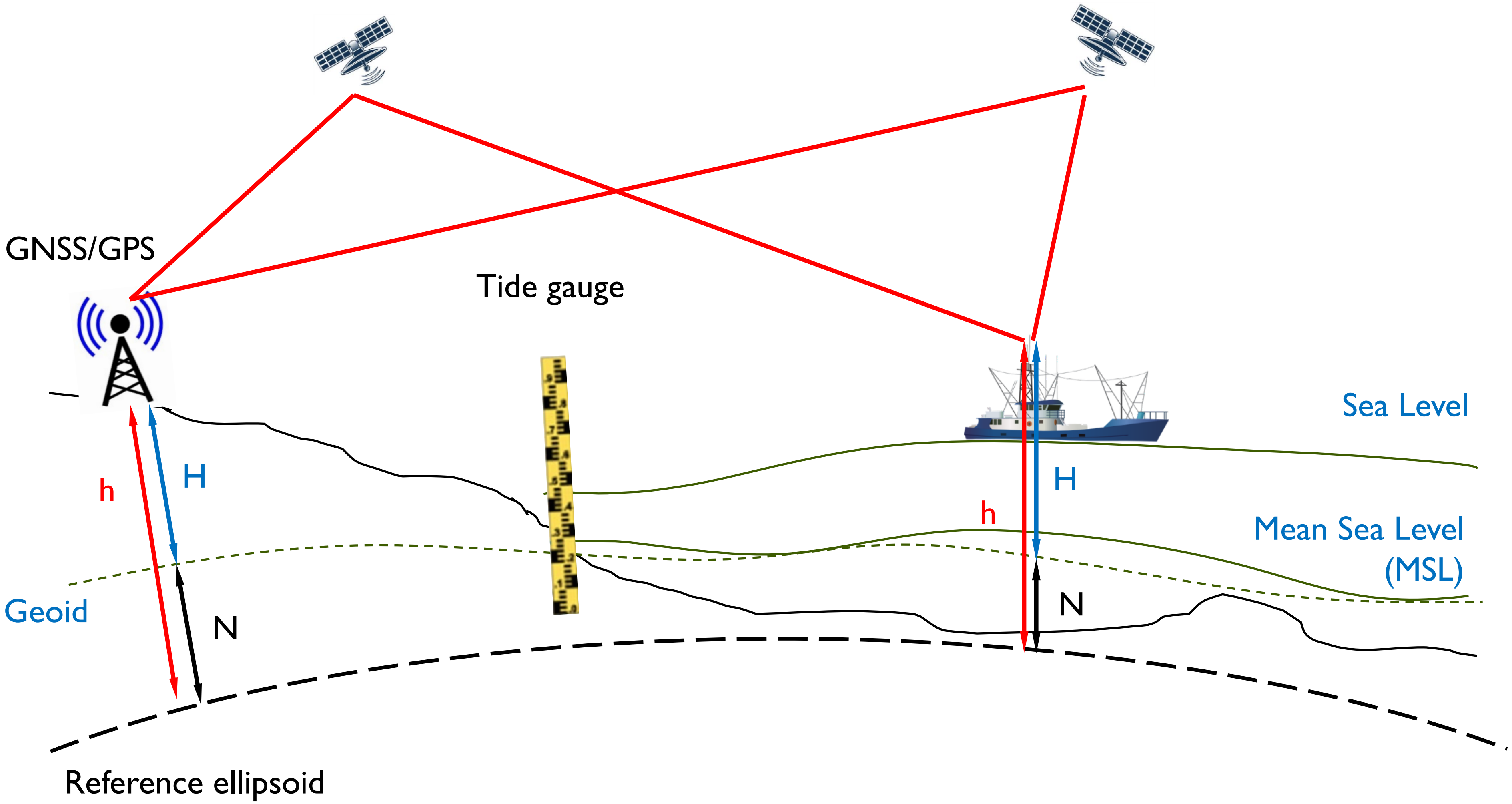


Hammarklint (2023)

TRADITIONAL HEIGHT/DEPTH DETERMINATION USING TIDE GAUGES



HEIGHT/DEPTH DETERMINATION IN A GEODETIC HEIGHT SYSTEM USING GNSS AND A GEOID MODEL



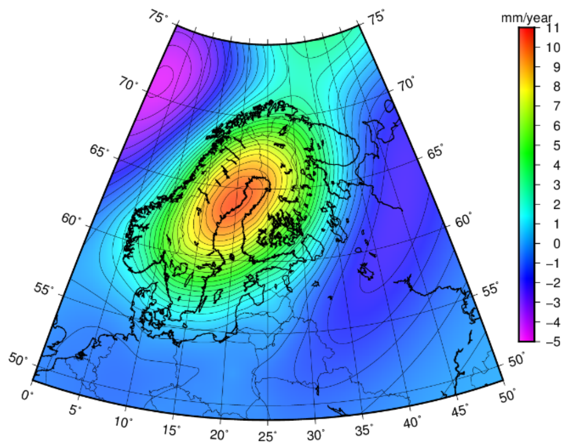
SPECIFICATION OF BSCD2000 – DEFINITION

- BSCD2000 is a geodetic height system that is defined in accordance with the European Vertical Reference System (EVRS) with land uplift epoch 2000.0
  - Zero level: The equipotential surface that agree with the NAP-level
  - Permanent tide system: Zero
  - Postglacial land-uplift epoch: 2000.0 (considered in areas with significant land uplift)

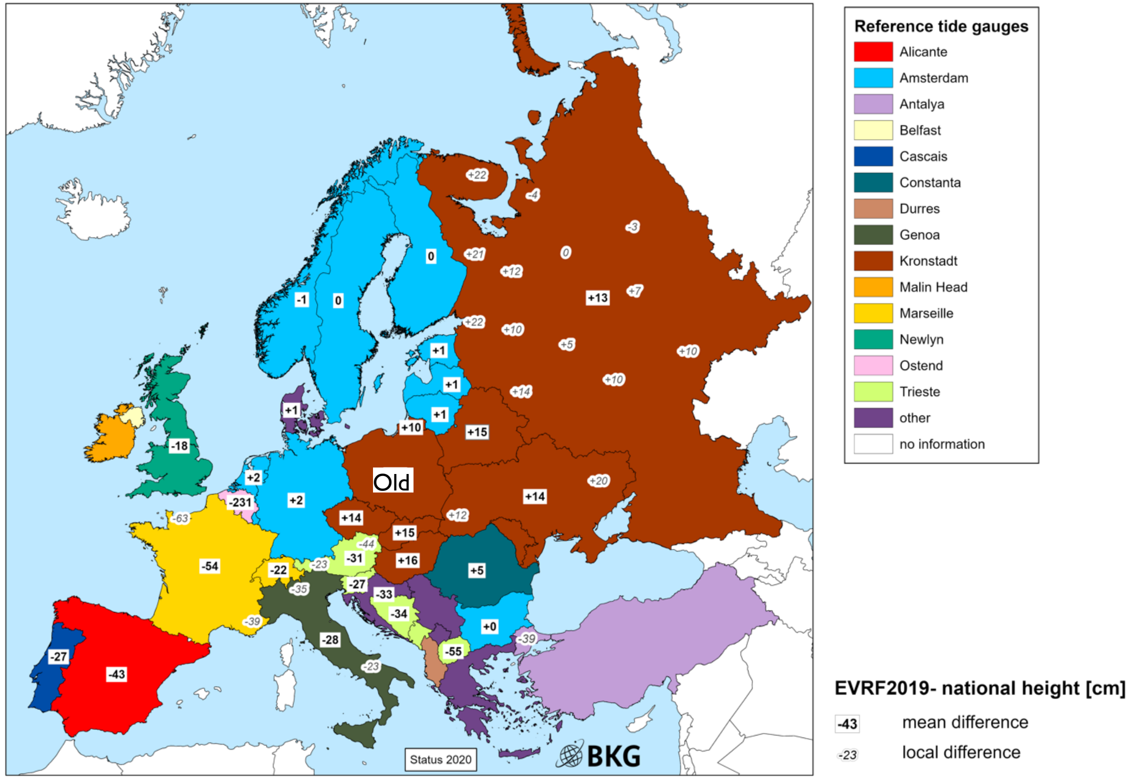
  ✓ All national height frames around the Baltic Sea except Danish DVR90 and German DHHN2016 fulfil this definition

  ✓ DVR90 and DHHN2016 use different tide systems (tide-free and mean, respectively), but the corresponding frames agree well with the "EVRS level" anyhow and are allowed by the BSCD2000 specification; se the next slide

- For GNSS/geoid-based realisation, BSCD2000 is defined in accordance with ETRS89 for the spatial 3D coordinates



Postglacial land uplift model: NKG2016LU



Ågren et al. (2023) *Specification of the Baltic Sea Chart Datum 2000* + Hammarklint (2023)

SPECIFICATION OF BSCD2000 – REALISATION

- The realisation of BSCD2000 is specified to make use of the existing national geodetic infrastructure, i.e. the official national vertical and spatial (3D) reference frames and the corresponding services
- It is the goal that the geodetic infrastructure for the realization of BSCD2000 shall provide a standard uncertainty better than 5 cm over the whole Baltic Sea including the costal zones
- On land and along the coasts, BSCD2000 is realized by the official national height systems that are all levelling-based
- Offshore, BSCD2000 is realised based on
  - the national GNSS positioning services (e.g. SWEPOS, FINNREF and SAPOS),
  - the corresponding official national ETRS89 realisations (e.g. SWEREF 99, EUREF-FIN and ETRS89/DREF91) and
  - a consistent BSCD2000 height transformation grid based primarily on a gravimetric quasigeoid model
- The BSCD2000 height transformation grid shall take into account existing (small) differences in the definition and realisation of the different reference frames
- This means that we get
  - a levelling-based realisation on land and along the coasts
  - a GNSS/geoid-based realisation at sea
  - a smooth transition in between, out to around 10 km from the coast

Ågren et al. (2023), Specification of the Baltic Sea Chart Datum 2000

CURRENT NATIONAL GEODETIC REFERENCE SYSTEM REALISATIONS

	Country	National height realization	National ETRS89 realization	National height reference surface
DE	Germany	DHHN2016	ETRS89/DREF91/R2025	GCG2016
DK	Denmark	DVR90	EUREF-DK94	DVR90(2023)
EE	Estonia	EH2000	EUREF-EST97	EST-GEOID2017
FI	Finland	N2000	EUREF-FIN	FIN2023N2000
LT	Lithuania	LAS07	LKS-94 (EUREF-NKG-2003)	LIT20G
LV	Latvia	LAS-2000,5	LKS-92	LV'14
NO	Norway	NN2000	EUREF89	Href2018b
PL	Poland	PL-EVRF2007-NH	PL-ETRF2000	geoid2021-PL-EVRF2007-NH
SW	Sweden	RH2000	SWEREF99	SWEN17_RH2000

Schwabe et al. (2023a, b)

IMPLEMENTATION OF BSCD2000 (1)

- For many years, the BSHC Chart Datum Working Group (CDWG) was implementing BSCD2000
- Since 2024, this task is the responsibility of the new Chart Datum, Water level and Currents Working Group (CDWCWG)

Photo: Chart Datum, Water level and Currents Working Group 2nd meeting, 25-26 March 2025, Tallinn, Estonia

IMPLEMENTATION OF BSCD2000 (2)

- IHO BSHC has approved the name and the adoption of the Baltic Sea Chart Datum 2000

- The chart datum name to be shown in paper charts is:

  Mean Sea Level (Baltic Sea Chart Datum $2000^{\text{national realization name}}$)

  Mean Sea Level (Baltic Sea Chart Datum 2000)

- It was required by BSHC to have "Mean Sea Level" here, even though this is not correct
- BSCD2000 is currently being implemented by the Baltic Sea countries in their printed and digital charts
- The tide gauges are also connected to BSCD2000

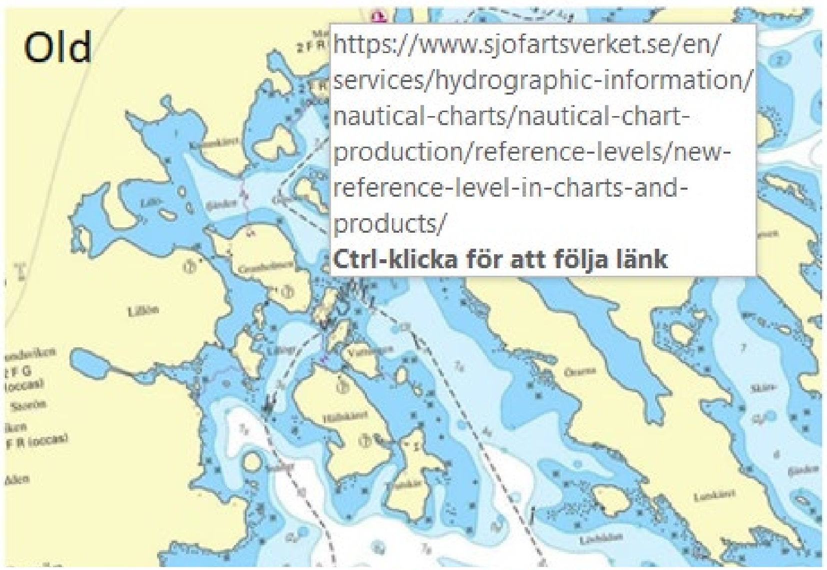
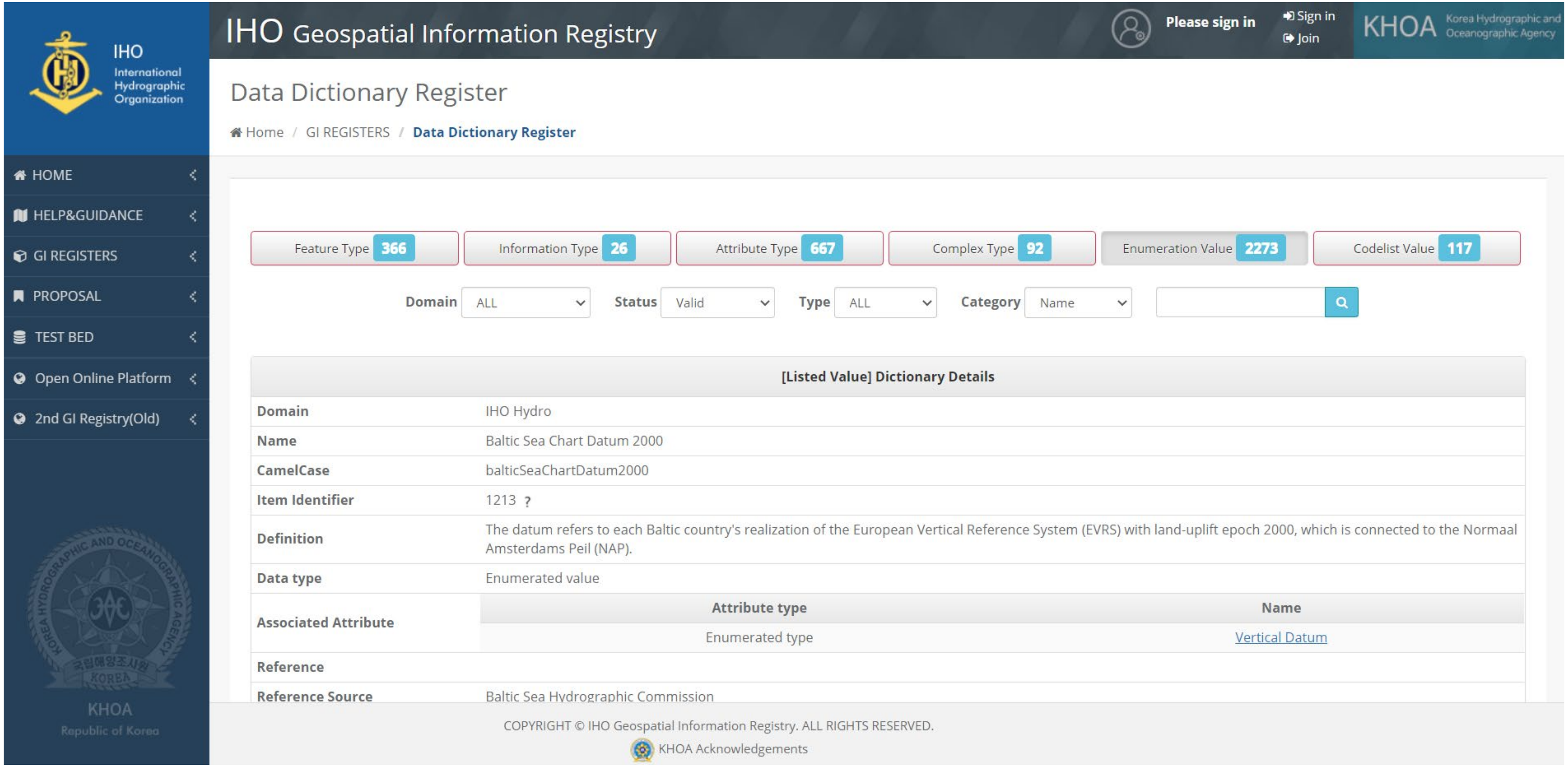


CHART DATUM: Mean Sea Level (Baltic Sea Chart Datum 2000^{RH2000})
REFERENSNIVÅ: Medelvattenyta (Baltic Sea Chart Datum 2000^{RH2000})
SYMBOLS and ABBREVIATIONS: see INT 1
BETECKNINGAR och FÖRKORTNINGAR: se KORT 1

Referensnivå

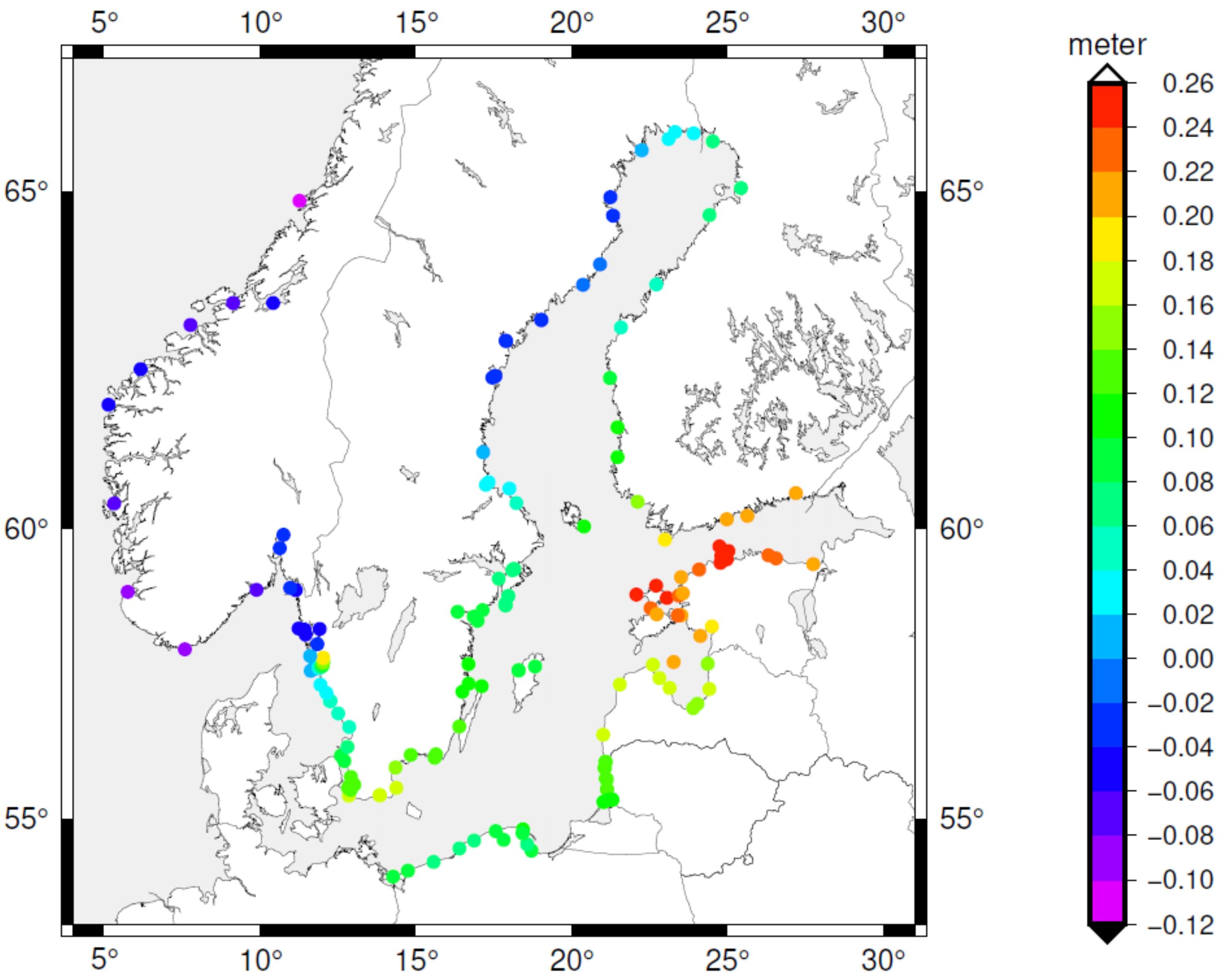
Hammarklint (2023)

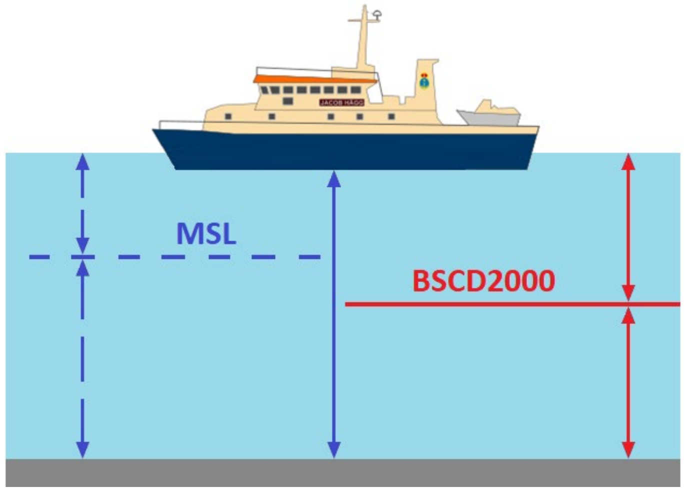
BSCD2000 IS NOW INCLUDED IN IHO GEOSPATIAL INFORMATION REGISTRY AS CHART DATUM NUMBER 44



Hammarklint (2023)

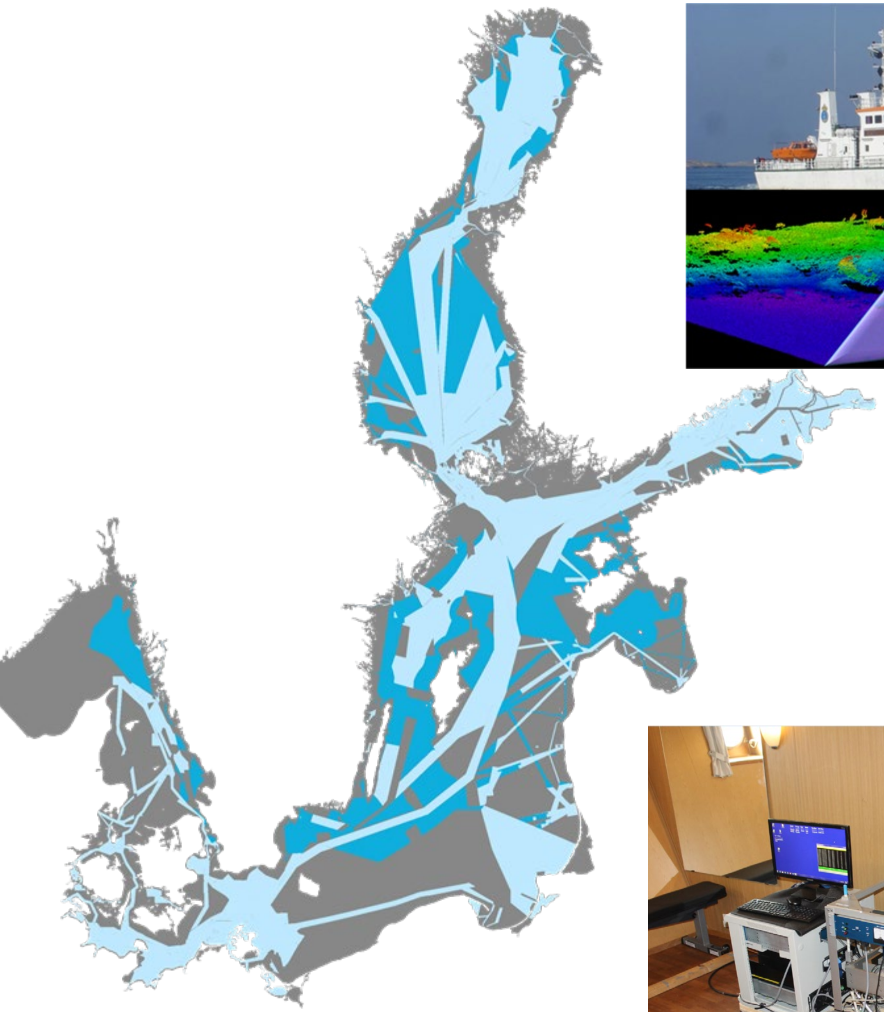
EXAMPLE: DIFFERENCE BETWEEN SOME OLD CHART DATUMS AND BSCD2000

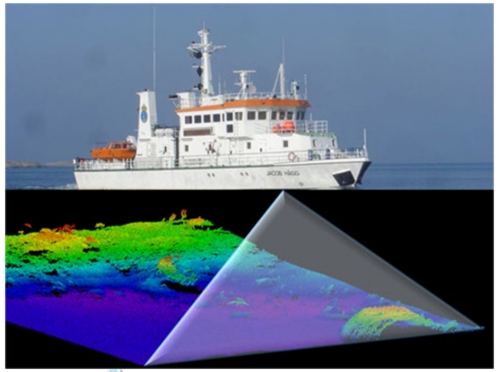


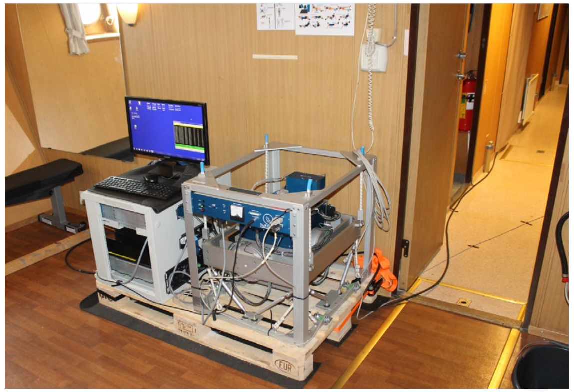


- In Sweden and Finland, the old reference levels are here (in this figure) equal to **Mean Sea Level transferred to year 2025** following the corresponding (different) national conventions
- The differences in Norway refer to the Mean Sea Level over the period 1996-2014, relative BSCD2000 (LAT–20 cm is otherwise used in Norway)
- In Estonia, Latvia and Lithuania, the Kronstadt reference level is the old chart datum
- In Poland, the local Polish Height System Amsterdam NN55 is the old chart datum

FAMOS – FINALISING SURVEYS FOR THE BALTIC MOTORWAYS OF THE SEA







- The official FAMOS project started in 2014 and ended in June 2019. It consisted of two sub-projects:
  - FAMOS Freja (2014−2016)
  - FAMOS Odin (2016−June 2019)
- The largest part of the EU project FAMOS (Activity 1) was to finalise hydrographic surveying in areas of the Baltic Sea of most interest for commercial shipping
- The major purpose of FAMOS Activity 2 was to support the introduction of BSCD2000
- The main tasks of Activity 2 were to collect marine gravity data and compute gravimetric quasigeoid models (and finally, to compute the BSCD2000 height transformation grid)
- Due to interrupted funding, Activity 2 could not be finalised in FAMOS
- Instead the work was completed in 2023 in the FAMOS finalisation action of the CDWG under the leadership of Joachim Schwabe, BKG

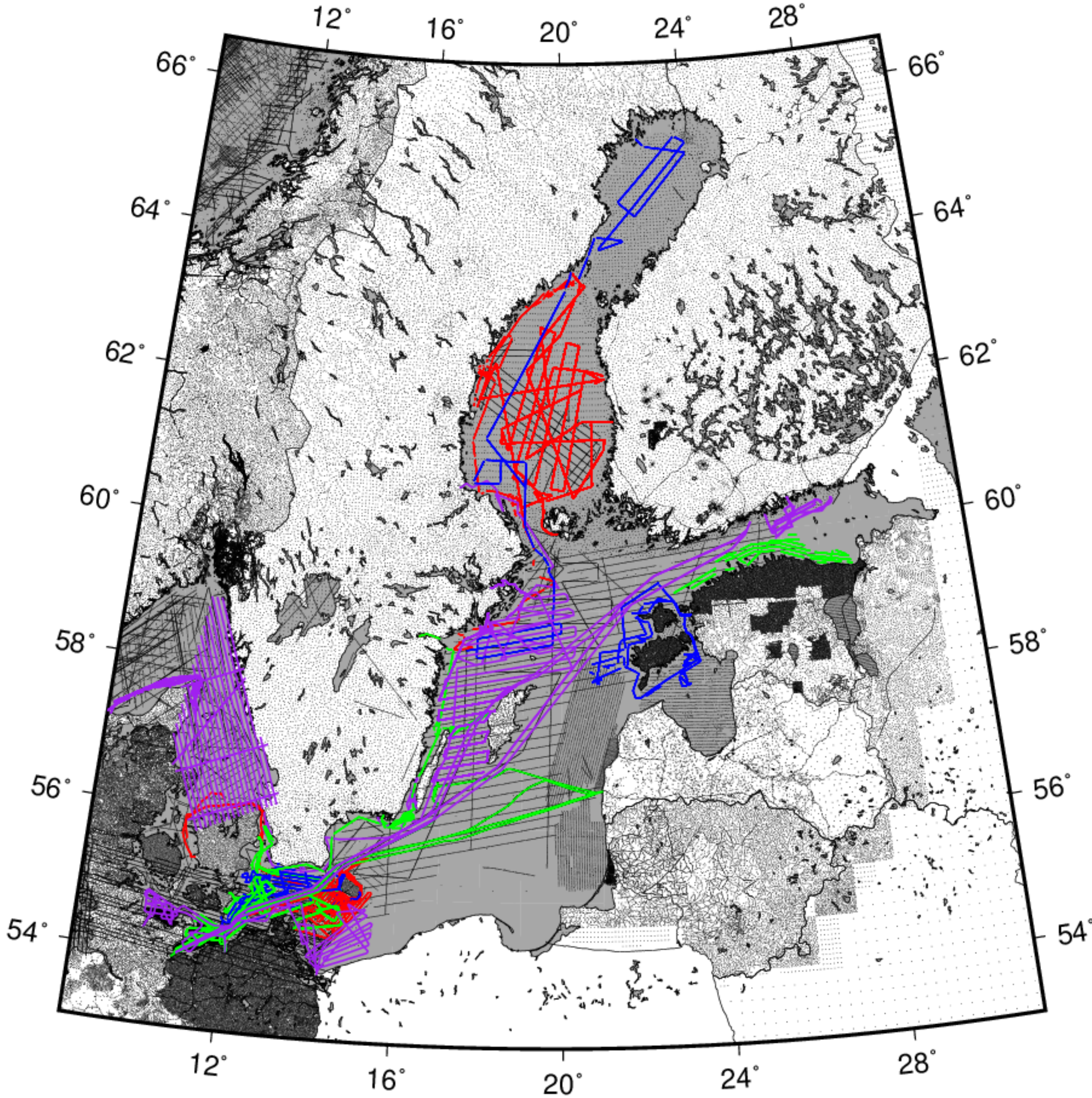
MARINE GRAVIMETRY CAMPAIGNS CONDUCTED **IN FAMOS** TILL THE END OF 2018

FAMOS Freja:

- Deneb 2015
- Airisto 2015
- Jacob Hägg 2015
- Jens Sørensen 2015
- Deneb 2016
- Jacob Prei 2016
- Jacob Hägg 2016
- Jens Sørensen 2016

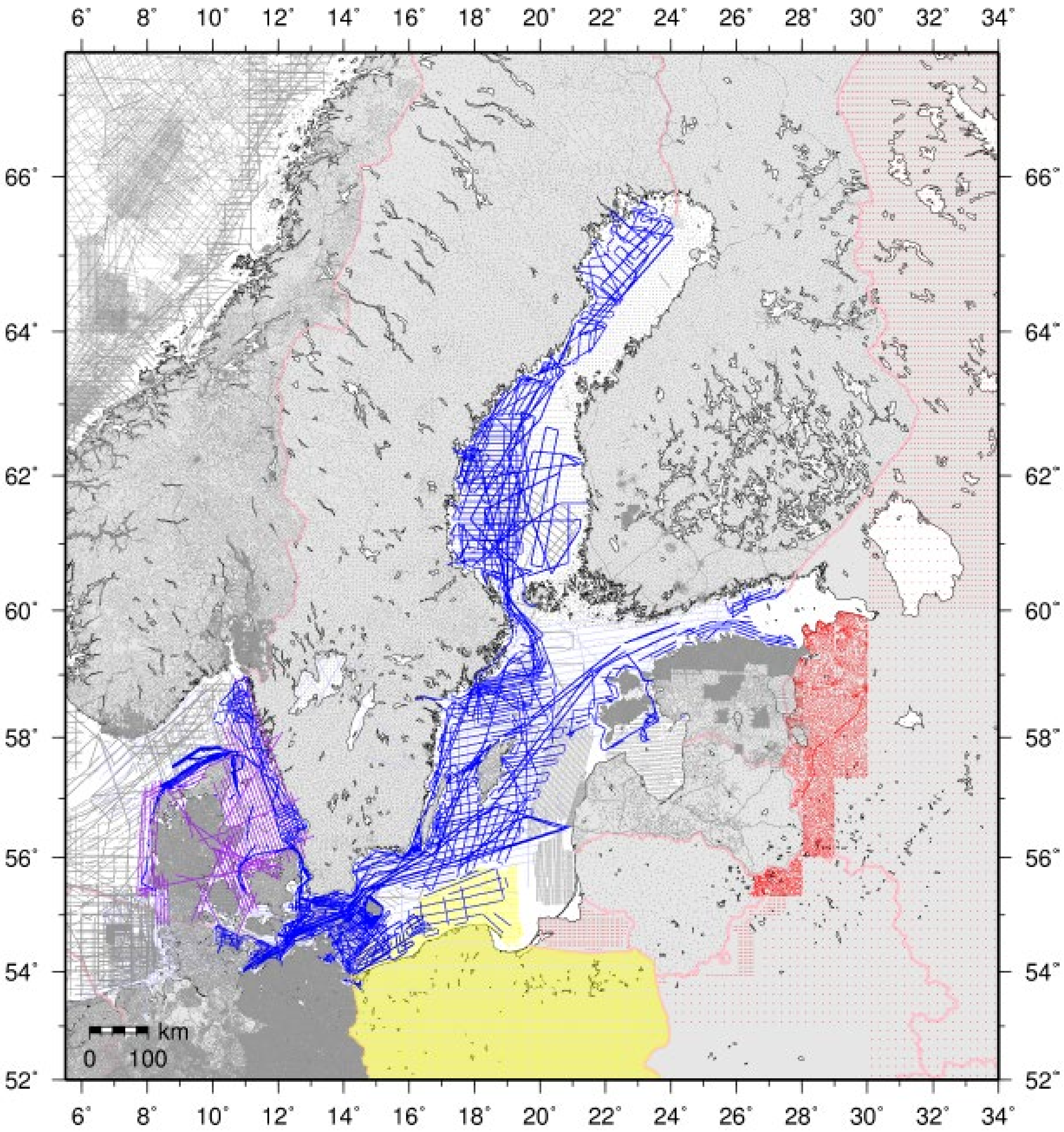
FAMOS Odin:

- Deneb 2017
- Sektori 2017
- Jacob Hägg 2017a
- Jacob Hägg 2017b
- Jens Sørensen 2017
- Urd 2017
- Deneb 2018
- Jakob Hägg 2018
- Geomari 2018
- Fyrbyggaren 2018a,b
- Jens Sørensen 2018
- Finnlady 2018
- Kattegatt 2018* (airborne)



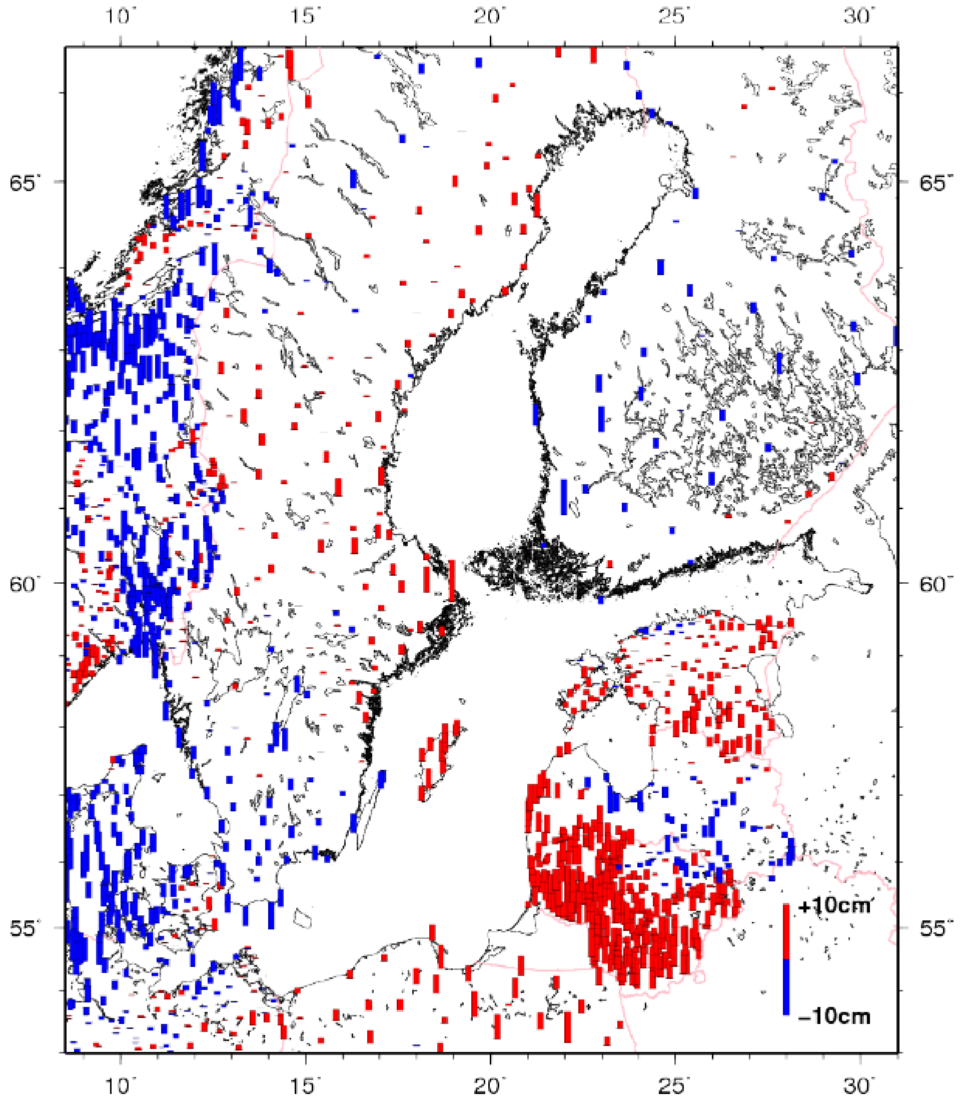
FAMOS GRAVITY DATABASE

- Version 4: December 26, 2022
- Managed by DTU Space in Copenhagen
- The figure shows:
  - Land gravity data in the NKG and BKG databases (black)
  - Marine data collected in the FAMOS project (blue)
  - Gridded data over Poland (yellow)
  - Less accurate data over Russia and Belarus (red)
  - New airborne data (purple)
- It has been a major task to arrange with the necessary permissions and agreements



Schwabe et al. (2023b)

FAMOS GNSS/LEVELLING DATABASE



Schwabe et al. (2023b)

- Originally created by merging the corresponding NKG and BKG GNSS/levelling databases
- After FAMOS before computing BSCD2000, new datasets have been added for Poland, Latvia and Lithuania
- Available in three different reference frames variants (lists) depending on the GNSS frame,
  - A. ITRF2008 with epoch 2000.0 (in countries with significant land uplift)
  - B. ETRF2000 with epoch 2000.0 (in countries with significant uplift)
  - C. National ETRS 89 realizations
- Levelled heights in national height reference frames (EVRS realizations with epoch 2000.0, DVR90, and DHHN2016)

COMPUTATION OF INTERIM GRAVIMETRIC QUASIGEOID MODELS

- An important part of FAMOS activity 2 and in the finalization effort was to compute interim gravimetric quasigeoid models using different regional geoid determination methods, mainly to evaluate the uncertainty stemming from the computation method
- Five computation centers (BKG, LM, TUT, DTU, FGI*) were computing interim geoid models, using basically three different well-established regional geoid modeling methods
  - Least Squares Modification of Stokes' formula with Additive corrections (LSMSA or KTH method) and RIR (Remove-Interpolate-Restore) interpolation. Least squares collocation used for gravity anomaly gridding in a Remove-Interpolate-Restore way
  - Stokes' formula (2D-FFT) with Wong and Gore kernel modification, Remove-Compute-Restore (RCR) with RTM topographic reduction
  - Point-mass modelling, RCR with RTM topographic reduction**

**) After the first stage of FAMOS, there were 4 computation centres (BKG, LM, TUT, DTU)*

EXAMPLES OF INTERIM GRAVIMETRIC QG MODELS COMPUTED IN THE FAMOS PROJECT BEFORE 2019

Partner	BKG1-PUMA	BKG3A	BKG3C	LM6A	LM6C	LM6E	TUT1A	TUT1C	FGI-1A
Method	Point-mass, RTM	RCR W&G, RTM	RCR W&G, RTM	LSMSA, RIR/LSC gridding	LSMSA, RIR/LSC gridding	LSMSA, RIR/LSC gridding	LSMSA, RIR/LSC gridding	LSMSA, RIR/LSC gridding	RCR W&G, RTM
Software implementation	BKG	BKG, SPFOUR	BKG, SPFOUR	LM/KTH	LM/KTH	LM/KTH	TUT/KTH	TUT/KTH	GRAVSOF
FAMOS g DB	Ver. 1	Ver. 2	Ver. 2	Ver. 2	Ver. 2	Ver. 2	Ver. 2	Ver. 2	Ver.2
EGM	GECO	GECO	GECO	DIR_R5	XGM2016	DIR_R5	GOCO05C	COCO05S	DIR_R5
Max. degree	2190	2190	2190	240/300	719	240/300	240	200	300
Bathymetry	GEBCO2014	GEBCO2014	GEBCO2014	No	No	GEBCO2019	No	No	No
Fill-in with altimetric gravity anom. (DTU13)	Yes, sel. method 1	No	In good areas, sel. method 2	No	No	No	No	No	No

FINAL GRAVIMETRIC SOLUTIONS OF THE COMPUTATION CENTERS

- Gravity DB version 4 (Dec. 26, 2022)

Solution	Method	GGM (d/o)	Kernel modification	Bathymetry
BKG5A	RCR 2D-FFT of Stokes integral (GRAVSOFT/SPFOUR)	GECO (2190) (Gilardoni et al., 2016)	20 – 200	Yes
BKG5B				
BKG5C				No
BKG5D				
DTU2A		XGM2019 (760) (Zingerle et al., 2019)	180 – 200	No
LM9F	LSMSA (“KTH method”)	DIR_R6 (300) (Förste et al., 2019)	$\leq$ 300	
TUT3A			$\leq$ 200	Yes
TUT3B		GOCO05c (720) (Pail et al., 2016)	$\leq$ 280	

Schwabe et al. (2023b)

GNSS/LEVELLING STATISTICS FOR THE GRAVIMETRIC QUASIGEOID (QG) MODELS

- A-list GNSS/levelling (ITRF2008 with epoch 2000.0)
- Table shows standard deviations
- ”Mean” is the merged gravimetric QG model computed as weighted mean of the 4 models (see below)

Country	BKG5D	LM9F	DTU2A	TUT3A	Mean	No.
DE	1.0	1.2	2.0	1.6	1.2	100
DK	1.6	1.8	1.9	1.9	1.8	617
NO	2.9	2.1	2.4	2.5	2.3	473
SW	1.9	1.7	2.0	2.1	1.8	182
FI	1.4	1.7	2.2	2.0	1.7	36
EE	0.9	0.8	1.0	1.1	0.7	131
LV	2.2	2.5	2.2	2.3	2.2	84
LT	1.9	1.9	2.0	1.9	1.9	250
PL	1.2	1.2	2.6	1.4	1.4	29
AR	2.0	1.8	2.0	2.1	1.9	1902
SR	**1.6**	**1.7**	**1.9**	**1.9**	**1.6**	812

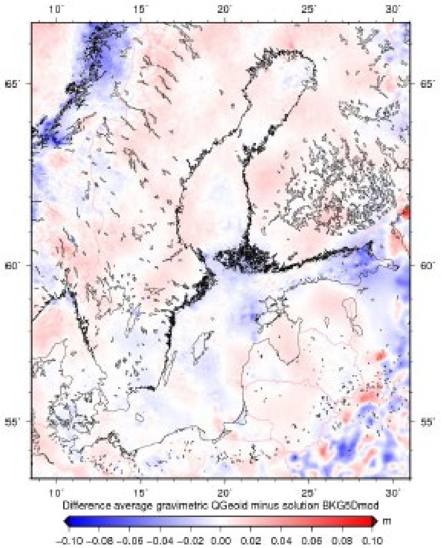
AR = merged residuals of all countries after one mean value per country has been subtracted

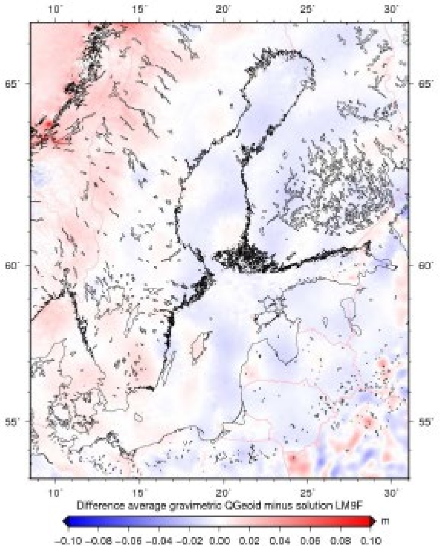
SR = merged residuals of selected countries (all except DK and NO) after one mean value per country has been subtracted

Schwabe et al. (2023b)

RESIDUALS OF INDIVIDUAL GRAVIMETRIC SOLUTIONS AGAINST THE WEIGHTED MEAN (SEE BELOW)

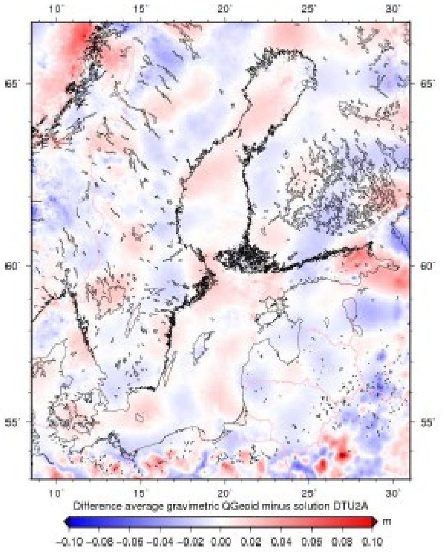
BKG5D

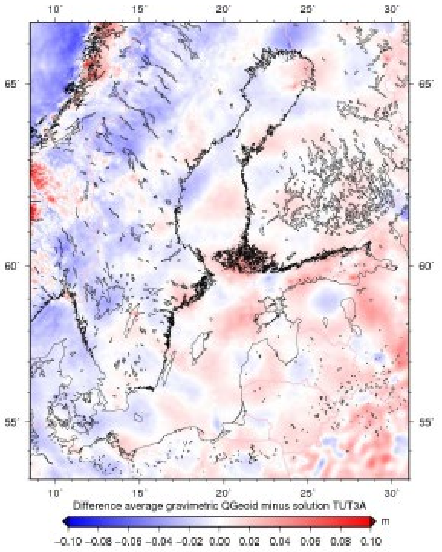




LM9F

DTU2A





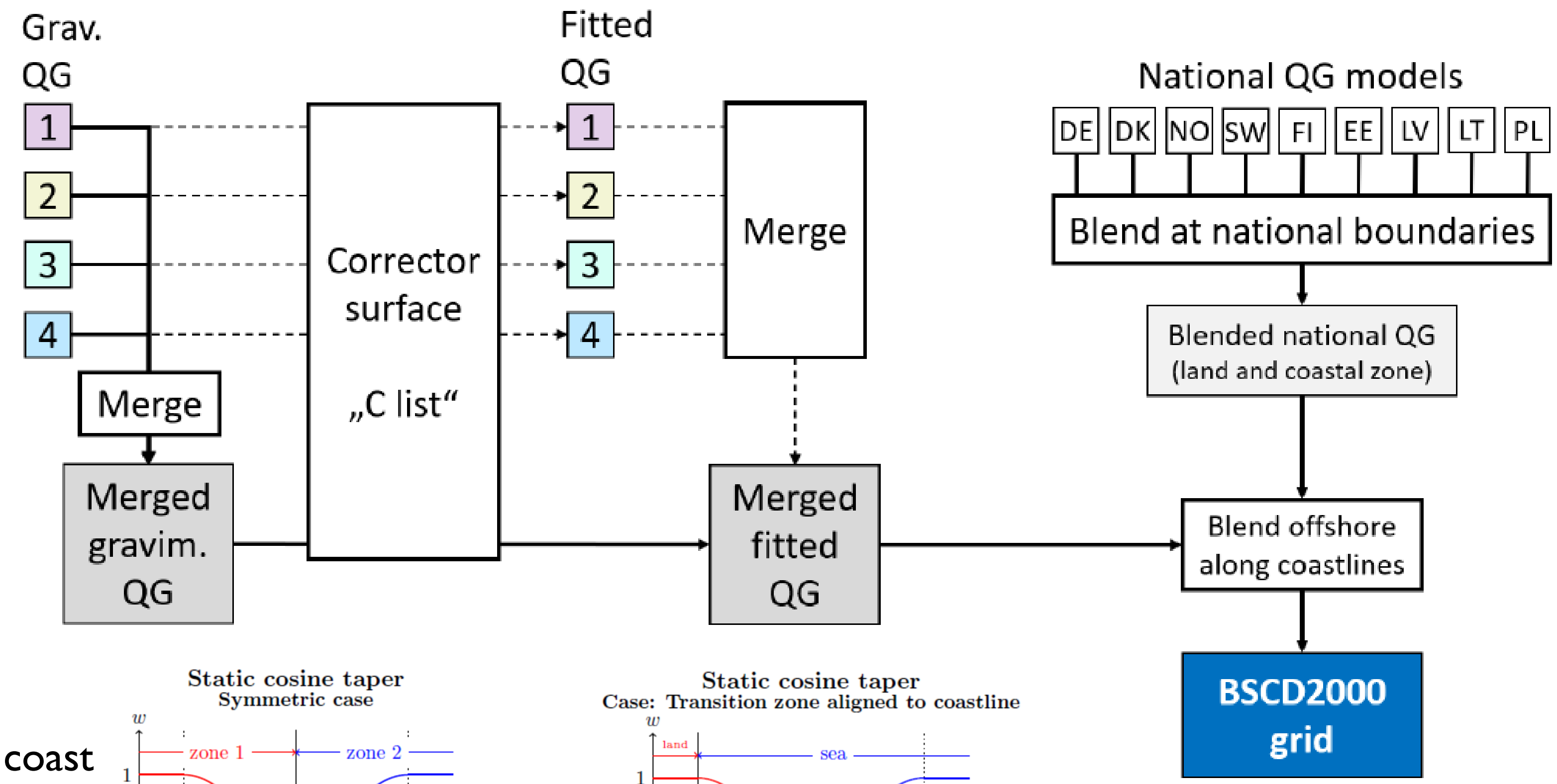
TUT3A

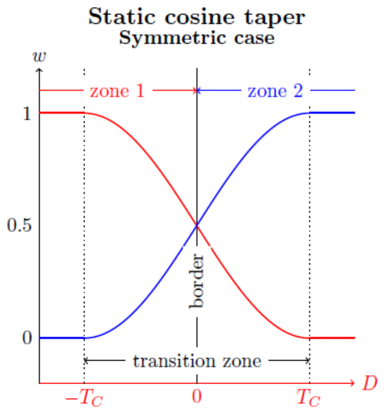
Schwabe et al. (2023b)

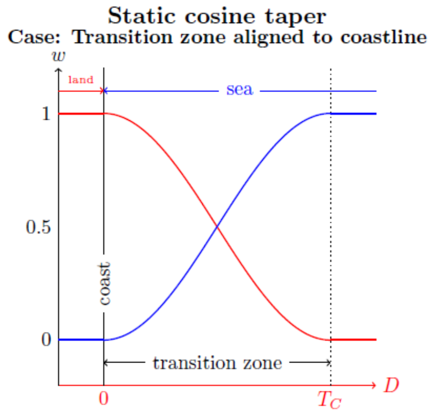
GENERAL SCHEME TO MERGE SOLUTIONS AND DERIVE THE BSCD2000 HEIGHT TRANSFORMATION GRID

The final BSCD2000 grid is derived using the following scheme, which is in agreement with the specification of BSCD2000 (see above):

1. The 4 gravimetric QG models are merged (weighted mean; see below)
2. A corrector surface is computed (Hirvonen covariance function with correlation length 122 km). This gives the merged fitted QG model
3. The official national geoid models (height reference surfaces) on land are blended with each other along the borders and outside the coast
4. The fitted QG model is then merged with the blended official national height correction models on land







Schwabe et al. (2023b)

1. MERGED GRAVIMETRIC QG MODEL

- The merged gravimetric QG model is computed as the weighted mean of the computation centre solutions
- The weights are computed based on the SR standard deviations above (different alternatives tested)

Solution	SD (cm)	Weight
BKG5D	1.64	1.16
LM9F	1.67	1.13
DTU2A	1.99	0.79
TUT3A	1.85	0.92

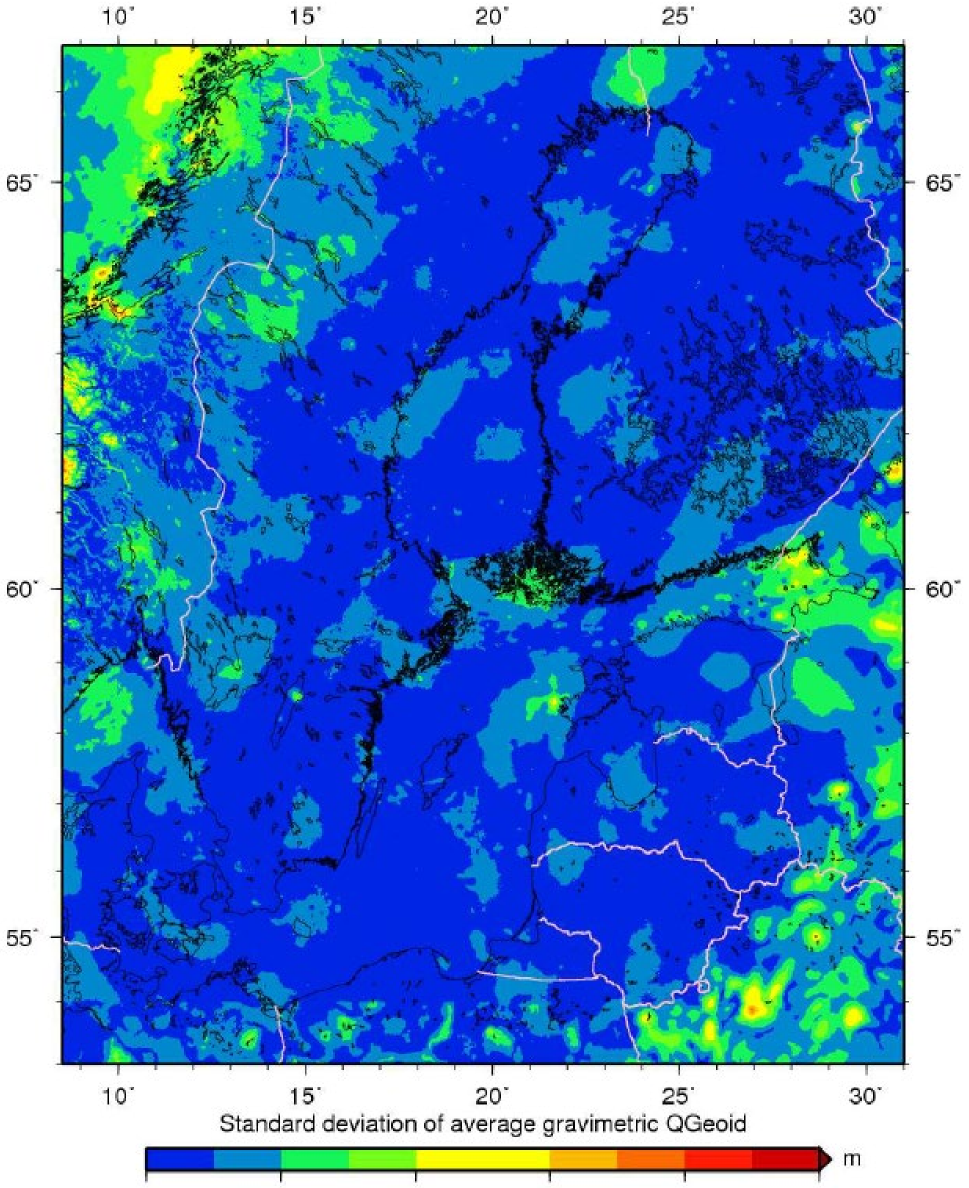
- Statistics of the residuals of the individual gravimetric quasigeoid solutions vs. weighted mean in meters; see also next slide

Model	Mean	SD	Min.	Max.
BKG5D	0.000	0.011	−0.105	+0.086
LM9F	0.000	0.008	−0.042	+0.091
DTU2A	0.000	0.012	−0.079	+0.114
TUT3A	0.000	0.014	−0.069	+0.143

- Standard deviation of the weighted mean were computed pointwise for each grid point ⇒ see the figure

Since the same synthetic data were used by all in the Russian part of the Gulf of Finland, this figure clearly underestimates the uncertainty there. It is most likely 15–20 cm or so

Internal validation: "Analyst noise"

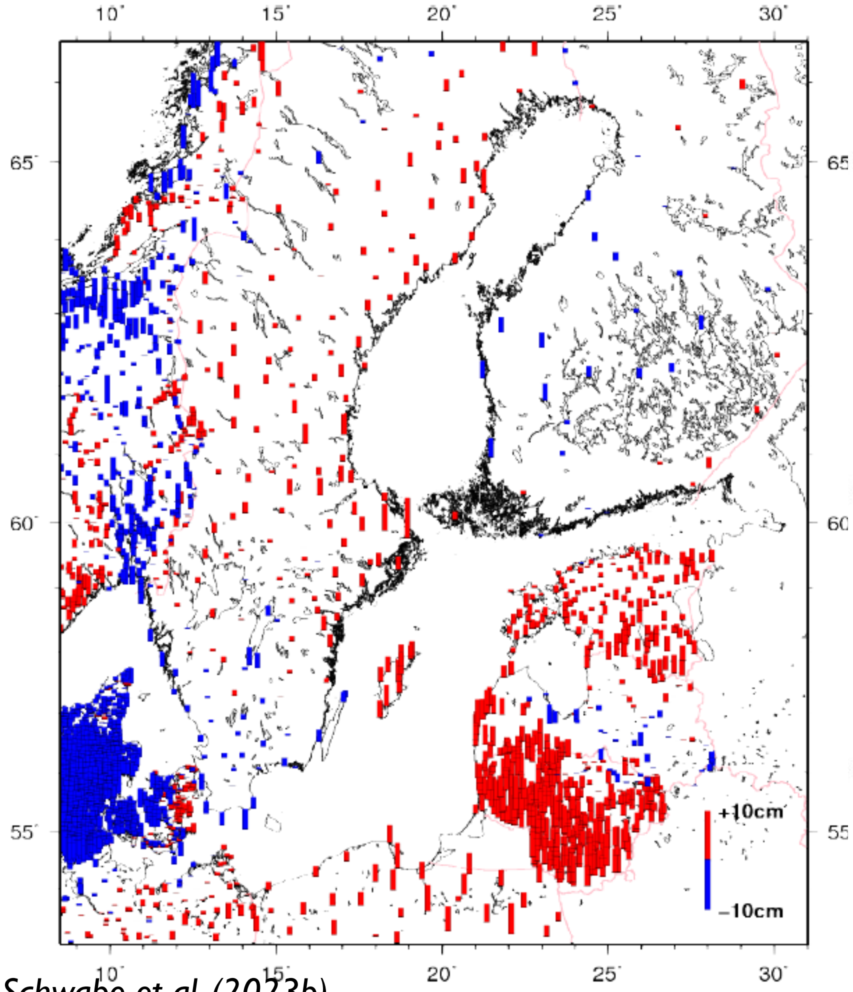


Schwabe et al. (2023b)

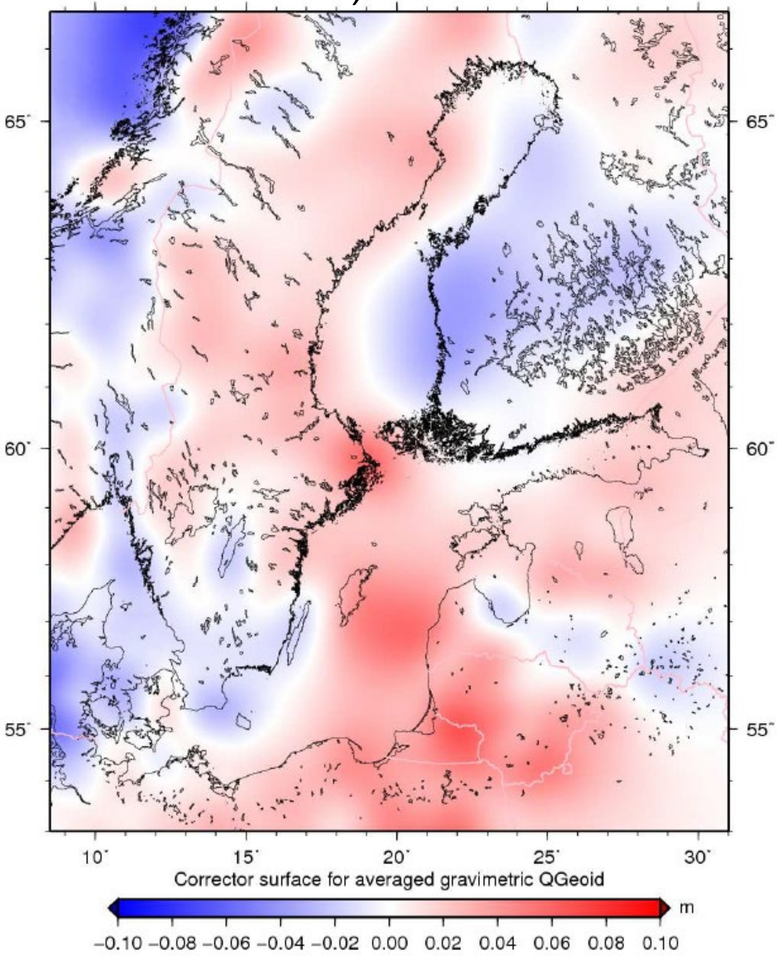
2. MERGED FITTED QG MODEL ("MARINE GRID")

C-list GNSS/levelling (national ETRS89 realisations)

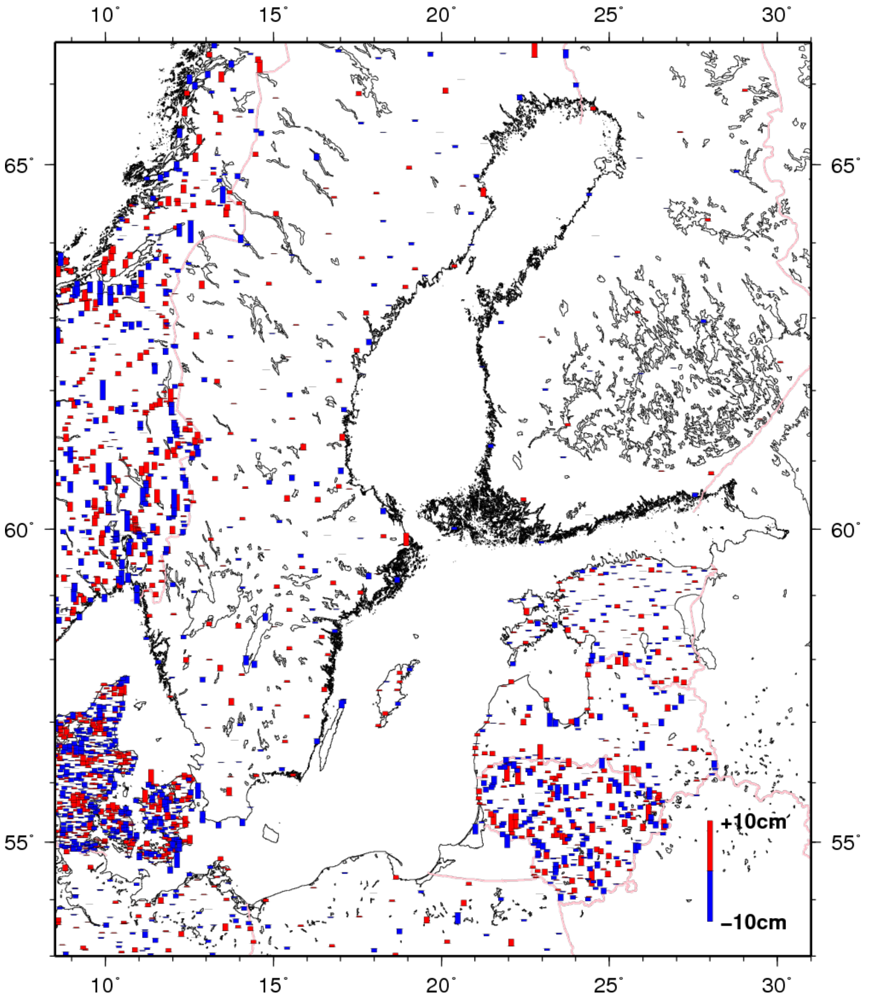
Residuals of merged gravimetric model



Residual surface (Hirvonen, 122 km, Varbla, 2023a, b)



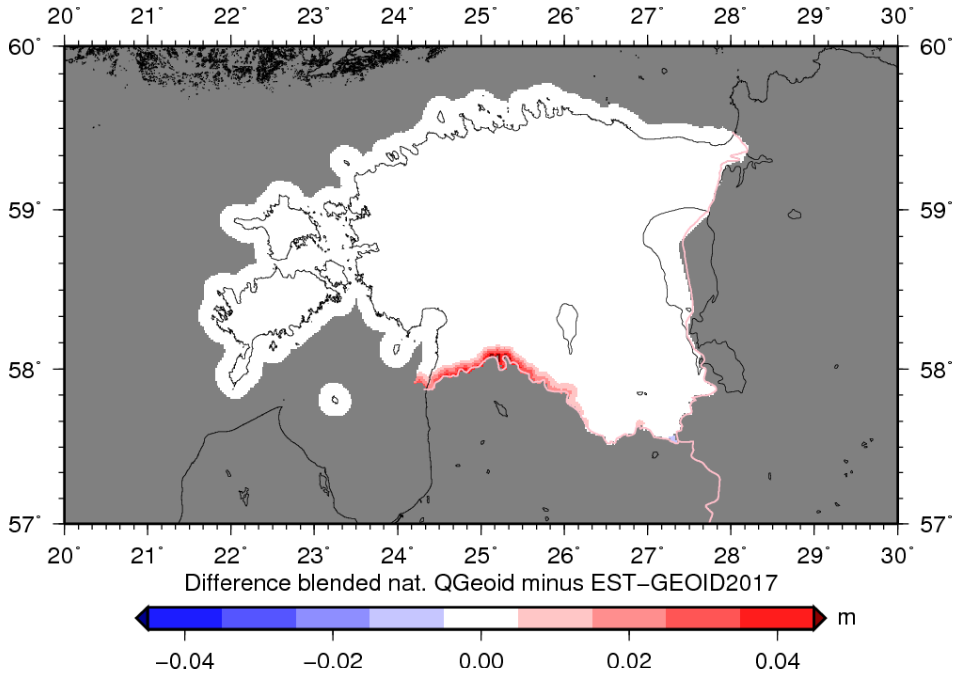
Residuals of merged fitted QG model

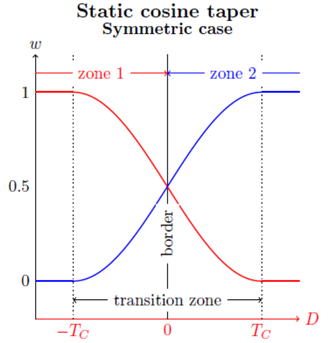


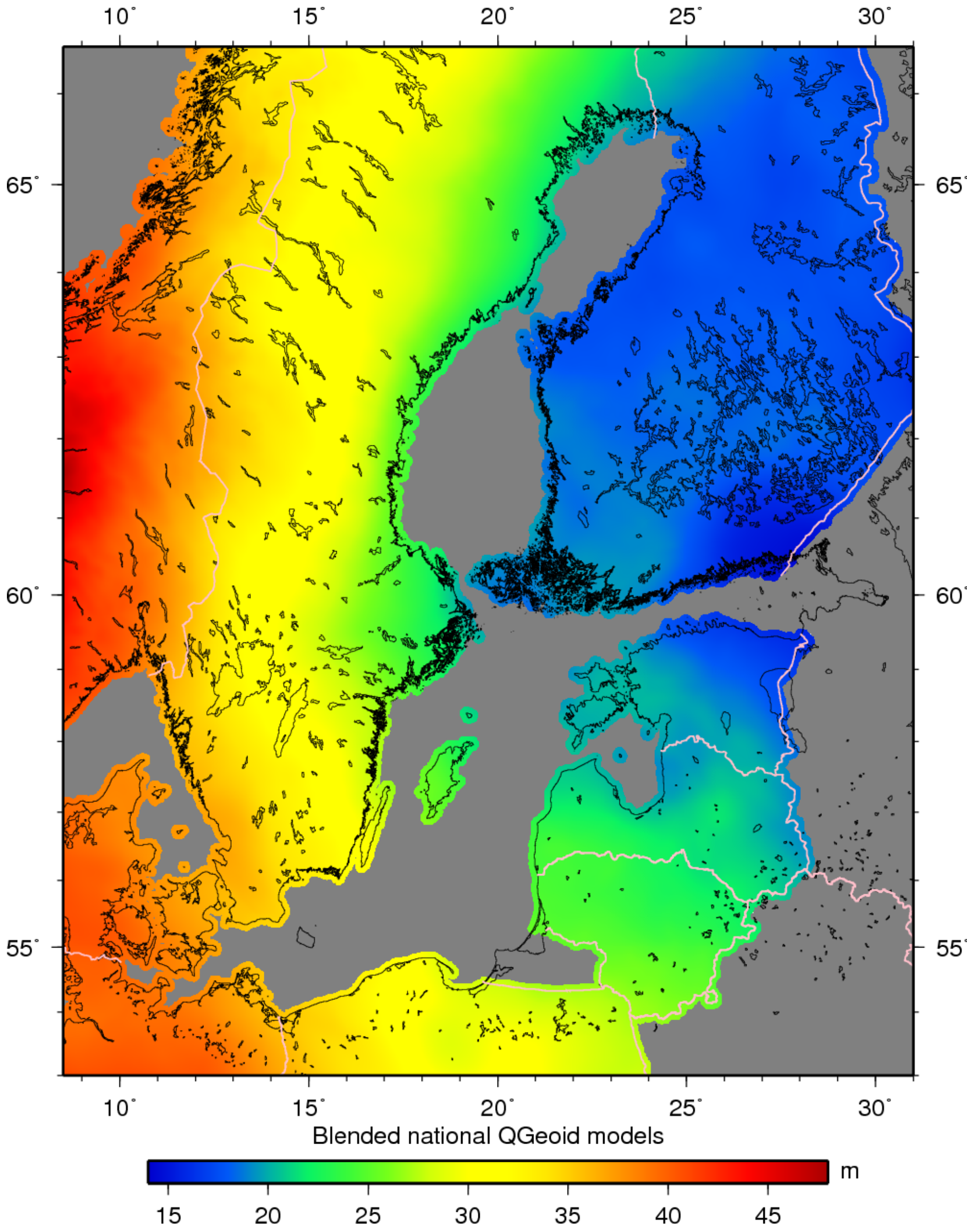
Schwabe et al. (2023b)

3. BLENDED NATIONAL QG MODELS ON LAND (1)

- Example: National quasigeoid models blended along the borders (10 km cosine taper on both sides)



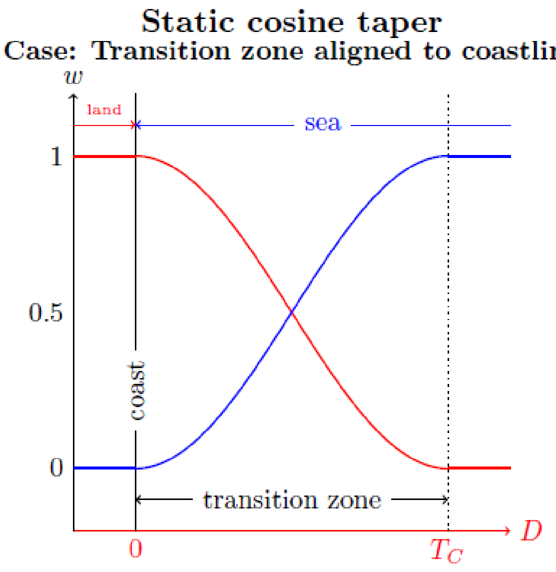


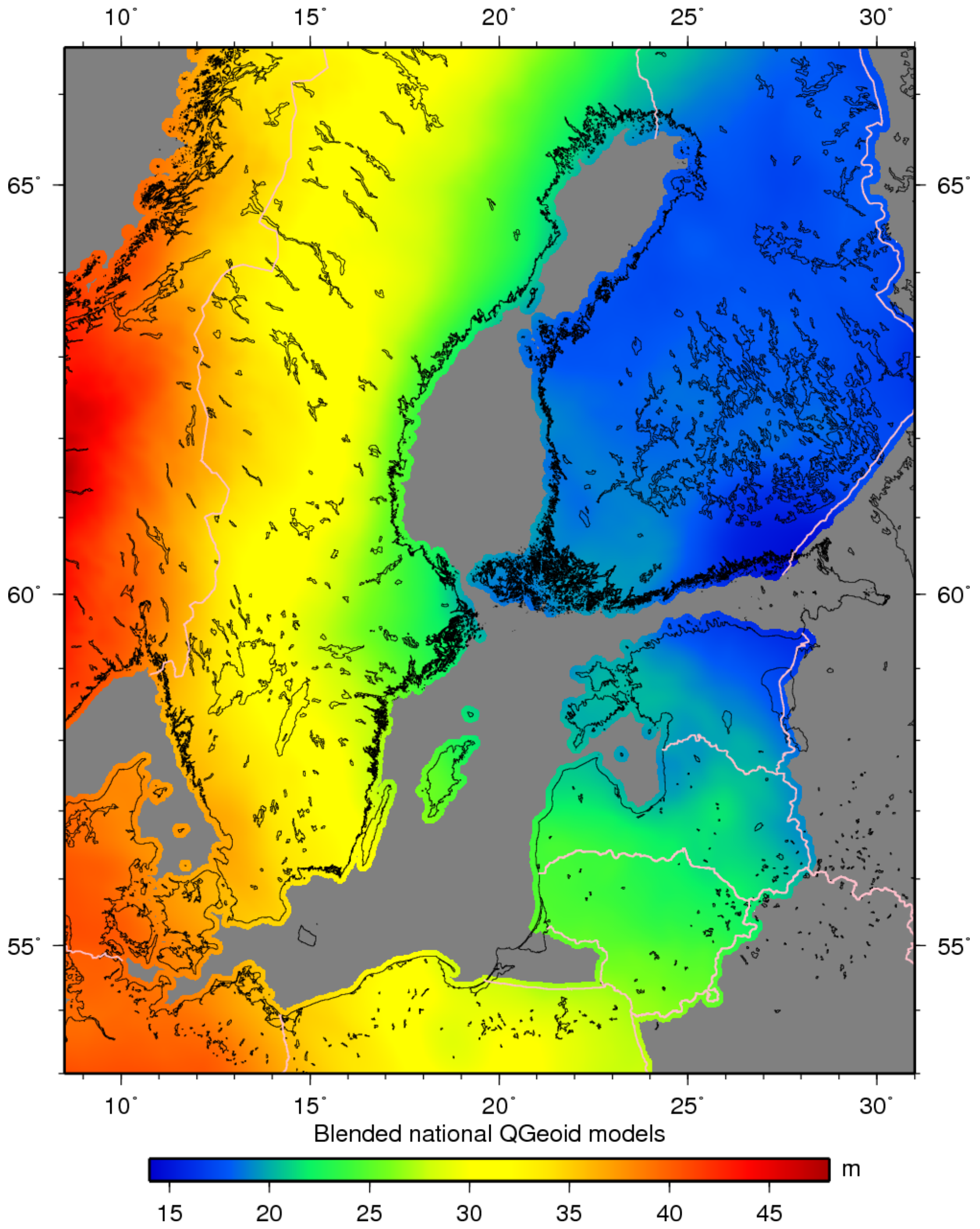


Schwabe et al. (2023b)

3. BLENDED NATIONAL QG MODELS ON LAND (2)

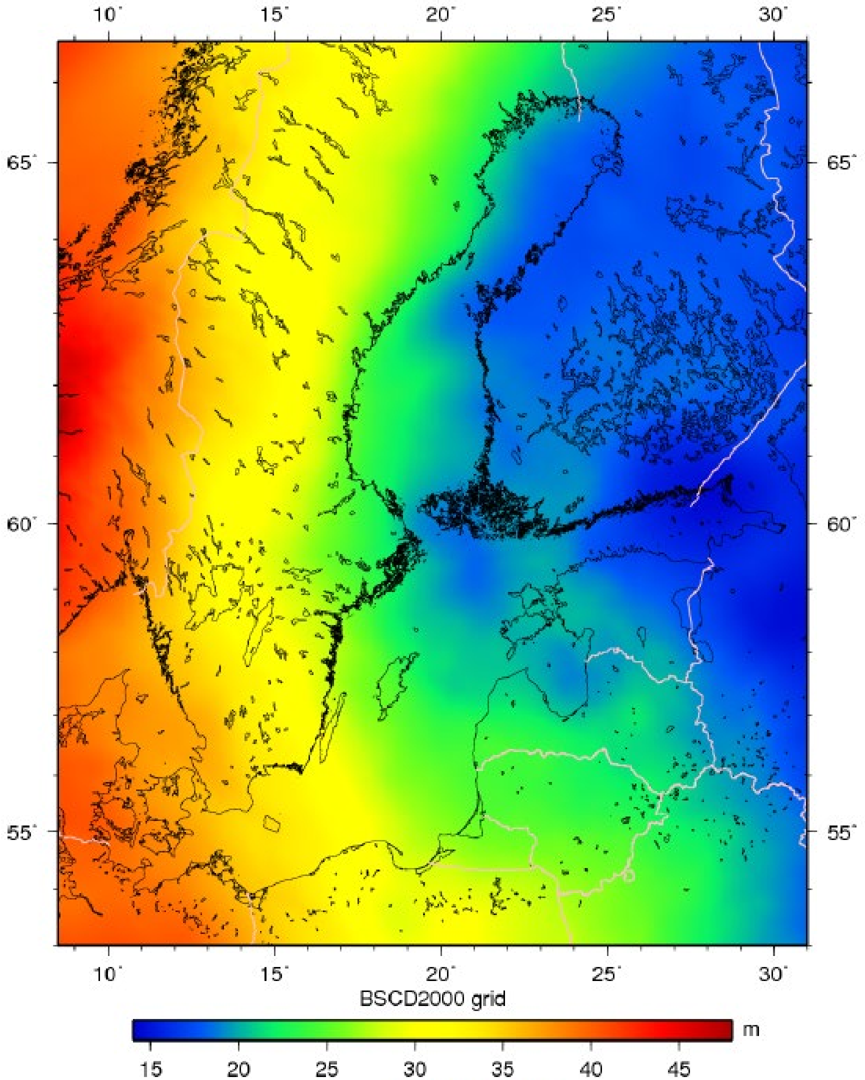
- National quasigeoid models blended along the coast (10 km cosine taper offshore, one-sided)



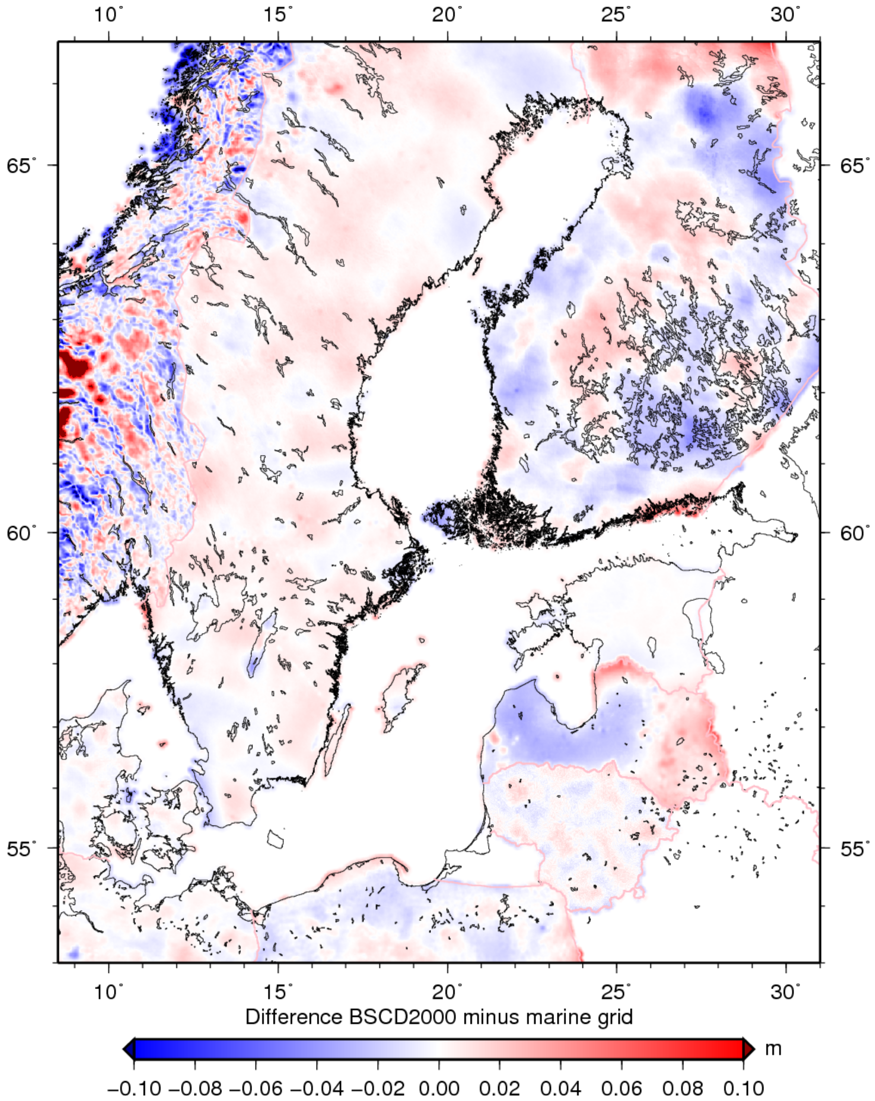


Schwabe et al. (2023b)

4. BSCD2000 = MERGED FITTED QG MODEL („MARINE GRID“) + BLENDED NATIONAL MODELS ON LAND



BSCD2000 grid



Difference BSCD2000 minus marine grid

Schwabe et al. (2023b

Figure 7: BSCD2000 grid

FINAL WORDS

- The BSCD2000 is currently being implemented in paper/digital charts, tide gauges, etc. This work is currently coordinated by the BSHC Chart Datum, Water Level and Currents Working Group (CDWCWG)
- The first version of the BSCD2000 height transformation grid was released in November 2023
- Landing page: https://www.bshc.pro/iho-bscd2000/ , DOI https://doi.org/10.58440/iho-bscd2000
- The BSCD2000 model is consistent with the specification of BSCD2000, and with European reference system standards (ETRS89, EVRS)
- It will make seamless GNSS based navigation possible for the first time in the Baltic Sea
- Note that the BSCD2000 grid is not a pure gravimetric quasigeoid model. It is a transformation surface that also absorbs various other effects (all < 5 cm), e.g.
  - different geodetic standards (tide systems) for GNSS coordinates and levelling heights
  - residual datum differences along the borders
  - regional differences between quasigeoid model, GNSS and levelling

Schwabe et al. (2023a, b, c)