Quantum Sensors:
Principles and ground applications

F. Pereira Dos Santos
LNE-SYRTE, Paris Observatory

NKG Science Week
Online, March 12th, 2024

Outline

1. Applications of inertial sensors / accelerometry
2. Principle of atom interferometry
3. Gravimeters: state of the art quantum accelerometers
4. Cold Atom Interferometer Gravity Gradiometer
5. Other sensors and applications

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Applications of inertial sensors

Inertial sensor = instrument which measures accelerations and rotations of an object

Inertial navigation

- Onboard accelerometers and gyrometers

→ planes, satellites, submarines, ...

bias ~ 10^{-5} $m.s^{-2}$

↓

Drift in position ~ 100 m after one hour of flight

Applications of inertial sensors

Inertial navigation

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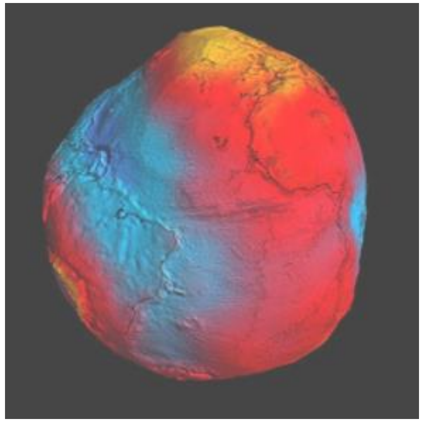
Drift in position ~ 100 m after one hour of flight

Geophysics

- Study of the underground (oil, gas …)
- Airborne gravimetry

→ uncertainty ~ 1 µg

- Determination of the geoid



ESA/GOCE, 2011

sensibilité: 10^{-12} $m.s^{-2}$

Applications of inertial sensors

Inertial navigation

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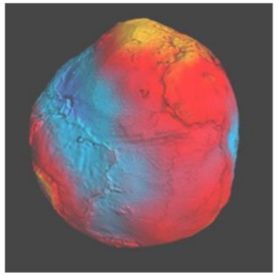
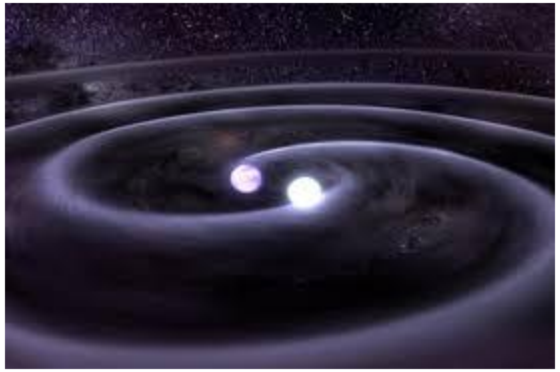


ESA/GOCE, 2011

sensibilité: 10^{-12} $m.s^{-2}$

Fundamental physics

- test of the equivalence principle
- detection of gravitational waves
- measurement of G and α
- metrology (Kibble balance)



Required sensitivity:

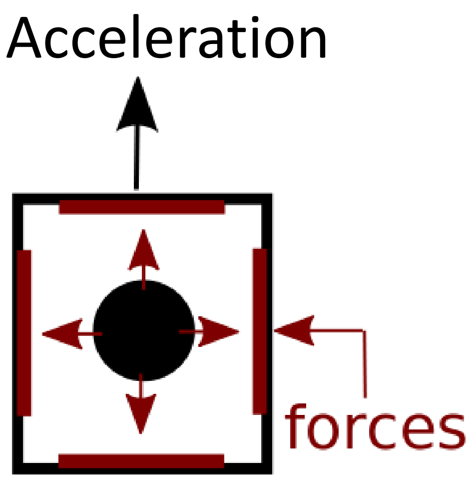
~ 10^{-15} $m.s^{-2}$ at f_{GW}

(~ 1 Hz for example)

Accelerometry

1. **Following the motion of an object which is linked to the accelerated frame**

- Piezoelectric accelerometers
- MEMS accelerometers
- Superconducting gravimeters, electrostatic accelerometers
  (the proof mass is maintained fixed in position)

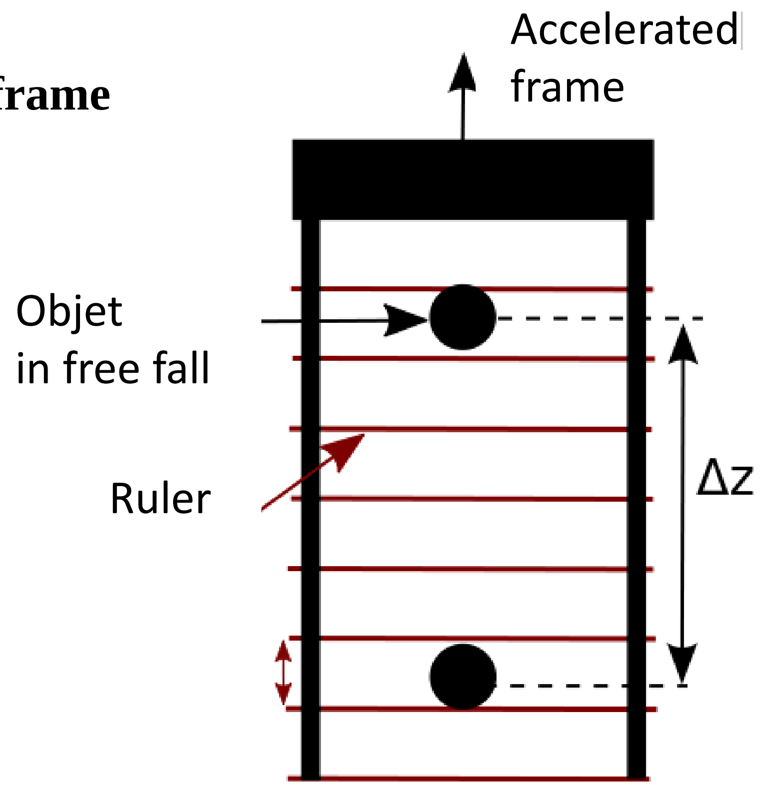


Accelerometry

2. **Measure the displacement of an accelerated frame with respect to a free falling object:**

- Corner cube gravimeters (Faller, 1967)
- Matter wave interferometer

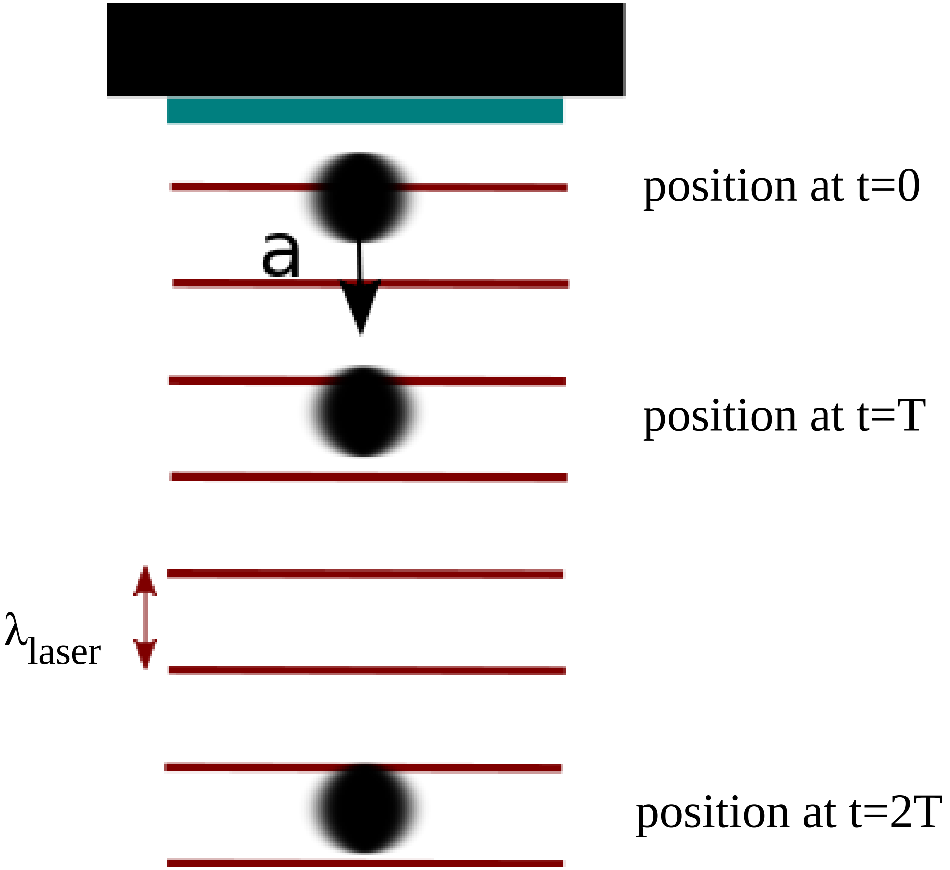
(neutrons, 1975 – atoms, 1991)



Quantum Accelerometry

Use cold atoms in free fall to read the phase of the laser linked to the accelerated frame

→ measurement of the distance in units of the wavelength



Number of scale divisions :

$$\sim \frac{aT^2}{\lambda_{laser}}$$

Orders of magnitude

Number of scale divisions $\sim \dfrac{aT^2}{\lambda_{laser}}$

- Ruler is a laser with scale division given by $\lambda_{laser} \sim 0.5$ µm
- Smallest measurable : λ_{laser} / 1000 (SNR = 1000)

Interrogation time 2T ~ 10 ms

→ Sensitivity to accelerations ~ 10^{-5} m.s^{-2}

Best airborne accelerometers

Interrogation time 2T ~ 10 s

→ Sensitivity to accelerations ~ 10^{-11} m.s^{-2}

Best gravity sensors

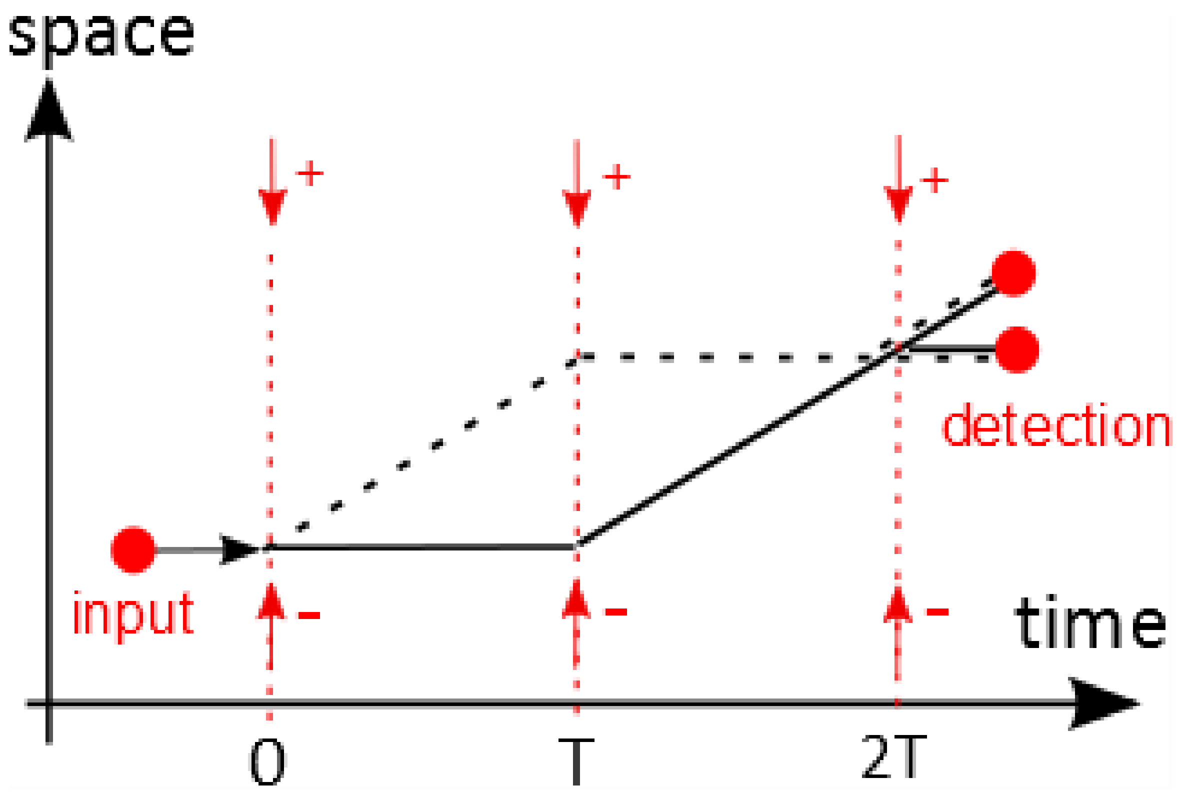
Two interesting applications
for atom interferometers

Outline

Principle of atom interferometry

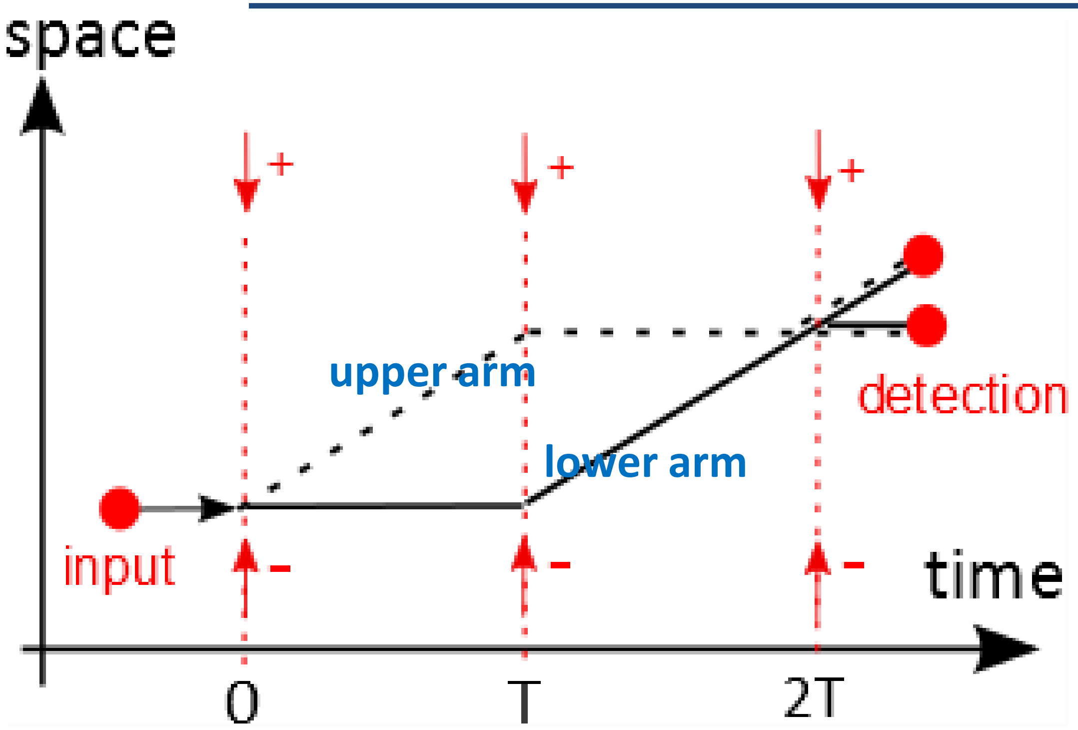
l'Observatoire de Paris — SYRTE
Systèmes de Référence Temps-Espace

- Analogy with optical interferometry
- Use atom-laser interaction to deflect matter waves



Interferometer based on a sequence of 3 laser pulses

Phase difference between the two paths



Phase of the interferometer : $\Delta\Phi = \Phi^{up} - \Phi^{down}$

Φ : phase of the quantum wavepacket

$$\Phi^{\text{up}} = \varphi(0) - \varphi(T) + \varphi(2T)$$

$$\Phi^{\text{down}} = 0 + \varphi(T) + 0$$

φ : laser phase : $\varphi(T) = kz(T) = \frac{4\pi}{\lambda} z(T) = 4\pi \frac{z(T)}{\lambda}$

$\Delta\Phi = \varphi(0) - 2\varphi(T) + \varphi(2T) = k(z(0) - 2z(T) + z(2T))$

Sampling of the positions at the three pulses

For a constant acceleration a, $z(t) = \frac{1}{2} a\, t^2 \quad \Rightarrow \quad \Delta\Phi = k\; a\; T^2$

Key advantages
scale factor scales as $T^2 \Rightarrow$ Benefit of cold atoms
depends on time (T) and frequency (k) $\Rightarrow$ SI traceable, accurate, bias free

Phase difference between the two paths

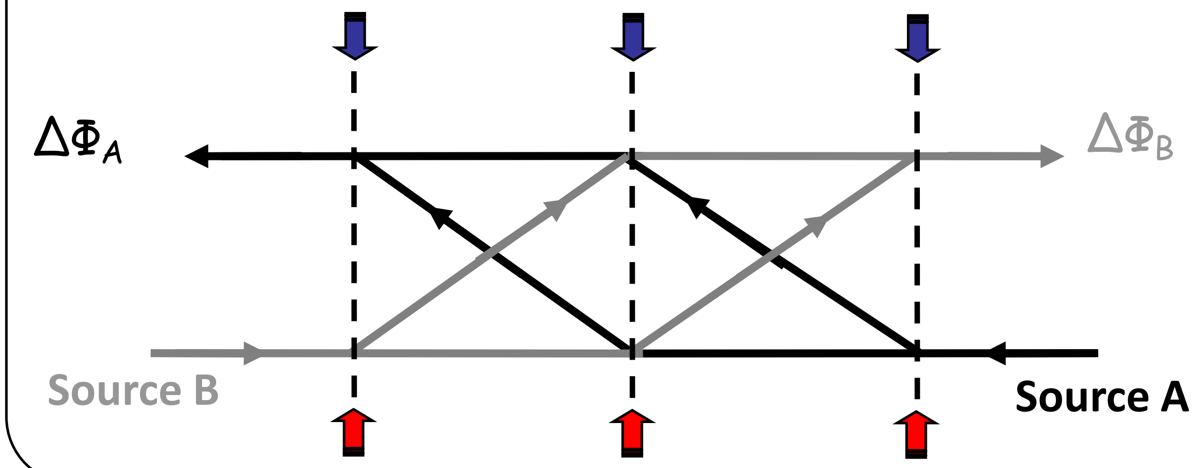
$$\Delta\Phi = \Delta\Phi_{acceleration} + \Delta\Phi_{rotation}$$

T L Gustavson et al.,
Class. Quantum Grav. **17** (2000).

$$\vec{k}_{eff}.\vec{a}T^2 \quad -2\vec{k}_{eff}.\vec{\Omega}\wedge\vec{V}\,T^2$$

Rotation phase shift:
Direction of atoms

Two atomic sources of opposite directions



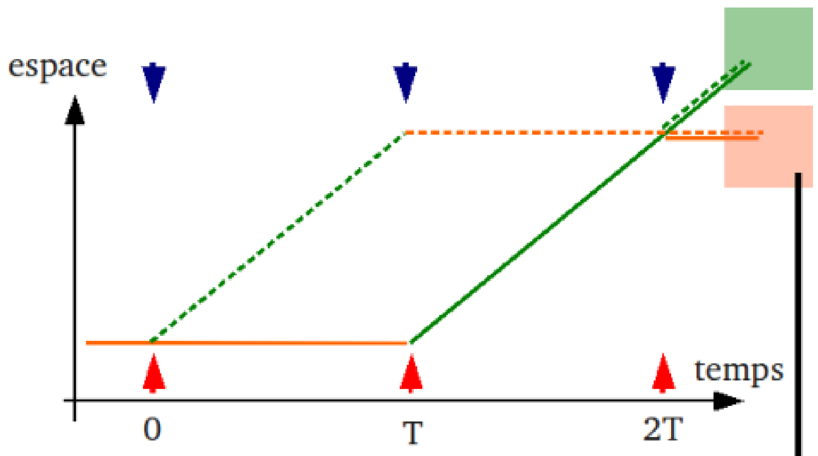
Sum: acceleration

$$\Delta\Phi_{acc} = \frac{\Delta\Phi_A + \Delta\Phi_B}{2}$$

Difference: rotation

$$\Delta\Phi_{rot} = \frac{\Delta\Phi_A - \Delta\Phi_B}{2}$$

Measurement of the phase difference



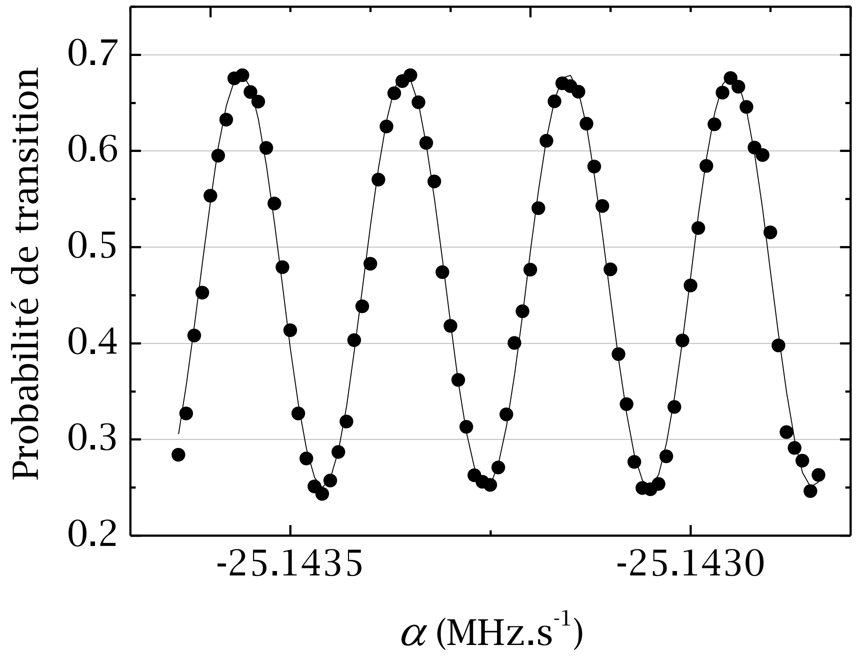
$$\Delta\Phi = k\, a\, T^2$$

Signal at the output of the interferometer

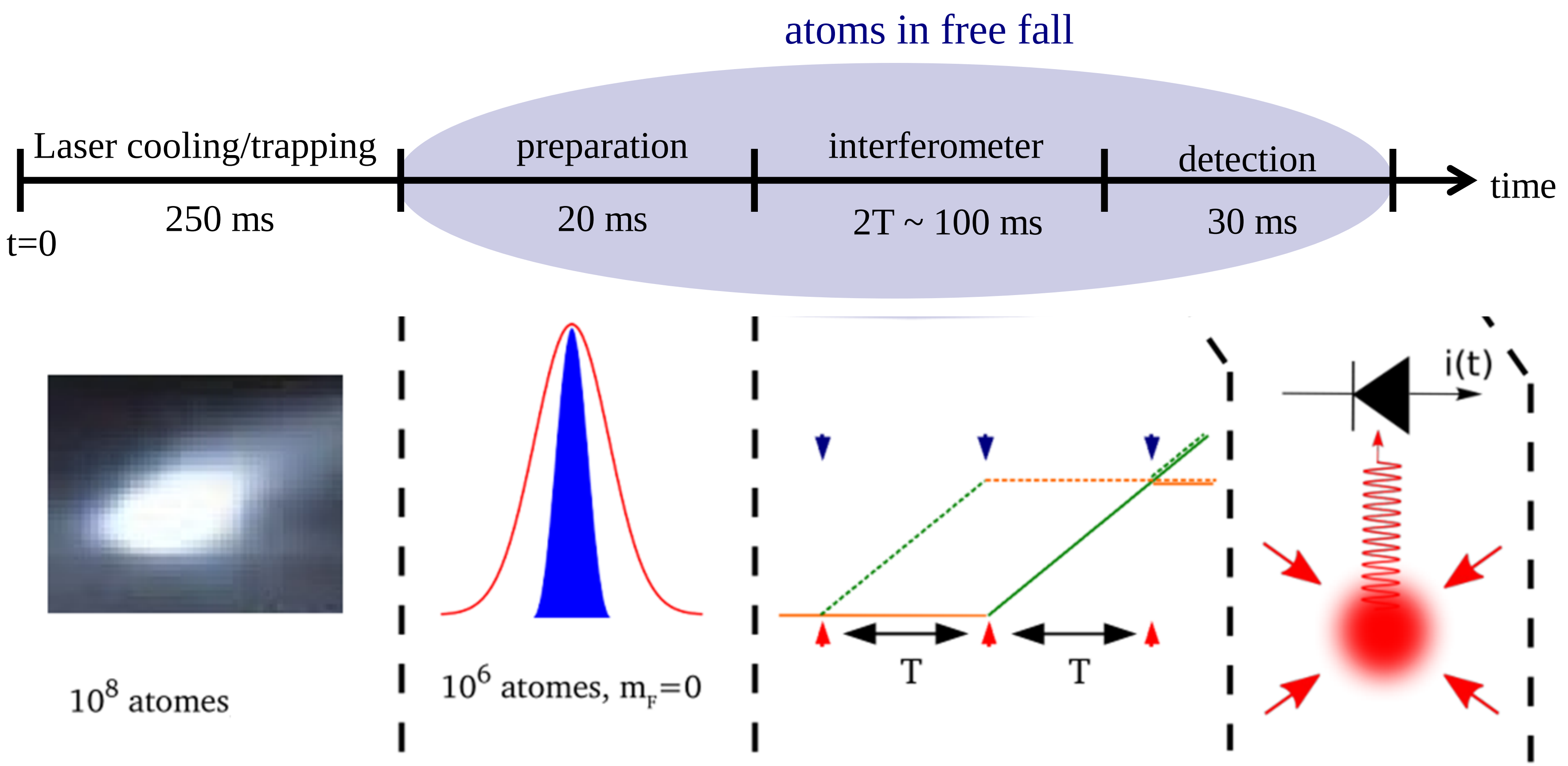
$$P = P_0 - A \cos \Delta\Phi$$

Normalized detection of the two populations by fluorescence

$$P = N_{up}/(N_{up}+N_{down})$$



Typical experimental sequence



Typical cycle time : 0.1 - 1 s
Repetition rate : 1 - 10 Hz range

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Systèmes de Référence Temps-Espace

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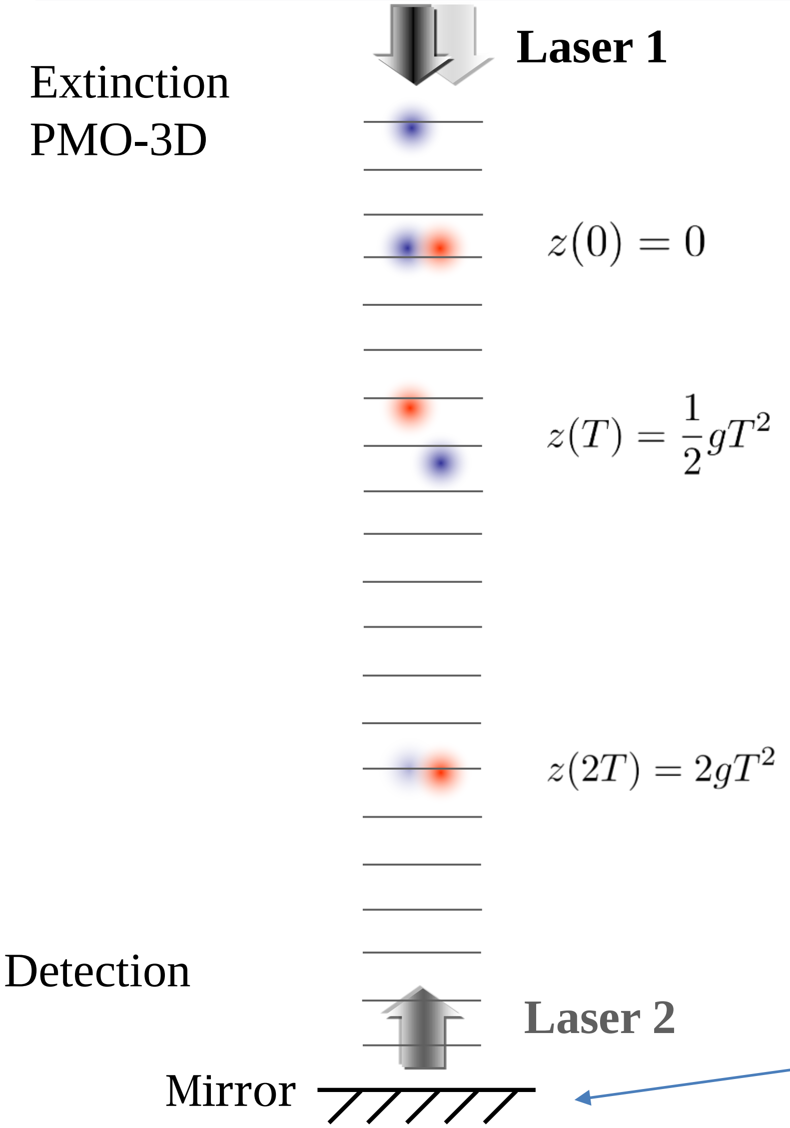
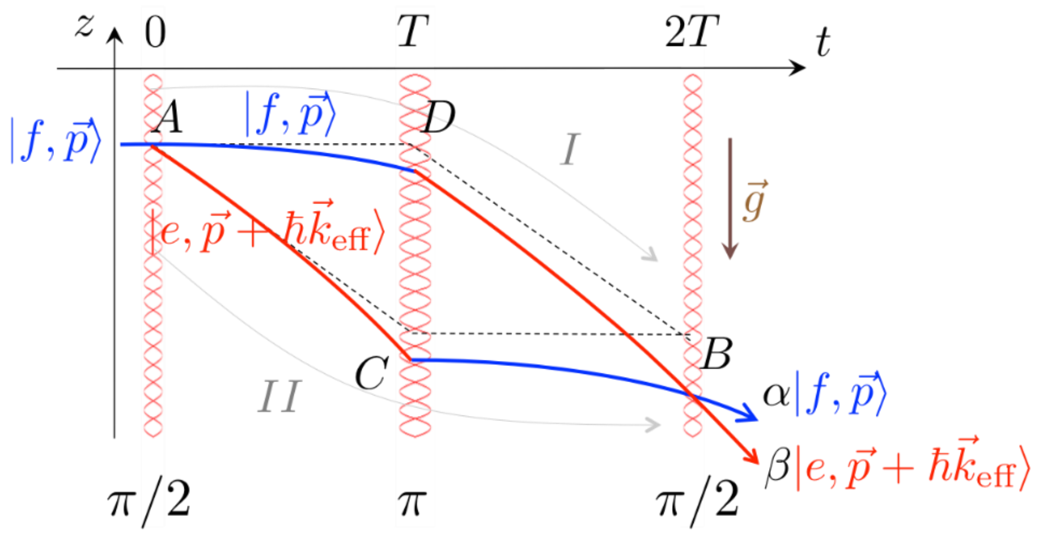
Gravimeter: a vertical accelerometer





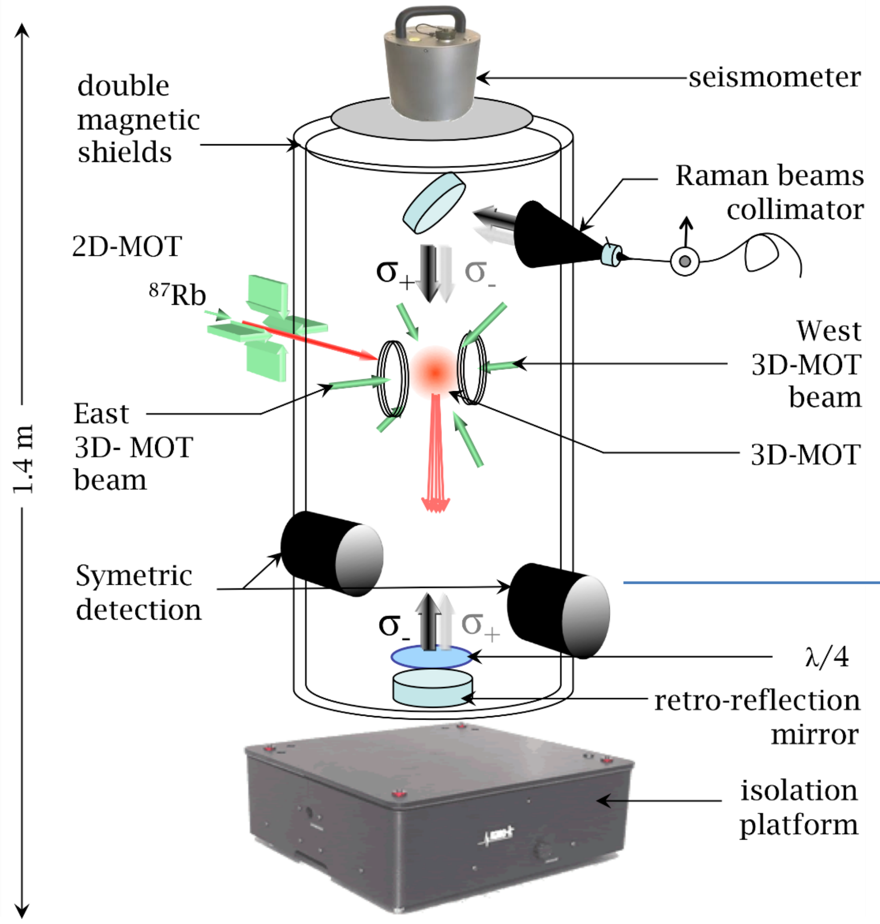
$$\Delta\Phi = k\, g\, T^2$$

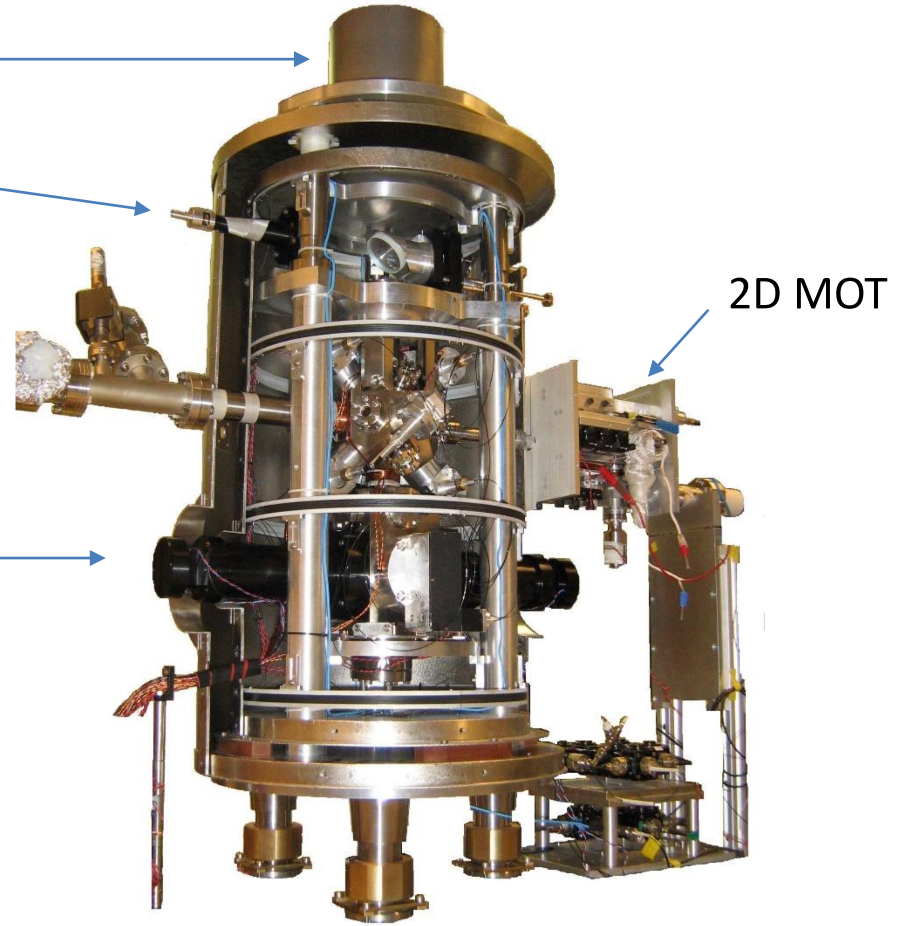
Important !

the positions of the laser equiphases are defined by the mirror position

Interferometer measurement = measure of the relative displacement atoms/mirror

An example : the SYRTE gravimeter

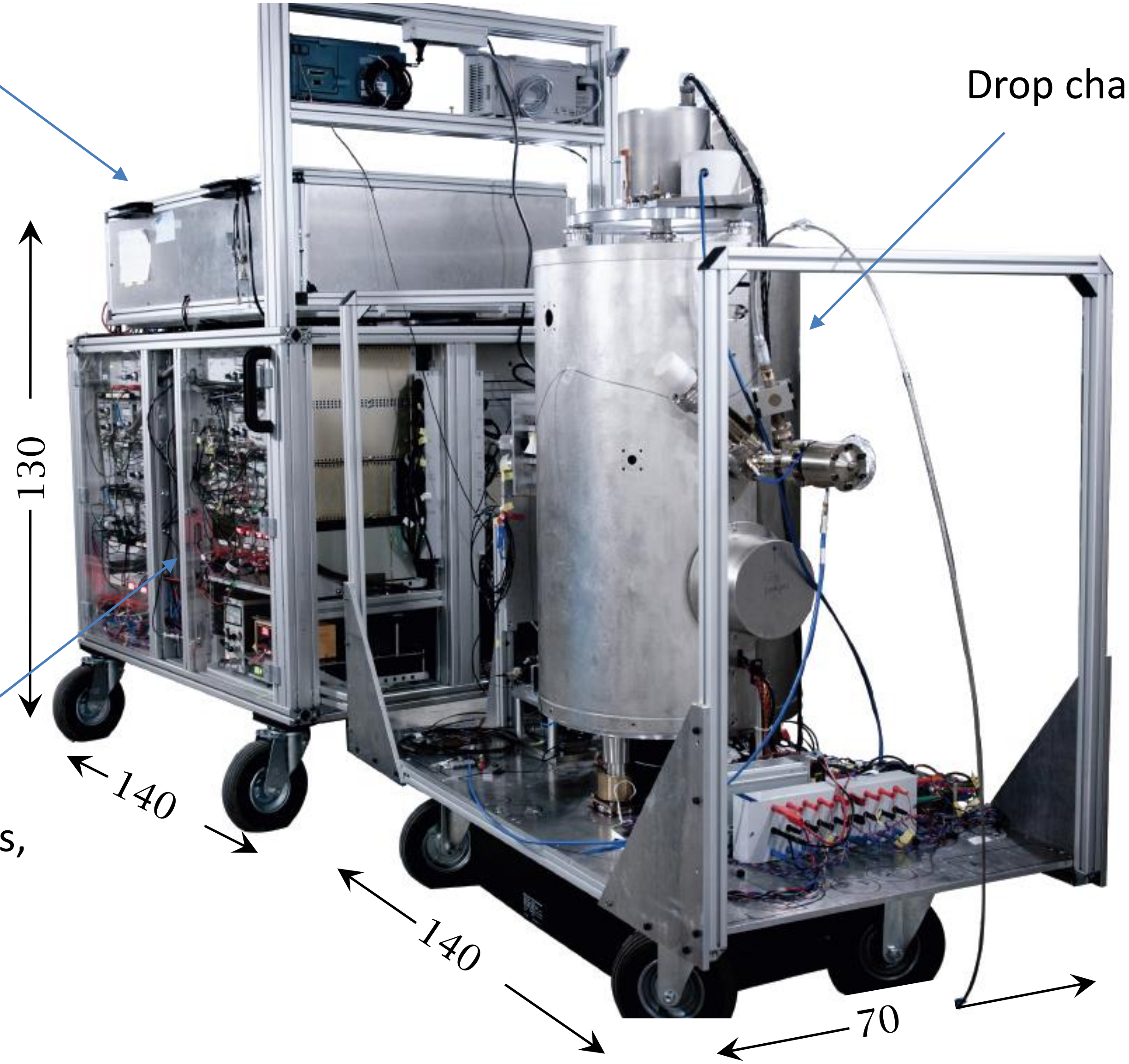




An example : the SYRTE gravimeter

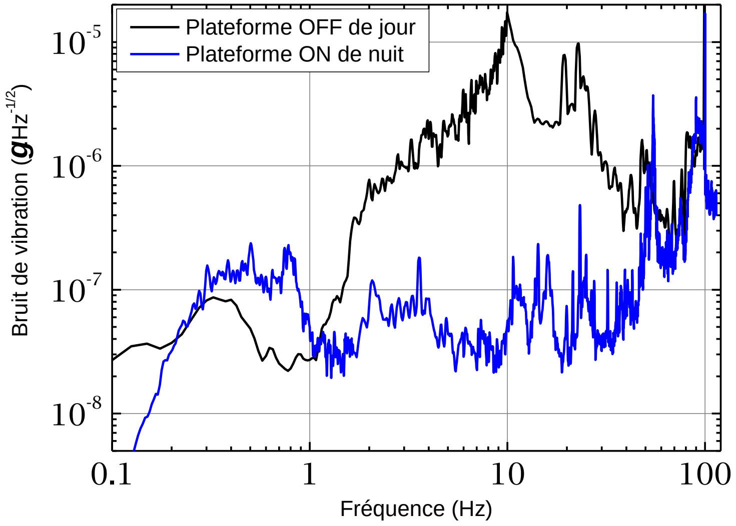
Laser system

Drop chamber



Control electronics,
Power supplies

Sensitivity limits: vibrations



Typical noise in urban environnement

Without isolation platform: 2.9 · 10^{-6} g/Hz$^{1/2}$

With platform: 7.6 · 10^{-8} g/Hz$^{1/2}$

GAIN : factor 40

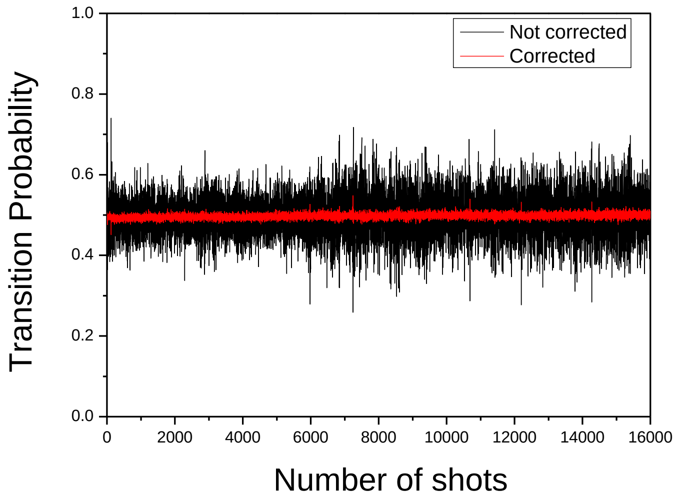
Post-correction

Seismometer → PC → $k_{eff}gT^2$

$v(t) \rightarrow \phi_{vib}^{S}$

Interferometer → PC

$k_{eff}gT^2 + \phi_{vib}^{S}$



Provides additional gain from 2 to 10 depending on vibration noise conditions:

6 10^{-9} g/Hz$^{1/2}$ (quiet, LUX) – 15 10^{-9} g/Hz$^{1/2}$ (urban, WB lab, Trappes)

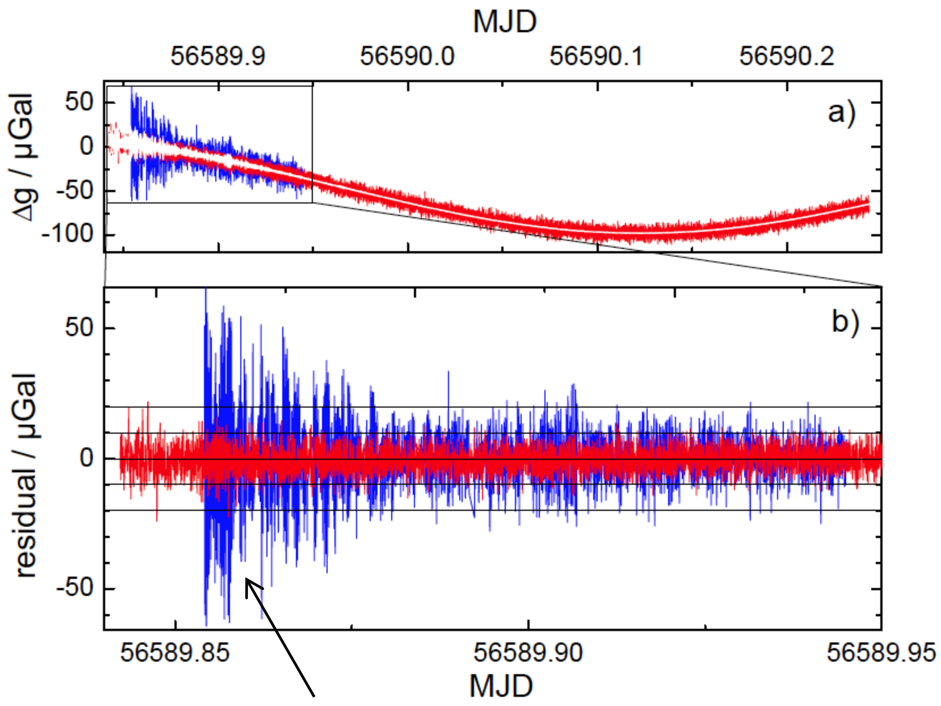
Sensitivity

Comparison with a corner cube gravimeter

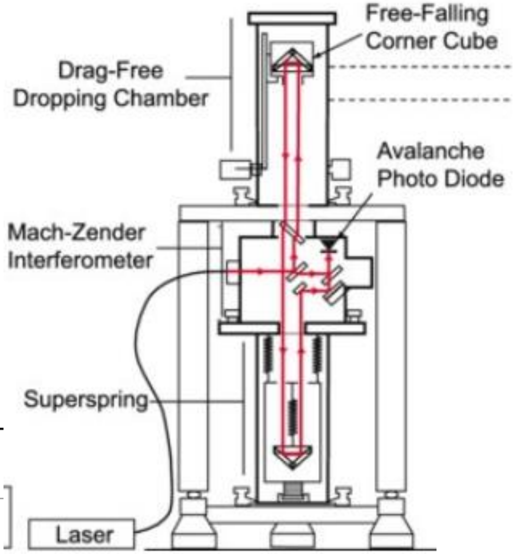
FG5X#216 (3 s cycle time)

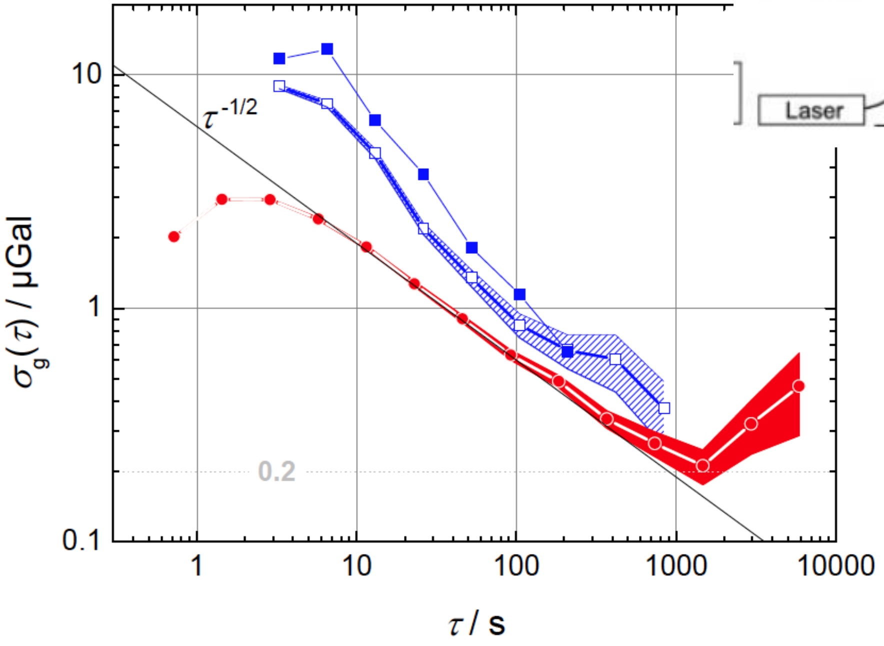
CAG (10 times faster measurement rate)

Unit of reference (used in geoscience): 1 µGal = 10^{-8} ms^{-2} ≈ 10^{-9} g



CAG : Better immunity vs earthquakes



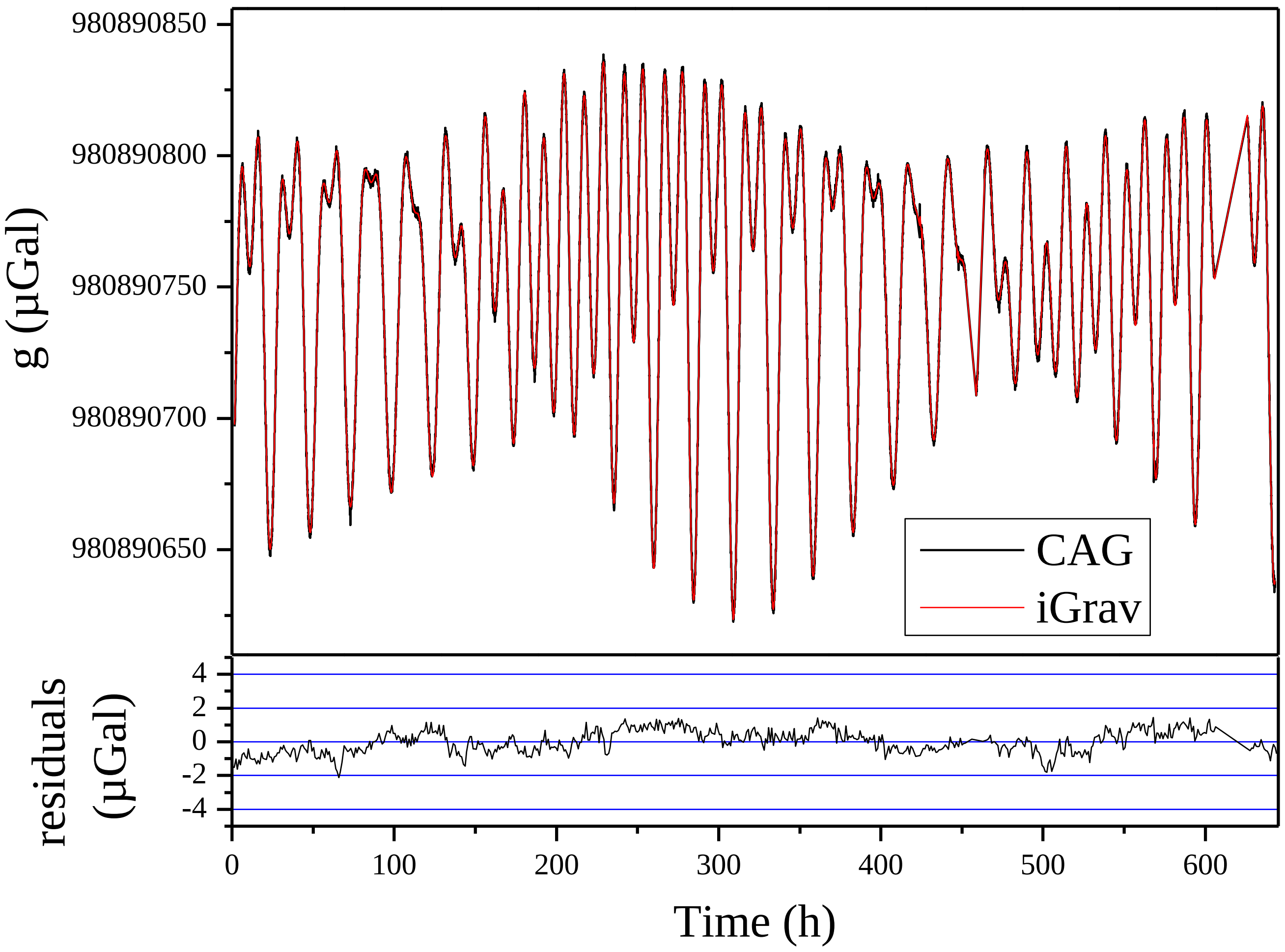


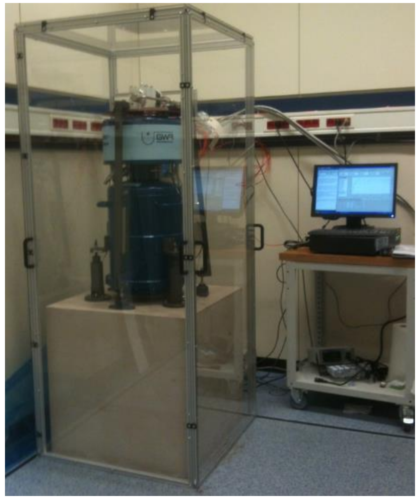
Better sensitivity than « conventional » instruments : 5.7µGal@1s

Continuous measurements

25 days of (almost) continuous measurement

Concurrent operation of two intruments : CAG and iGrav





Data averaged over 400 CAG shots = 176 s

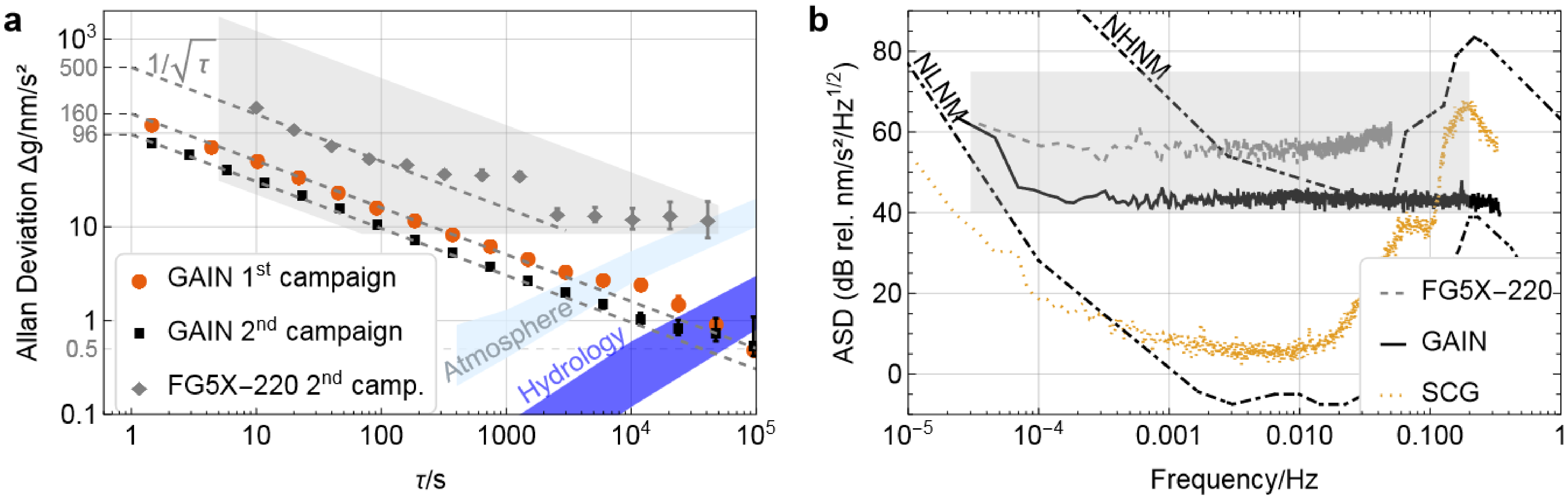
Averaged over 1h

Residuals 2 µGal ptp

1 µGal= 10^{-8} ms^{-2} ~ 10^{-9} g

Continuous measurements

Record long term stability by the GAIN, HU Berlin



Best stability: 0.5 nm/s^2 - 0.05 µGal

Accuracy

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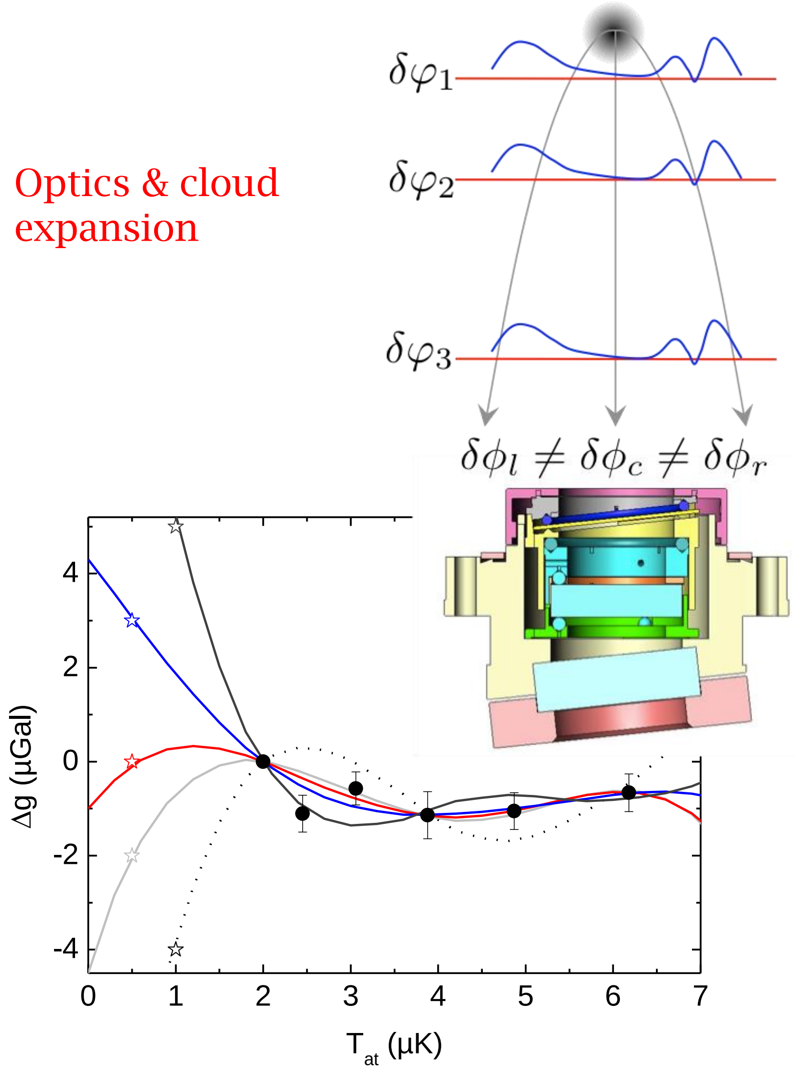
Effect	Bias μGal	u μGal
Alignments	2.4	0.5
Frequency reference	-4.6	<0.1
RF phase shift	0.0	<0.1
vgg	-10.3	<0.1
Self gravity effect	-1.3	0.1
Coriolis	1.3	0.8
Wavefront aberrations	0.0	4.0
LS1	0.0	<0.1
Zeeman	0.0	<0.1
LS2	-7.7	0.5
Detection offset	0.0	0.5
Optical power	0.0	1.0
Cloud indice	0.4	<0.1
Cold collisions	<0.1	<0.1
TOTAL	**-19.8**	**4.3**

Optics & cloud expansion



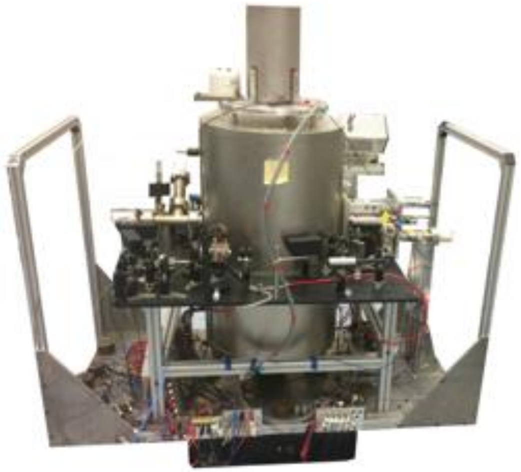
Need to limit the expansion of the cloud

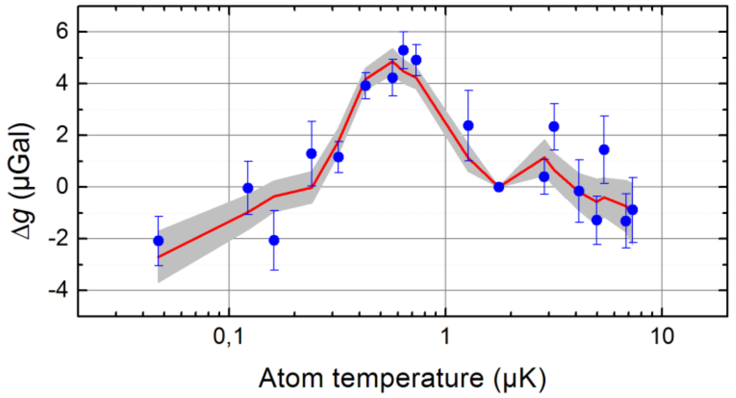
Accuracy

Effect	Bias μGal	u μGal
Alignments	0.3	0.5
Frequency reference	0.5	<0.1
RF phase shift	0.0	<0.1
vgg	-13.4	<0.1
Self gravity effect	-2.1	0.1
Coriolis	-5.3	0.8
Wavefront aberrations	-5.6	1.3
LS1	0.0	<0.1
Zeeman	0.0	<0.1
LS2	-3.6	0.8
Detection offset	0.0	0.5
Optical power	0.0	0.5
Cloud indice	0.4	<0.1
Cold collisions	<0.1	<0.1
CPT	0.0	<0.1
Raman α LS	0.3	<0.1
Finite Speed of Light	0.0	<0.1
TOTAL	**-28.5**	**2.0**

Evaporative cooling
in a dipole trap

Temperature range: [50 nK -7 µK]

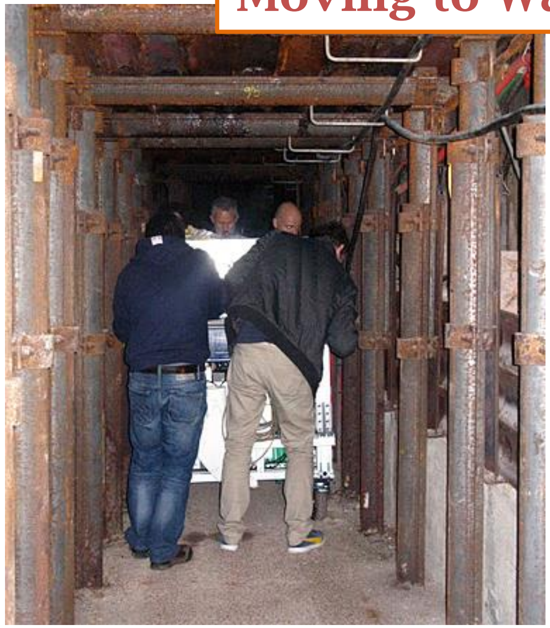


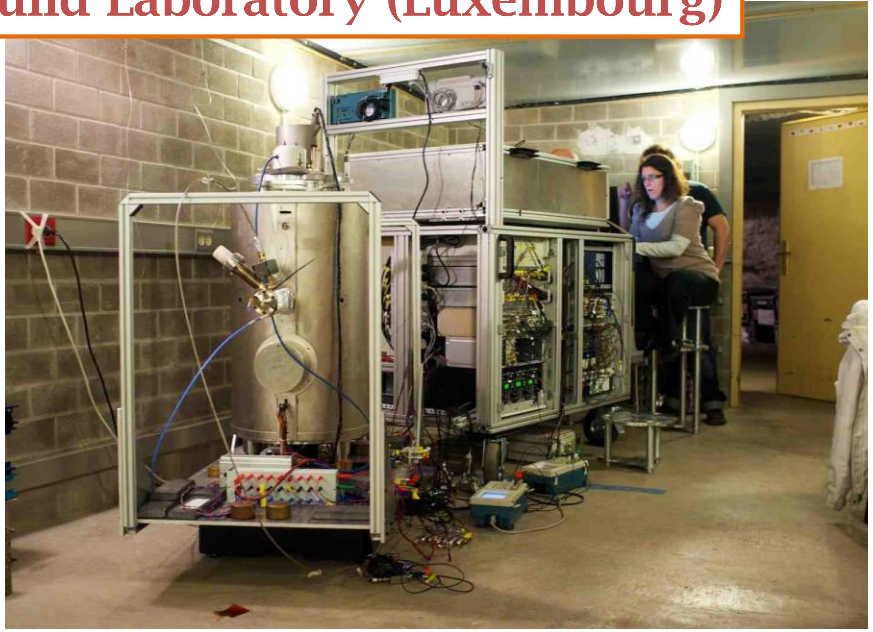


Transport out of the laboratory

l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace

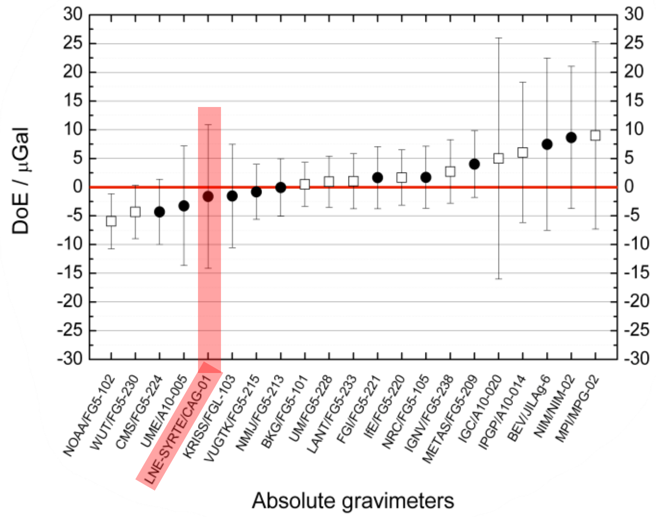
Moving to Walferdange Underground Laboratory (Luxembourg)



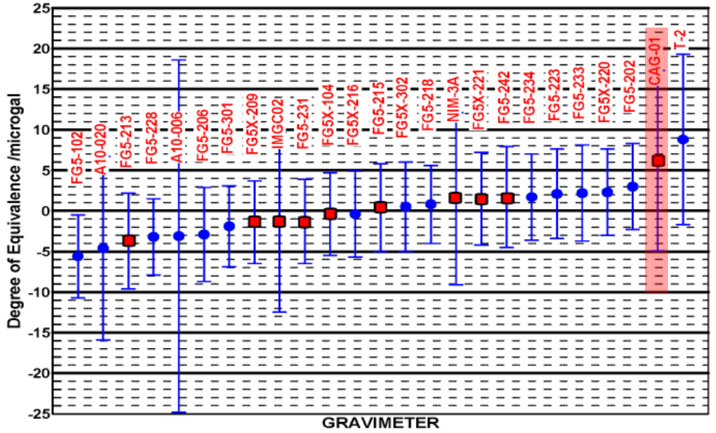


International comparisons

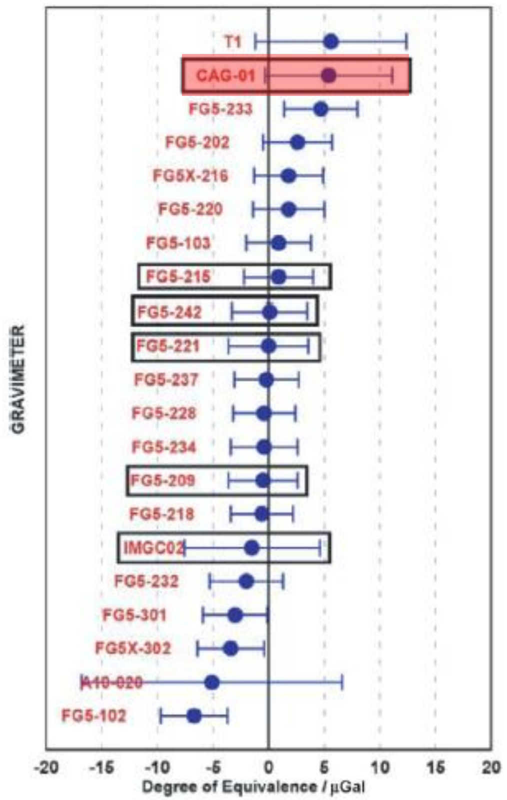
ICAG'09 (BIPM) : CCM.G-K1



CCM.G-K2 (2013, LUX)



ECAG (2011, LUX)



Results

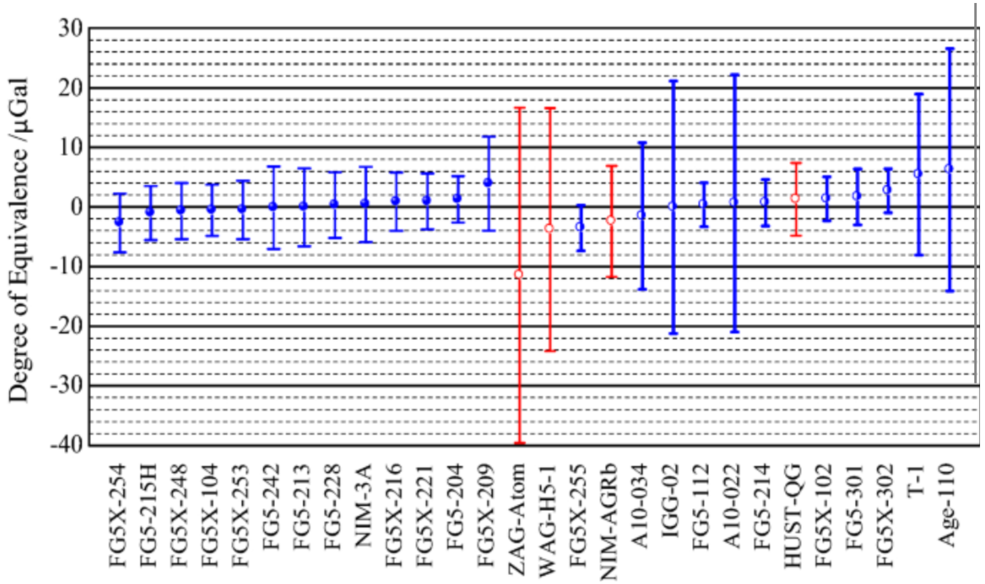
Date	Difference g_{CAG}-g_{REF} (µGal)
2009	- 1.6 (7.8)
2011	+ 5.4 (5.7)
2013	+ 6.2 (5.5)

Two « key » comparisons 2009 & 2013
CAG : 1st atom gravimeter to participate
All other instruments: corner cube gravimeters

CCM.G-K3

Third key comparison in 2017 in China

6 atom gravimeters
out of 30 instruments in total
(4 have contributed)
All from chinese groups !



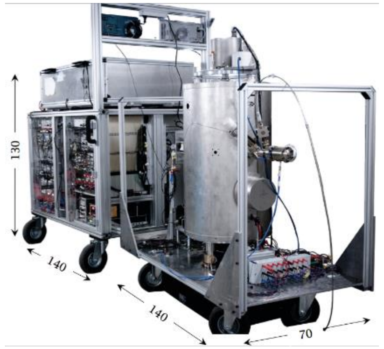
Atom gravimeters anticipated
to be reference standards
in the future

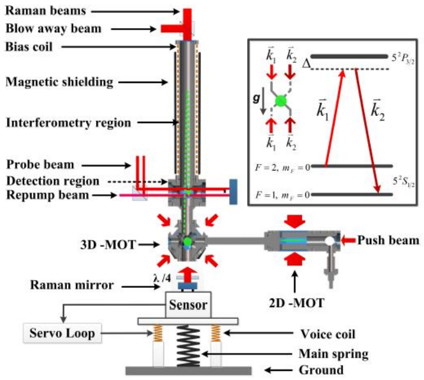
Since then, a new key comparison in 2023

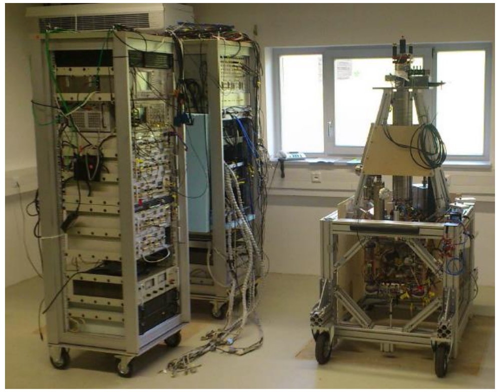
Quantum gravimeters (on the ground)

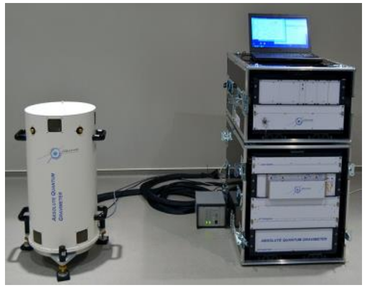
l'Observatoire de Paris — SYRTE — Systèmes de Référence Temps-Espace

State of the art quantum (vertical) accelerometers = quantum gravimeters









	LNE SYRTE	WUHAN	HUB	eXail (ex Muquans)
2T (ms)	160	600	600	120
Sensitivity ($/Hz^{1/2}$)	5.7 10^{-9}g	**4.2 10^{-9}g** (2 10^{-9}g)	10 10^{-9}g	50 10^{-9}g
Long term stability	< 10^{-10}g	5 10^{-10}g	**5 10^{-11}g**	10^{-9}g
Accuracy	**2 10^{-9}g**	3 10^{-9}g	3 10^{-9}g	A few 10^{-9}g

Differences in T (and thus in the scale factor) do not necessarily correlate with the performances

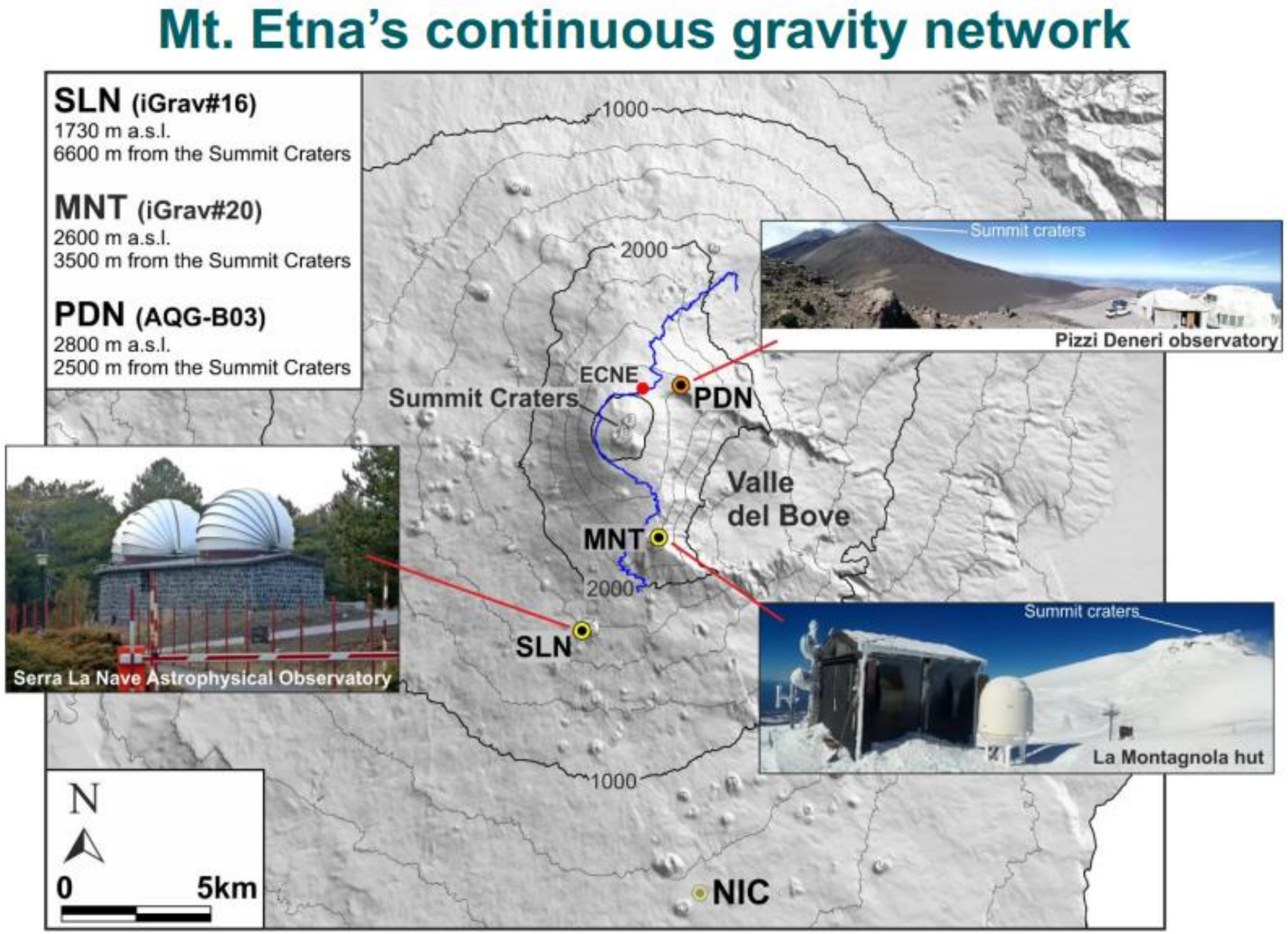
Deployment on the field by INGV

l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace

eXail AQG = compact gravimeter
for in lab and on-field operation

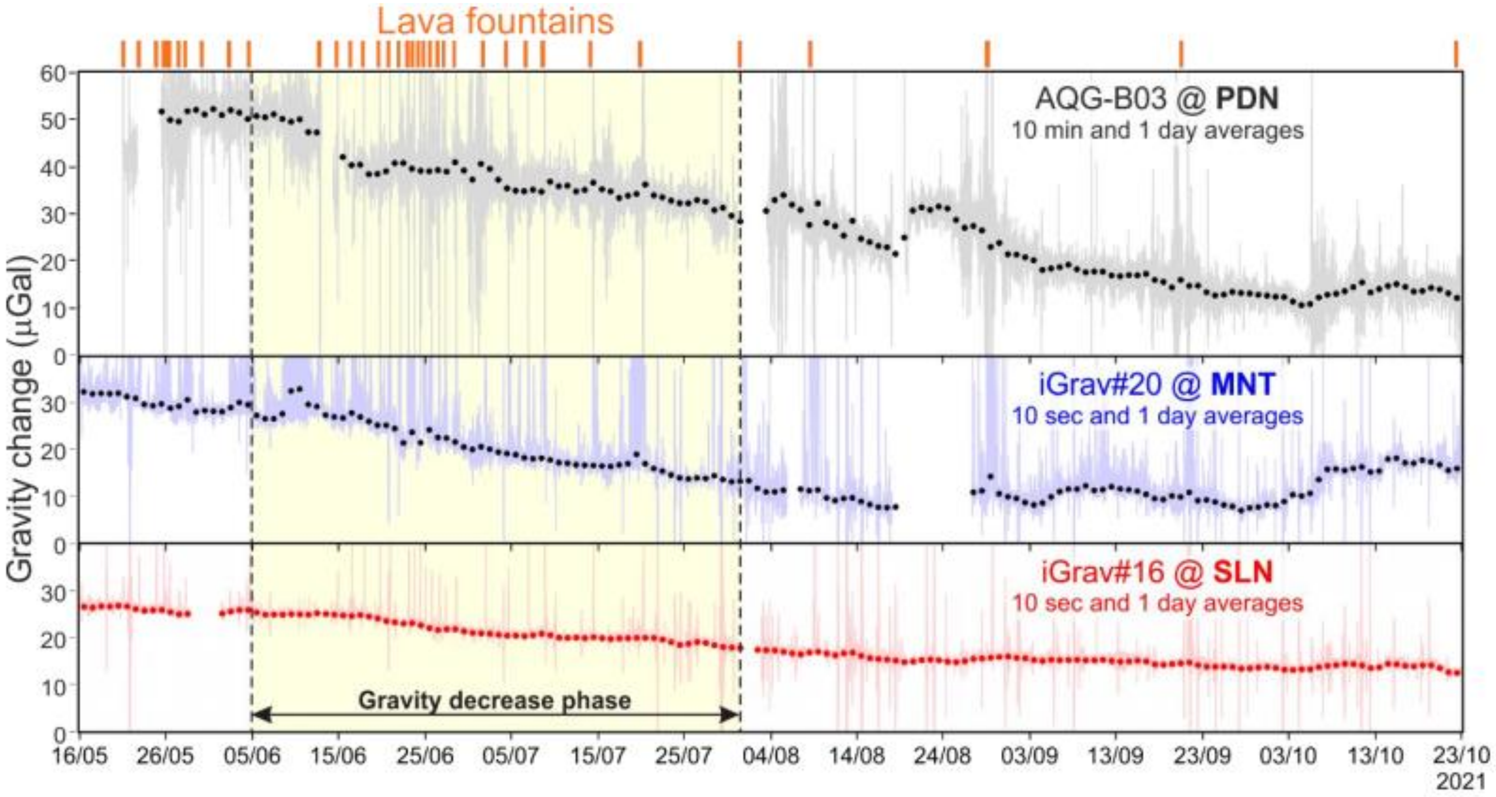
Specifications & Characteristics

Sensitivity:	50 µGal/√Hz at a quiet place
Measurement frequency:	2 Hz
Long-term stability:	< 1 µGal
Accuracy:	under evaluation
Dimensions :	Sensor head: h = 70 cm / D = 38 cm
Laser & electronics:	100 x 50 x 70 cm³
Mass Sensor head:	25 kg, control unit : 75 kg
Power consumption:	250 W typical



Deployment on the field by INGV

Recording of high-quality time series, allowing to track volcano related gravity changes



2021 lava fountains at Mt. Etna

Combined analysis of gravity changes and deformation field by GNSS allows obtaining insight on the dynamics

Deployment on the field by GFZ & BKG

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Gravimetric field survey with an Absolute Quantum Gravimeter

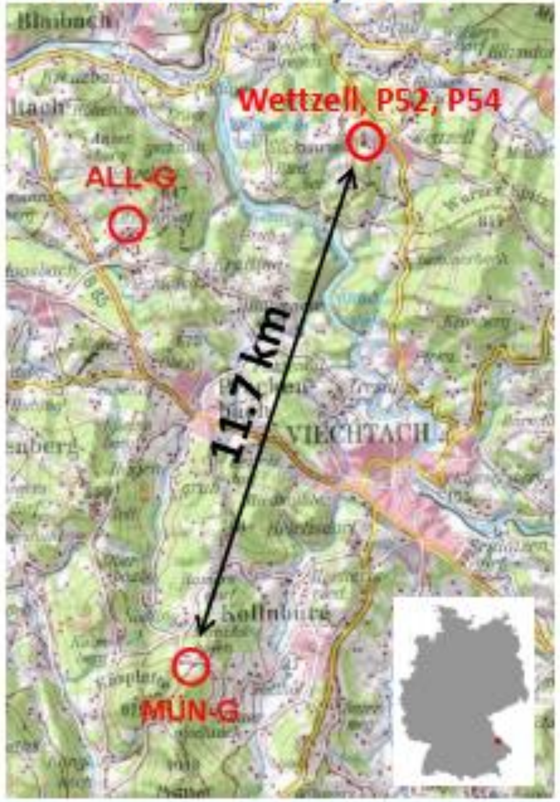
Instrument: Muquans AQG#B02

Study area: Gravimetric footprint network of the Geodetic Observatory Wettzell, Germany

Sites: 4 field sites (forest/grassland), 1 observatory site

564 m amsl,
gravity difference to
Wettzell Δg_w= +5.13 mGal

807 m amsl,
Δg_w= -48.03 mGal



643 m amsl,
Δg_w= -7.56 mGal

Wettzell observatory

607 m amsl
g= 980837.1 mGal

Deployment on the field by GFZ & BKG

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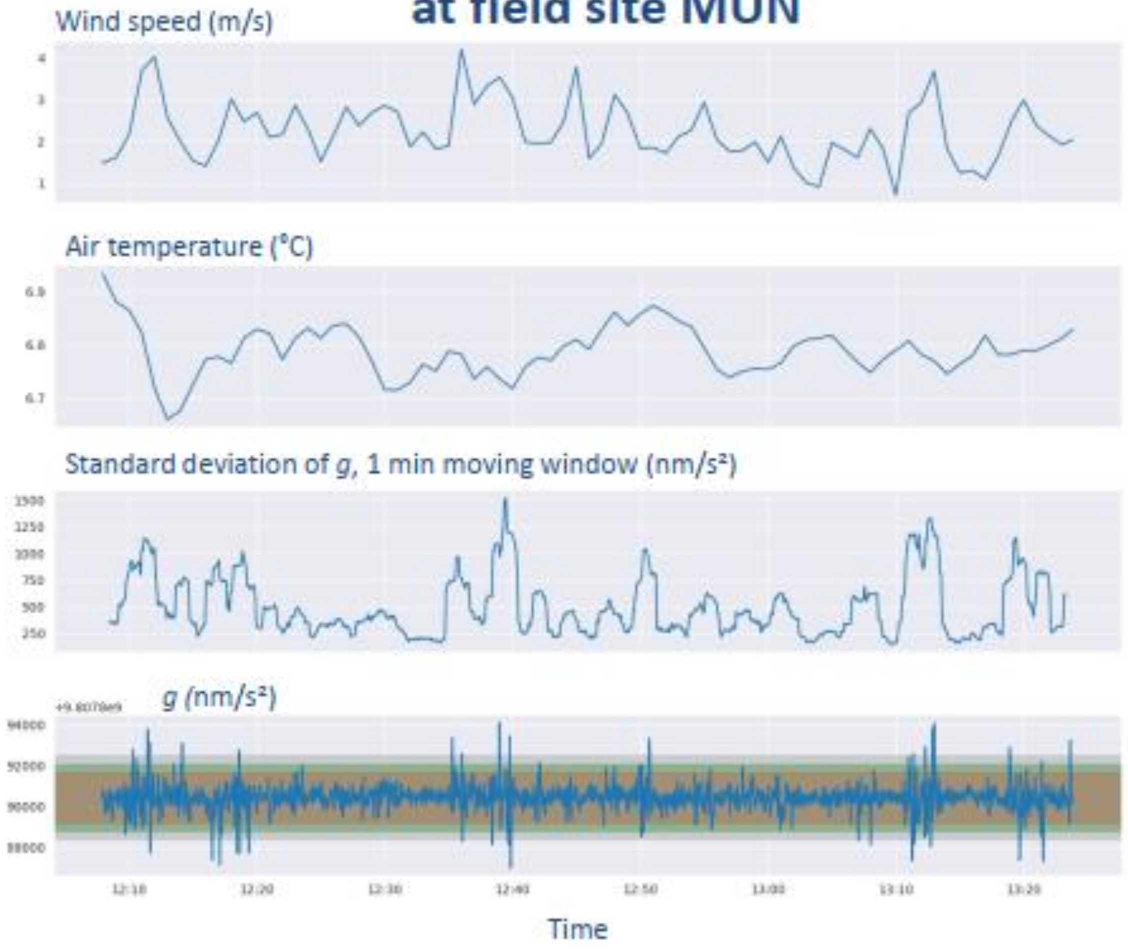
AQG field survey results – Sensitivity and precision

A total of 15 measurements have been performed at the different sites within a 4-day survey period, with a duration of at least about 60 minutes for each measurement.

Statistics of absolute gravity *g* after 60 min
(as mean of all 15 measurements):

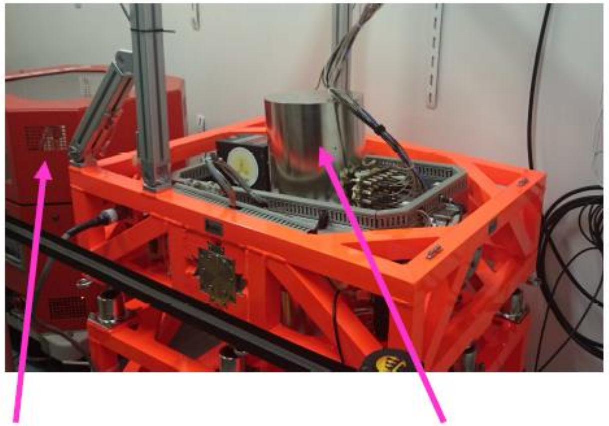
- Standard deviation of raw data (individual drops of about 2 Hz): **273 nm/s²**
- Standard error of mean: **11 nm/s²**
- Slight dependence of precision on wind speed (turbulence causes movements of truck with AQG laser unit and of laser fibres), see example for MUN

Example of AQG measurements at field site MUN



On board measurements

Development of a compact gravimeter for marine gravimetry by ONERA
Measurement campaigns on the Beautemps-Beaupré (French Navy)



KSS32 relative Marine Gravimeter (Bodenseewerk)

Cold Atom Gravimeter (Onera)

Better performance for gravity mapping with the absolute atom gravimeter
Suppression of calibration errors and drift corrections

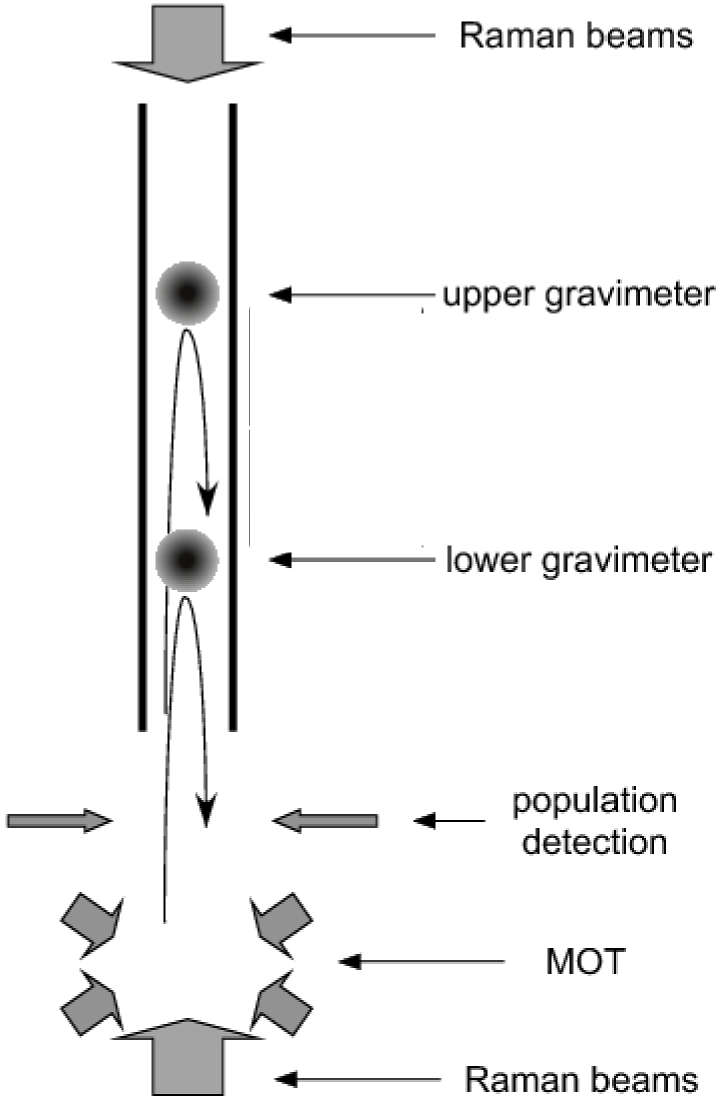
→ Gain of a factor 2-3 on the uncertainty (mGal level)

+ Plane campaign

Outline

1. Applications of inertial sensors / accelerometry
2. Principle of atom interferometry
3. Gravimeters: state of the art quantum accelerometers
4. Cold Atom Interferometer Gravity Gradiometer
5. Other sensors and applications

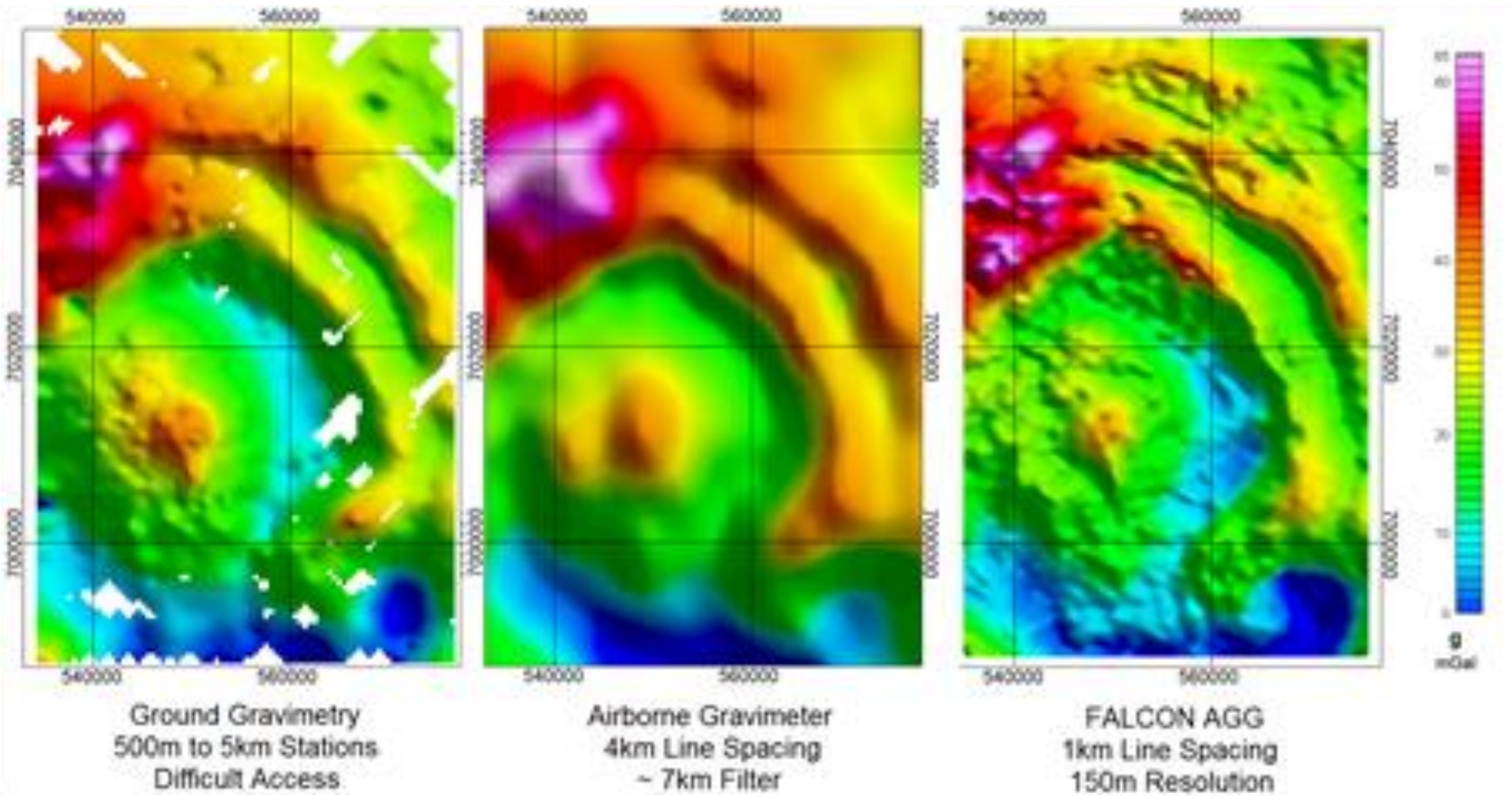
CAI Gravity Gradiometer



- **Simultaneous interferometers on two cold atom clouds with common Raman lasers**
- **Differential measurement allows for extracting the acceleration difference and thus the Earth gravity gradient**
- **Suppression of common mode noise, and in particular of the vibration noise**
- **Adapted for onboard measurements**

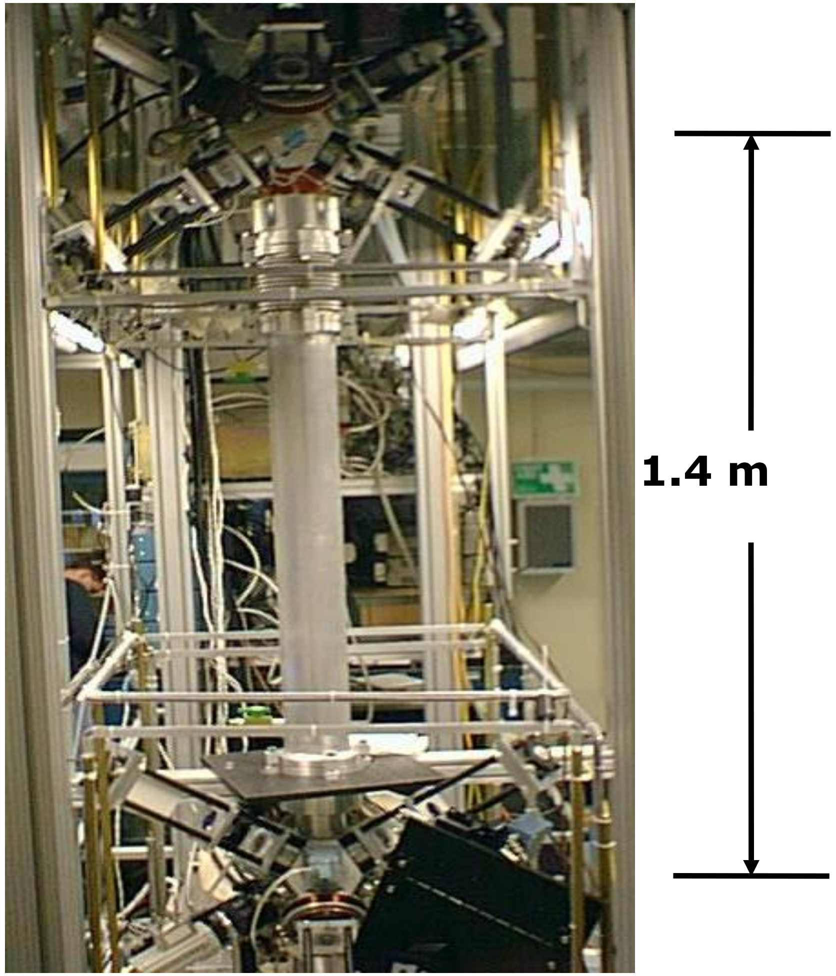
Gravity gradiometers

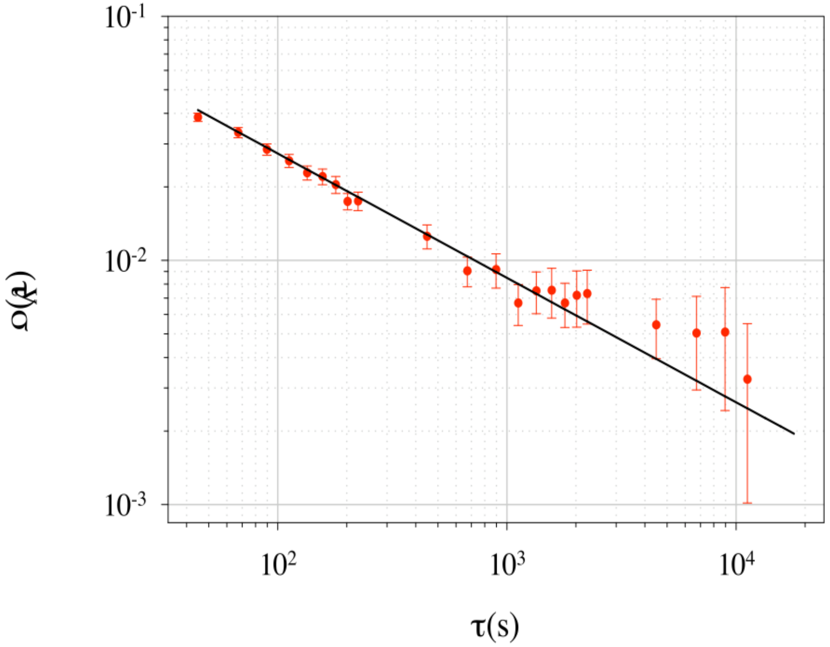
Comparisons of gravity maps realized with
1) ground-based gravimeters
2) airborne gravimeters
3) airborne gradiometers



CAI Gravity Gradiometer

Yale/Stanford, around 2000





Demonstrated differential acceleration sensitivity:

$4\text{x}10^{-9}$ g/Hz$^{1/2}$

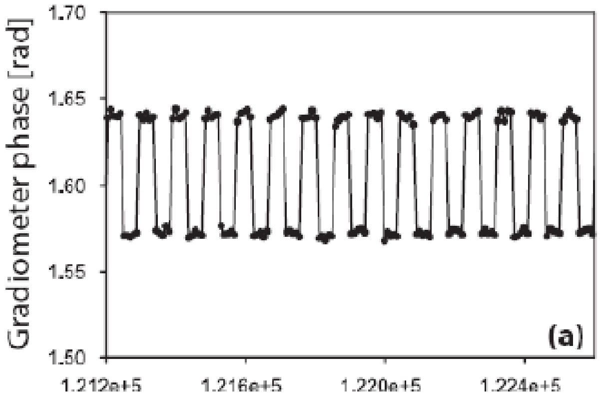
(2.8x10^{-9} g/Hz$^{1/2}$ per accelerometer)

Gradient sensitivity: ~ 30 E/Hz$^{1/2}$

Measurement of G

l'Observatoire de Paris SYRTE
Systèmes de Référence Temps-Espace

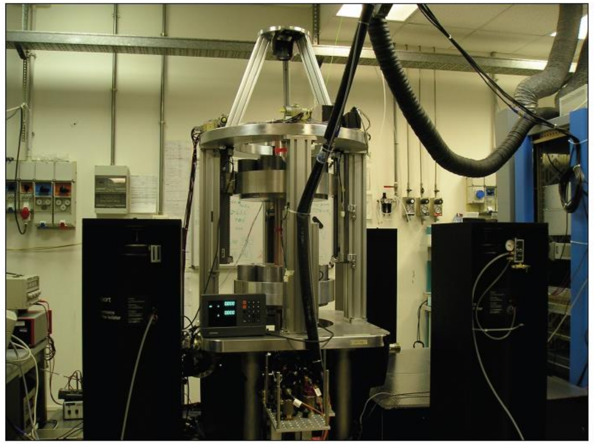
Stanford (M. Kasevich)

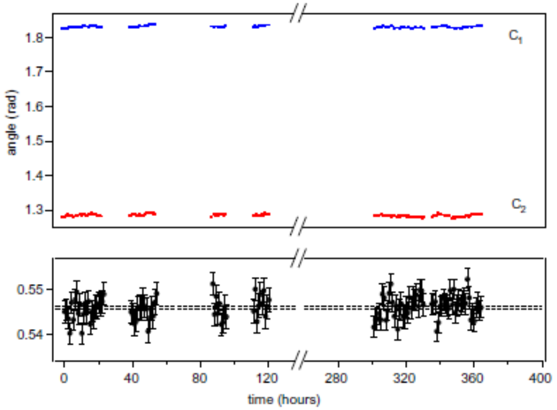


Differential acceleration sensitivity demonstrated: 10^{-11} g

Statistical uncertainty: 2 10^{-4} on G

Florence (G. Tino)

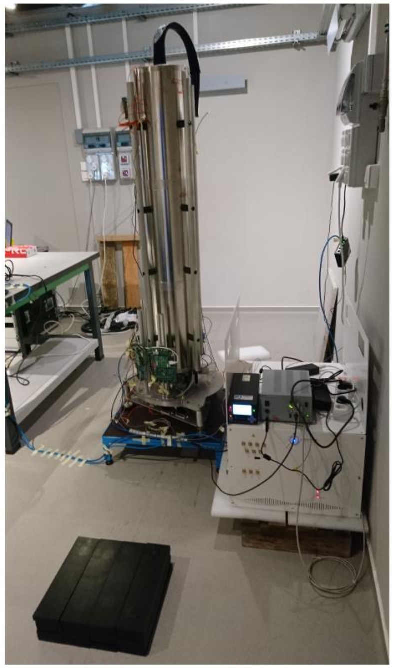


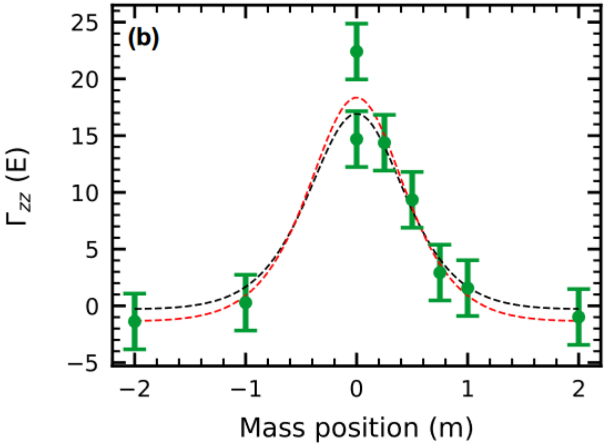


Combined uncertainty: 1.5 10^{-4} on G

More recent achievements

eXail gradiometer
Sensitivity : 50 E @ 1 s
Long term stability : 0.1 E





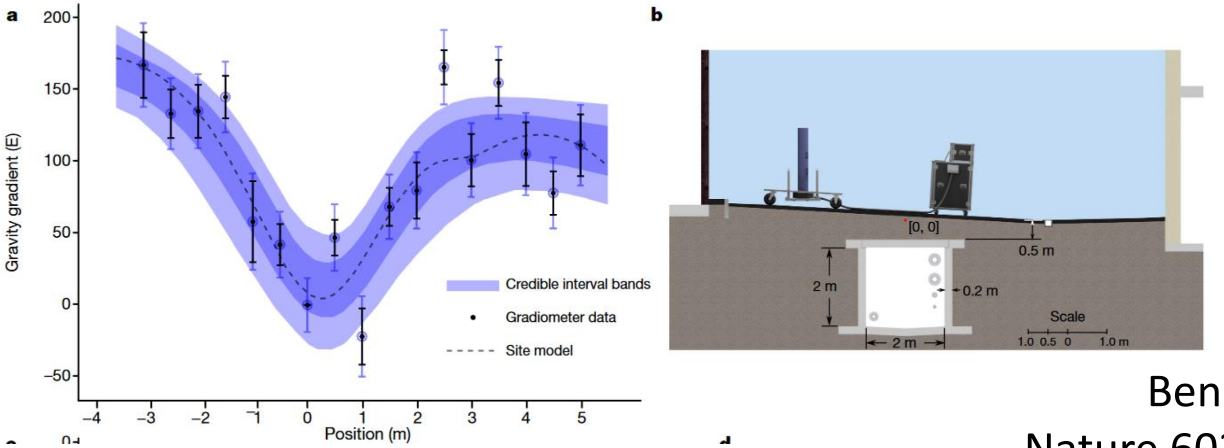
Camille Janvier et al.
Phys. Rev. A 105, 022801 (2022)

Second generation instrument currently under development

European project FIQUgS

https://www.fiqugs.eu/

U Bham gradiometer
Sensitivity : 500 E @ 1 s
Long term stability : 20 E



Ben Stray et al.,
Nature 602, 590–594 (2022)

A wealth of activities (academic +industry)

Gradiometer at sea
Bham

Hangzhou
Gravimeters
at sea
in a truck

Vector Atomic
Gravimeter at sea

Nomad Atomics
Portable gravimeter

Outline

l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace

Inertial navigation

Most « natural » field of application

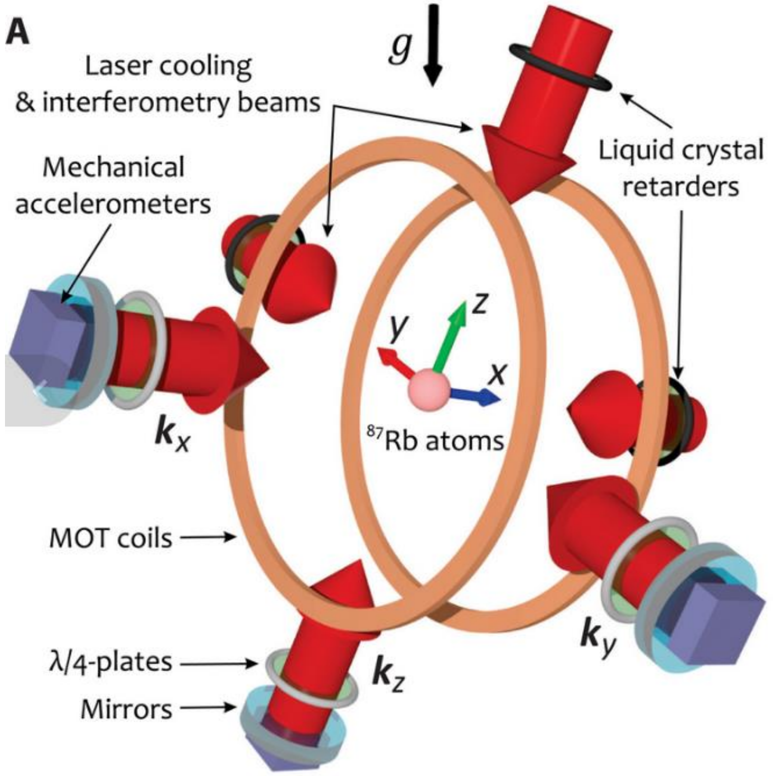
But there are difficulties

- ✓ Conventional sensors have (very) high level of performances
- ✓ Constraints due to the harsh environment
- ✓ Size, Weight, Power budget is demanding
- ✓ Incompatible with dead times

One possible solution to overcome dead times: Hybridization

Inertial navigation

S. Templier et al, **Tracking the vector acceleration with a hybrid quantum accelerometer triad**
Science Advances Vol 8, Issue 45 (2022)



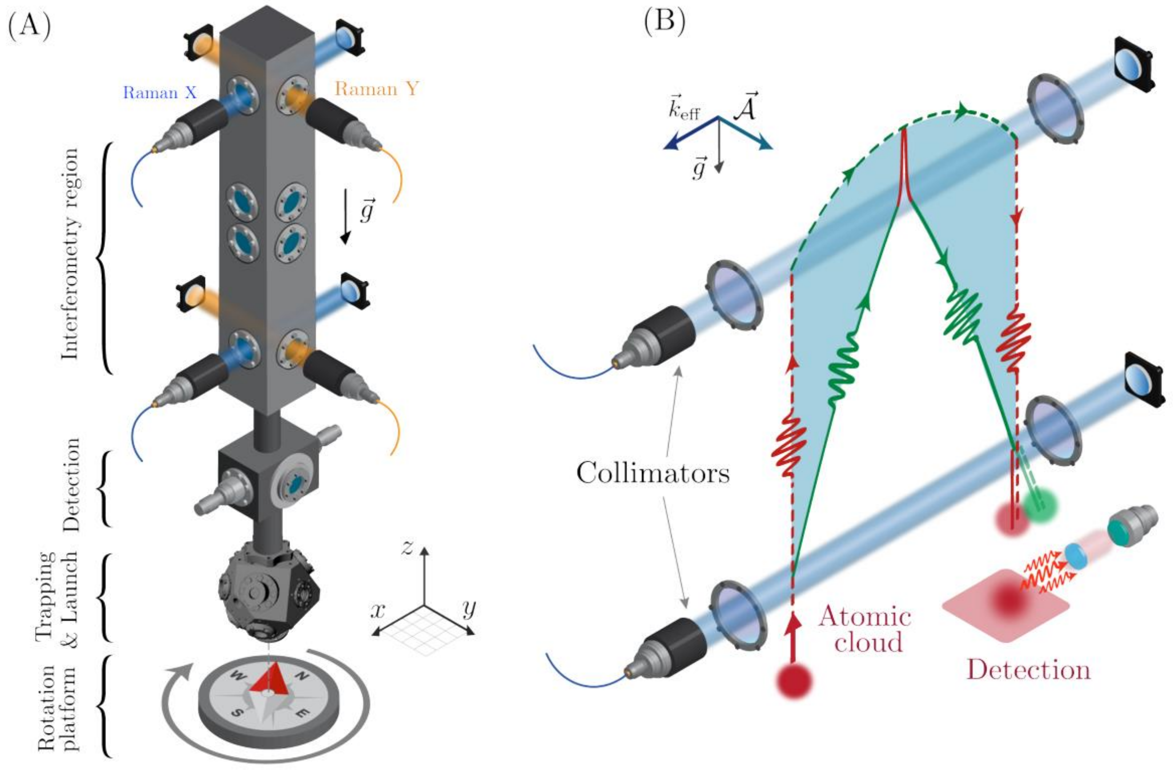
Laboratoire commun iXAtom (LP2N-iXblue)

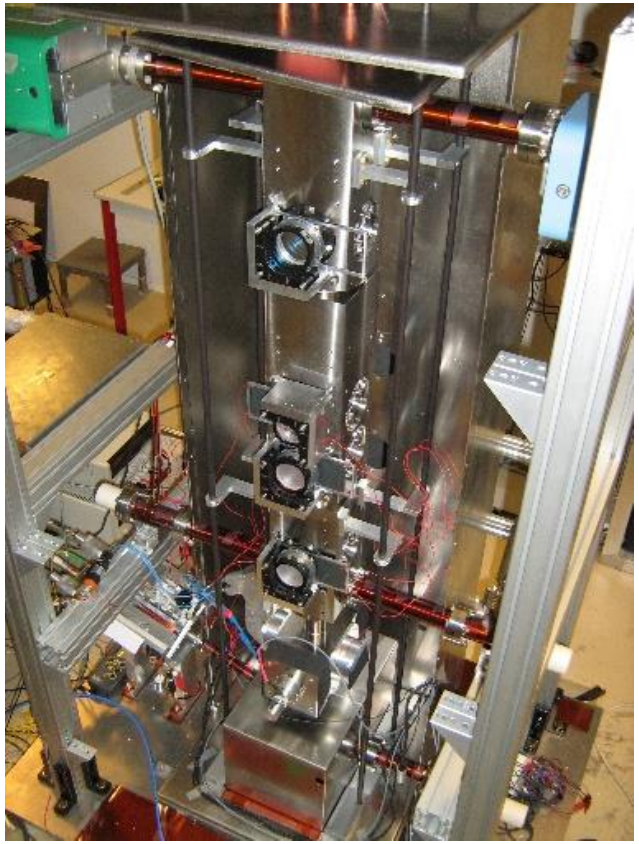
Demonstration of the first quantum accelerometer triad (QuAT)
Measures accelerations along three mutually orthogonal directions

Long-term stability of 60 ng
High data rate (1 kHz)
absolute magnitude accuracy below 10 µg
pointing accuracy of 4 µrad

Cold atom gyroscope

- Based on a 4-pulse sequence
- No DC acceleration sensitivity
- Pure gyroscope





Main results

- Zero dead time operation
- **Stability : 0.2 nrad/s**

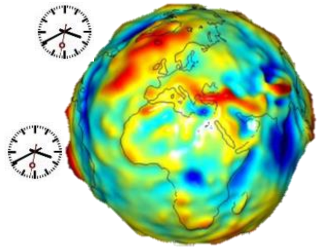
(state of the art atomic gyro)

Chronometric geodesy

❑ Principle

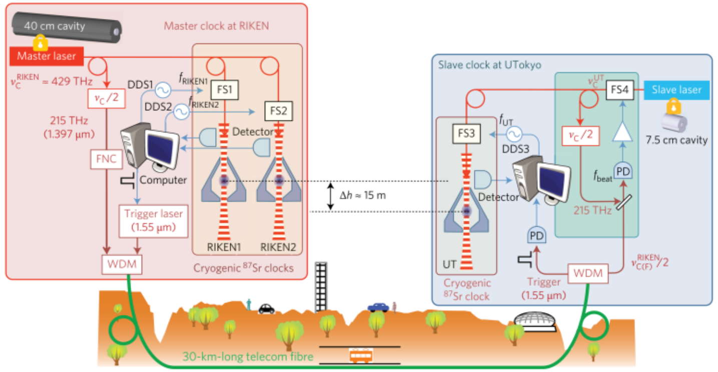
- Compare distant clocks to determine differences in the gravitational potential
- Sensitivity : 10^{-18} ⇔ 1 cm

$$\frac{\nu_2}{\nu_1} = \left(1 - \frac{U_2 - U_1}{c^2}\right)$$



❑ Specificity

- Direct measure of the potential
- Local measurement
- Sensitivity to mass sources in 1/r



Remote comparison of two Sr optical clocks

❑ Towards applications in earth sciences

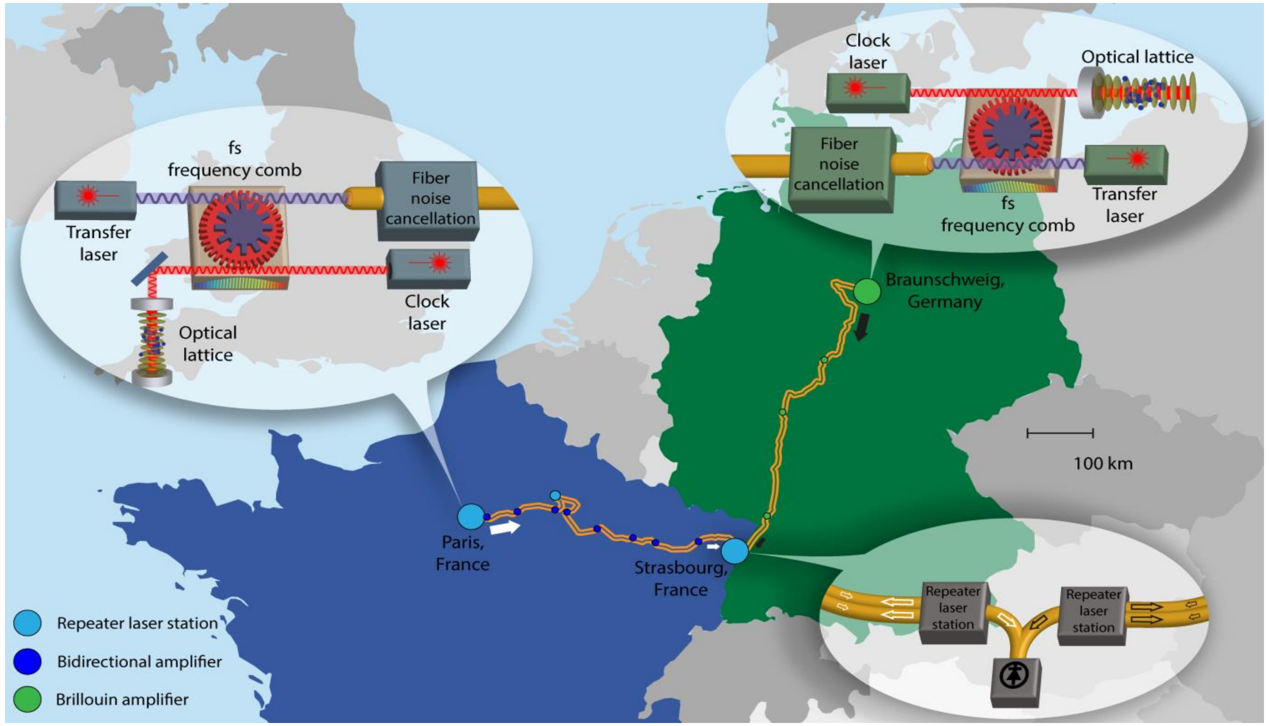
- First demontrations realized
- Objective: allow to reach the 10^{-18} level anywhere
  - Transportable clocks
  - Develop fiber and free space links

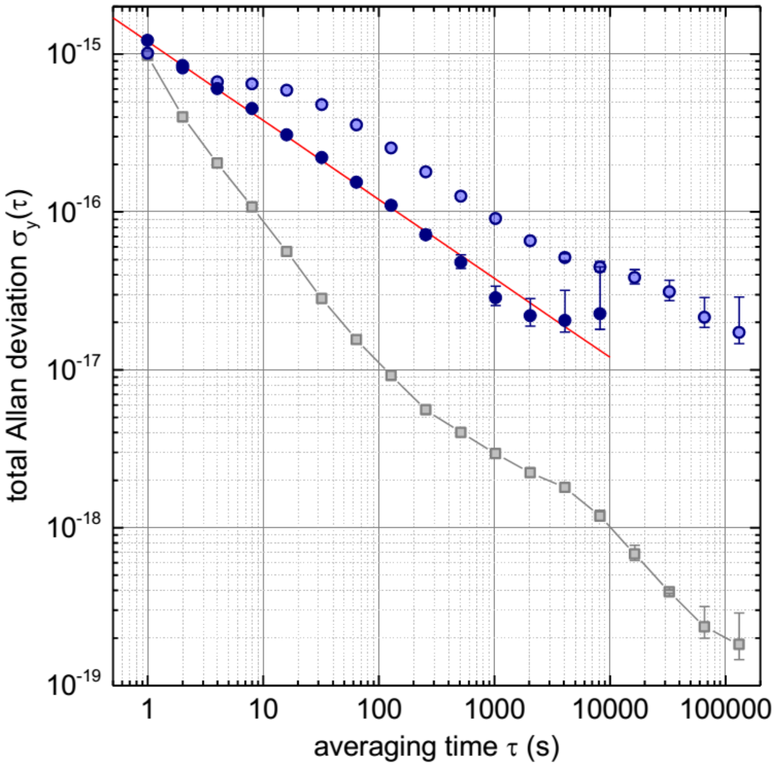
PTB Transportable Clock

Fiber links

❑ Example of the Paris (SYRTE) – Braunschweig (PTB) fiber link

- In 1s, the link compares as well as satellite methods in a day
- Allows to compare to better than 10^{-18} over continental distances





❑ Construction of an european network for clock comparisons and the dissemination of stable time and frequency references

Conclusion

- **Maturity of atom interferometry techniques for the development of inertial sensors**
- **Performances compare, or overpass, classical technologies**
- **Industrial transfer realized and commercial products available**
- **Current efforts towards reduced SWAP and improved on the field operability**
- **Clear perspectives for improvements (LMT, squeezing …)**
- **A variety of fields of applications:**
  fundamental physics, geosciences, reservoir monitoring, exploration, civil engineering, navigation …

Increasing the interaction time

Increasing T increases

- the intrinsic sensitivity (the scale factor)
- the size of the experimental set up

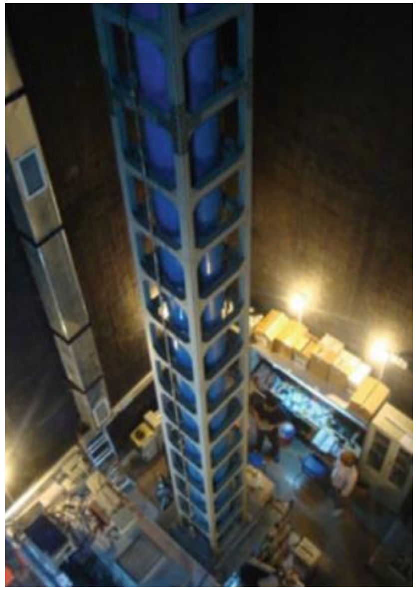
Release Vs Launched (fountain geometry)

T	H, release	H, fountain
10 ms	0.5 cm	0.1 cm
80 ms	**12.5 cm**	3 cm
300 ms	1.8 m	**45 cm**
1 s	20 m	5 m
1.4 s	40 m	**10 m**

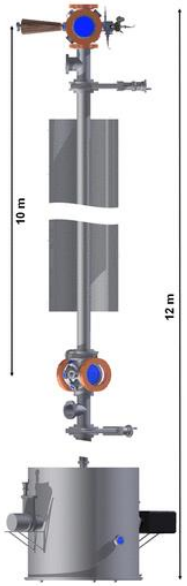
A few ongoing projects of 10 m tall chambers

Stanford

Wuhan



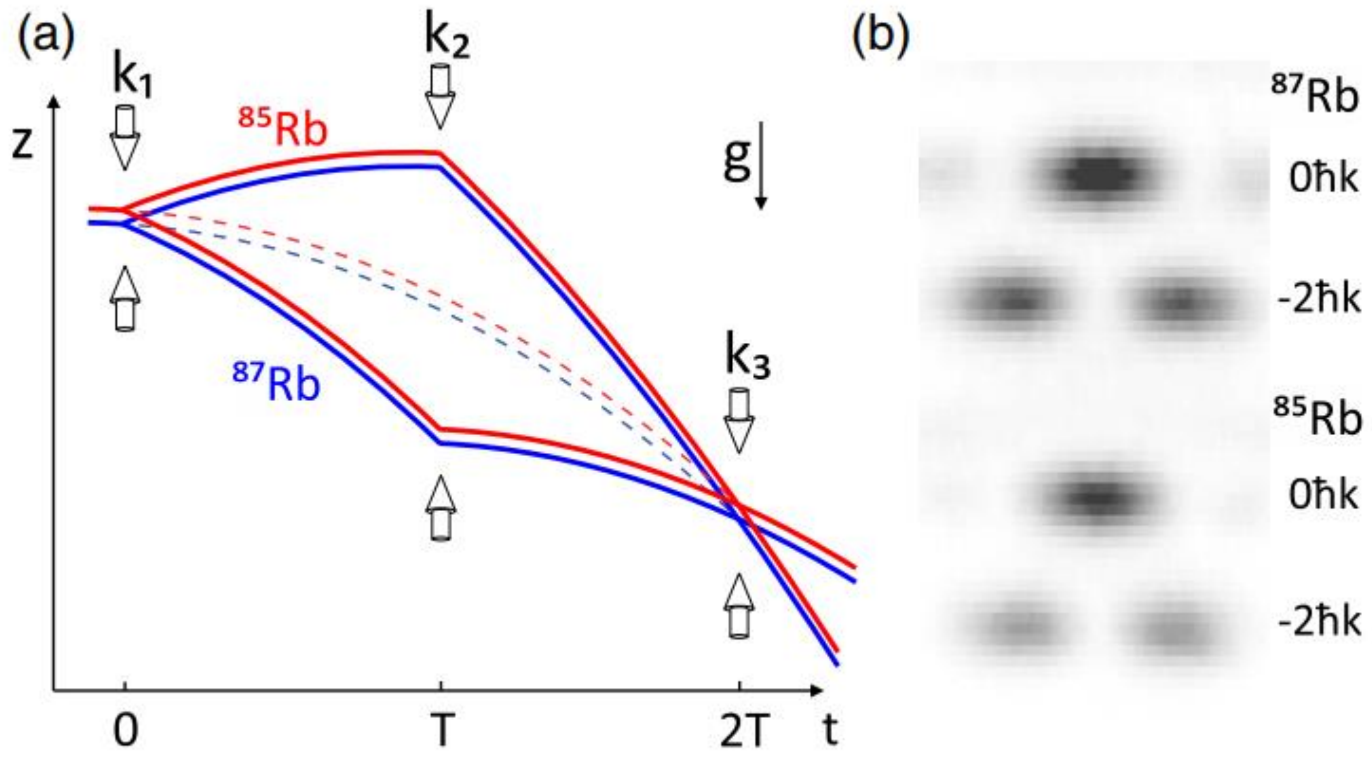
Hannover



Test of the WEP

Comparing gravity acceleration experienced by 85Rb and 87Rb

Asenbaum et al, PL 125, 191101 (2020)

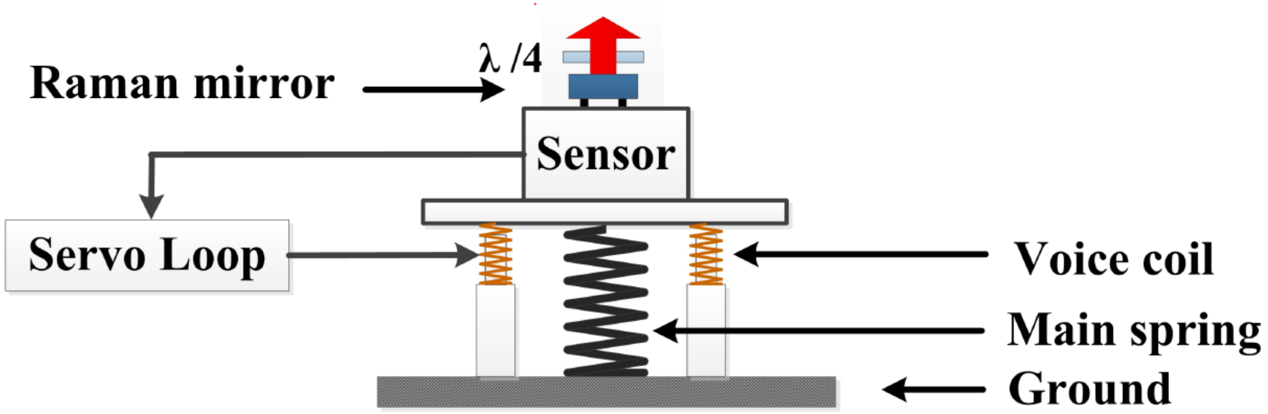


Total interferometer duration 2T = 1910 ms

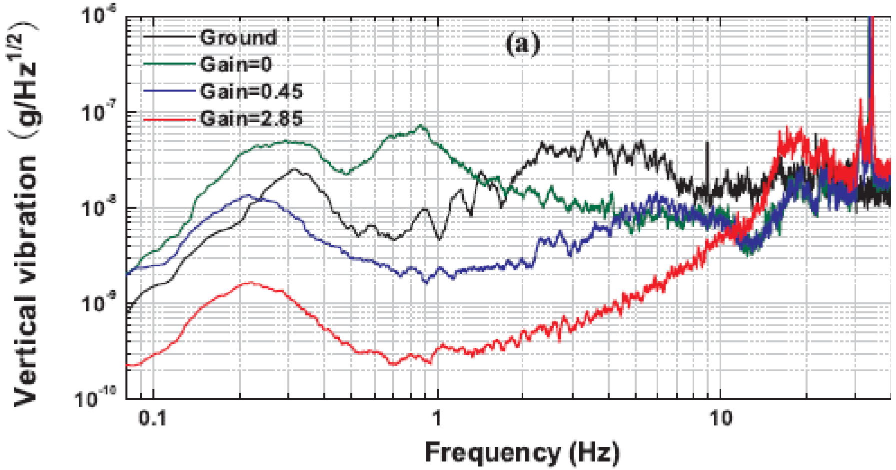
Eotvos parameter $\eta = \frac{\Delta a}{g} = (1.6 \pm 1.8(\text{stat}) \pm 3.4(\text{syst})) \times 10^{-12}$

Sensitivity limits: vibrations

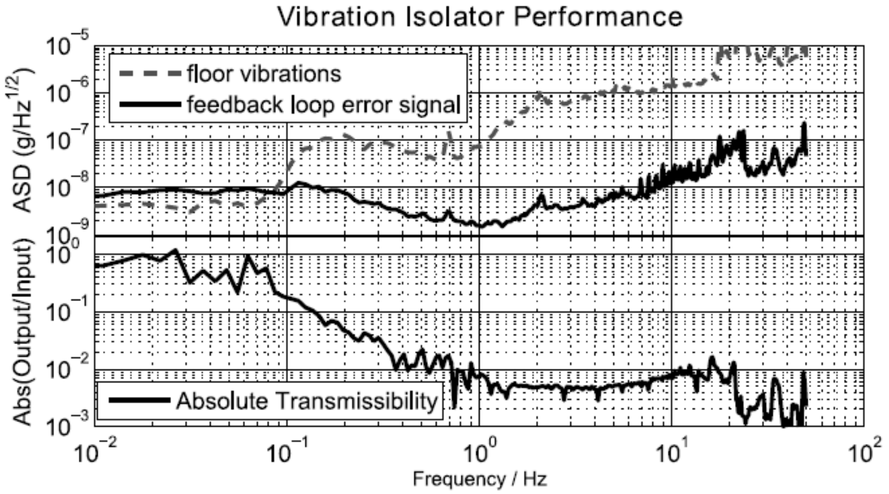
Active vibration isolation



WUHAN



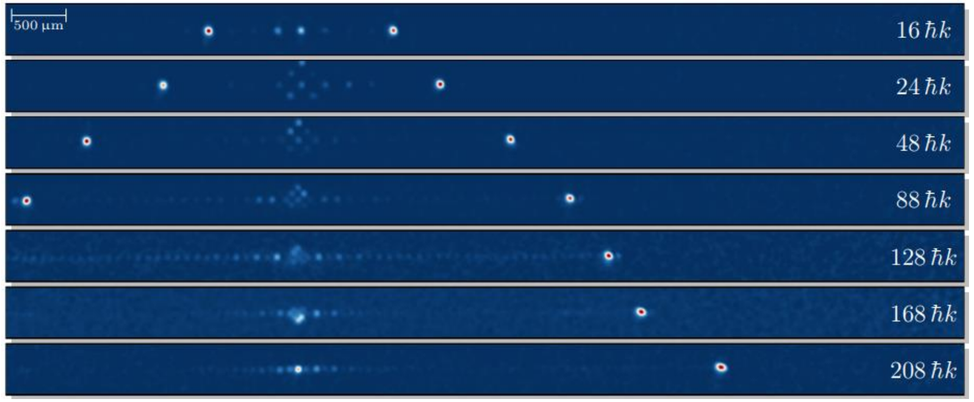
BERLIN



=> Similar Transmissibilities, but less (ground) noise in Wuhan

Large momentum transfer beamsplitters

Bragg diffraction in optical lattices

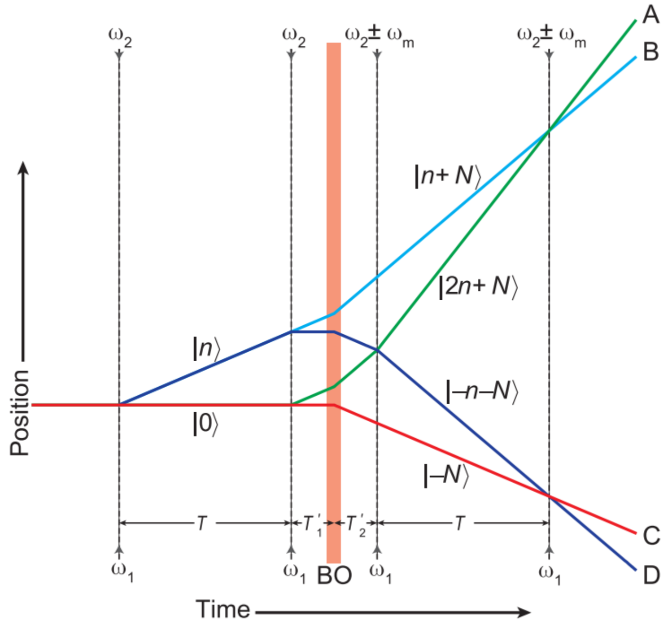


Courtesy: S. Abend, LUH

Allows momentum separations from $2\hbar k$ to $2N\hbar k$, with $N \sim 100s$ if not $1000s$

Pioneering work in the group of H. Müller

Application:
Measurement of the fine structure constant with 2 10^{-10} relative uncertainty
Parker et al., Science 360, 191–195 (2018)



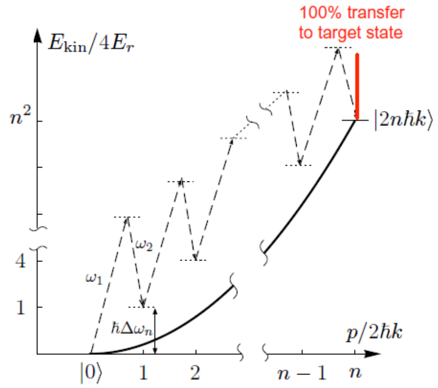
Gravity gradiometer at SYRTE

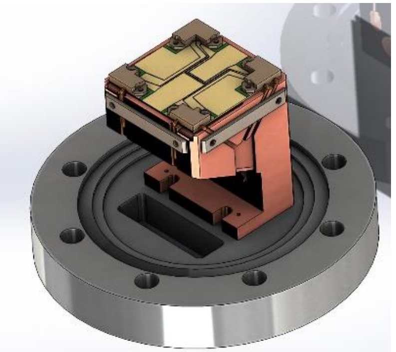
Ideal test bed for the development of new methods

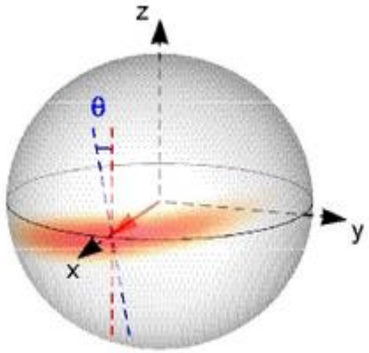
High order Bragg diffraction
Gain x 100 separation

Ultracold atoms
(atom chips)

Spin squeezing



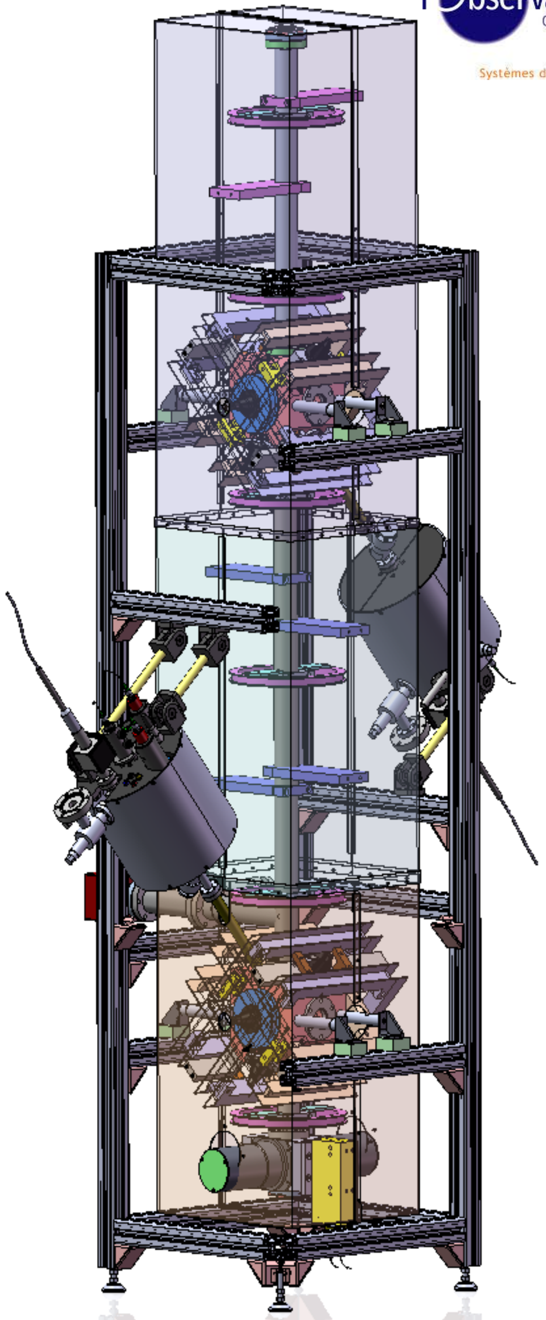




Expected sensitivity: 10^{-10} s^{-2} = 0.1 E at 1s

More than one order of magnitude better than

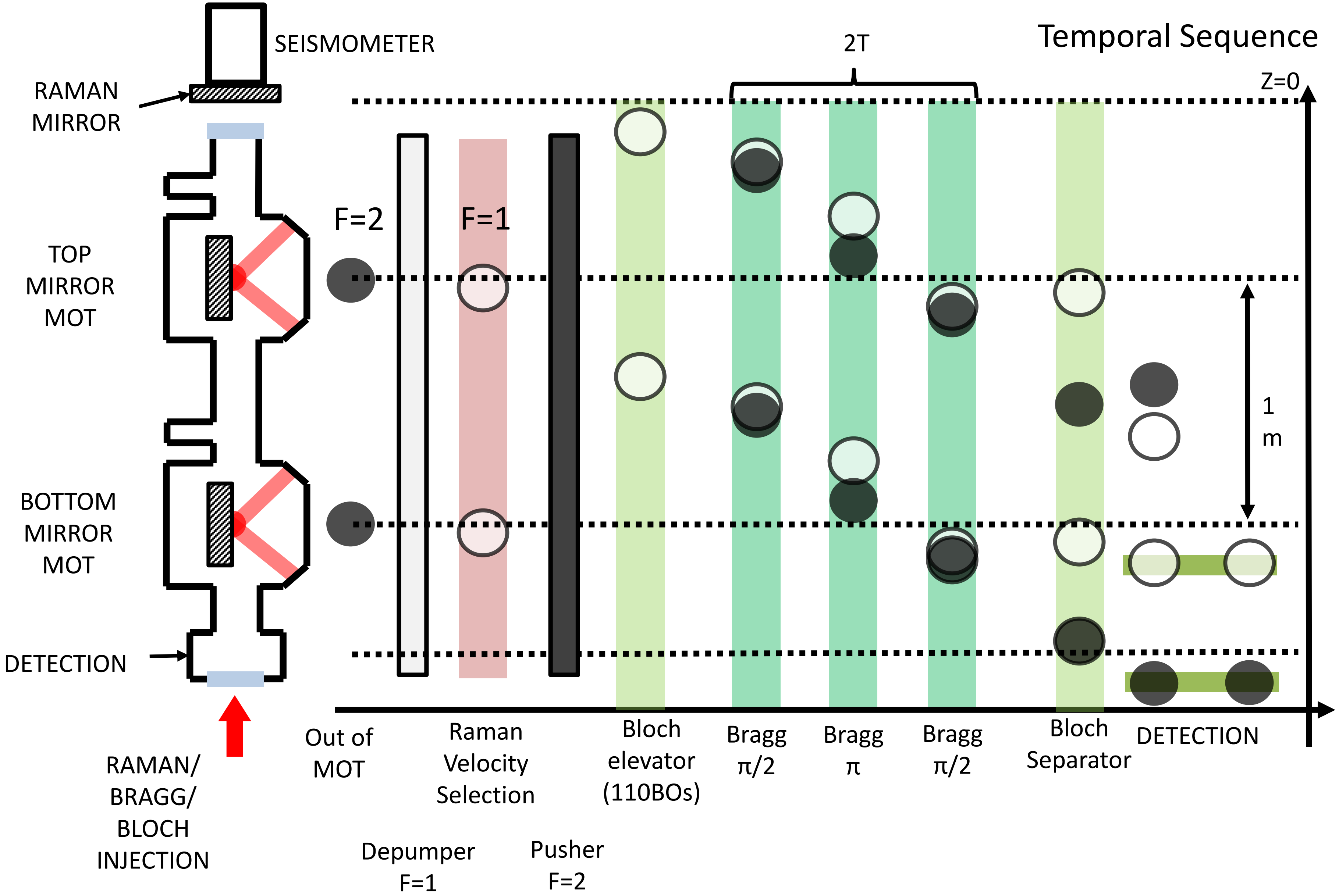
state of the art sensors based on differential accelerometry



l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace

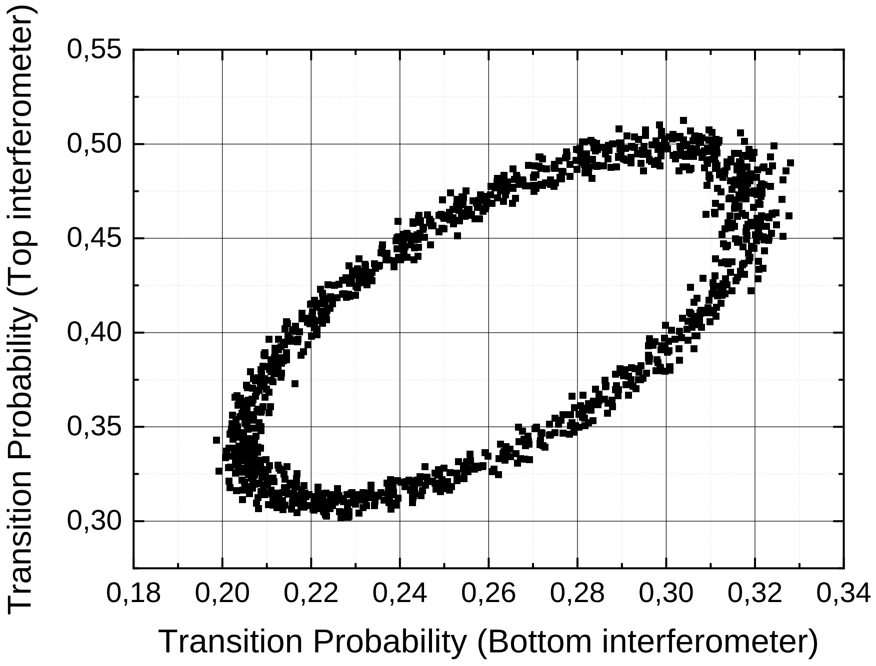
Gravity gradiometer

l'Observatoire de Paris | SYRTE
Systèmes de Référence Temps-Espace



Gravity gradiometer

Parametric plot of transition probabilities top/bottom clouds = ellipse



$2\hbar k$ Bragg transitions

2T = 260 ms

Current sensitivity:
210 E/shot

1 E = 10^{-9} s^{-2}

Best (Raman) gradiometer
28 E@1s

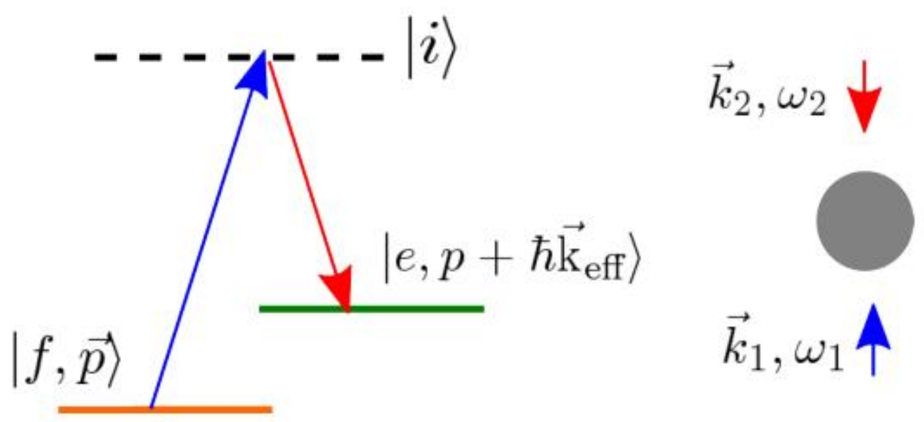
Bragg gradiometer (20ħk)
Asembaum et al, PRL 118, 183602 (2017)
14 E@1s
But ultracold atoms @ 50nK

Bragg gradiometer (6ħk)
with laser-cooled atoms
D'Amico et al, 2014
1160 E@1s

To be improved by
- Increasing the number of atoms → more laser cooling power
- Improving the contrast → Lower temperature
- Increasing 2T up to 500 ms
- Operating at mid-fringe → reduce vibration noise
- Increasing the separation to $N\hbar k$ → more Bragg power

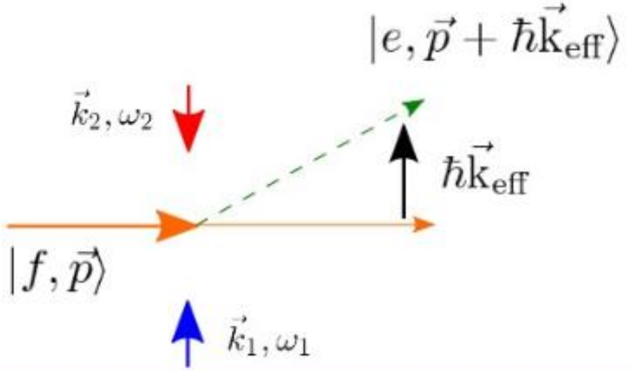
Stimulated Raman transitions

Three level atom
Interacting with two
counter propagating lasers



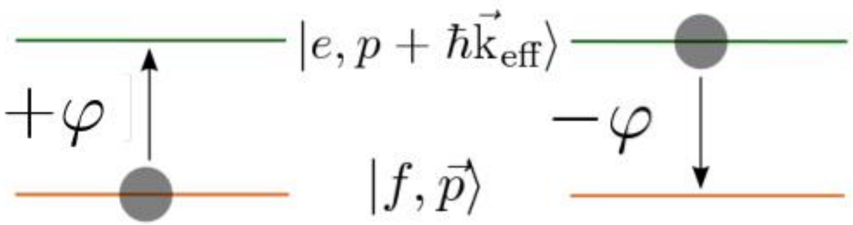
Momentum transfer (~1 cm/s)

$$k_{\mathrm{eff}} = k_1 + k_2$$

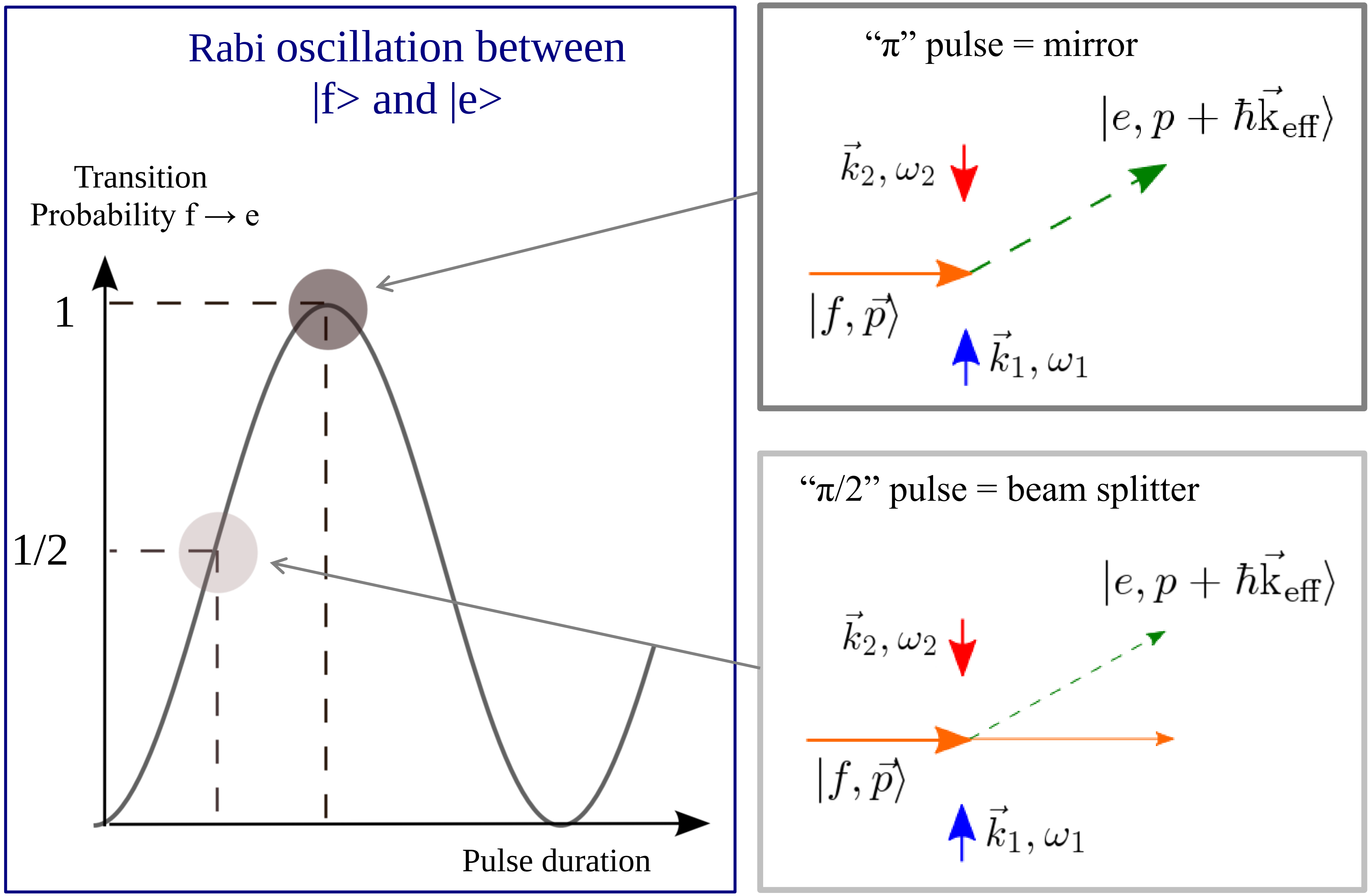


Phase difference between the lasers gets imprinted

$$\varphi = \phi_1 - \phi_2 = \vec{\mathrm{k}}_{\mathrm{eff}} \cdot \vec{r}(t)$$



Rabi oscillations

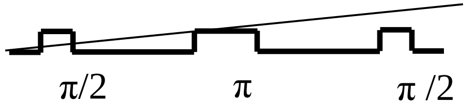


Principle of measurement

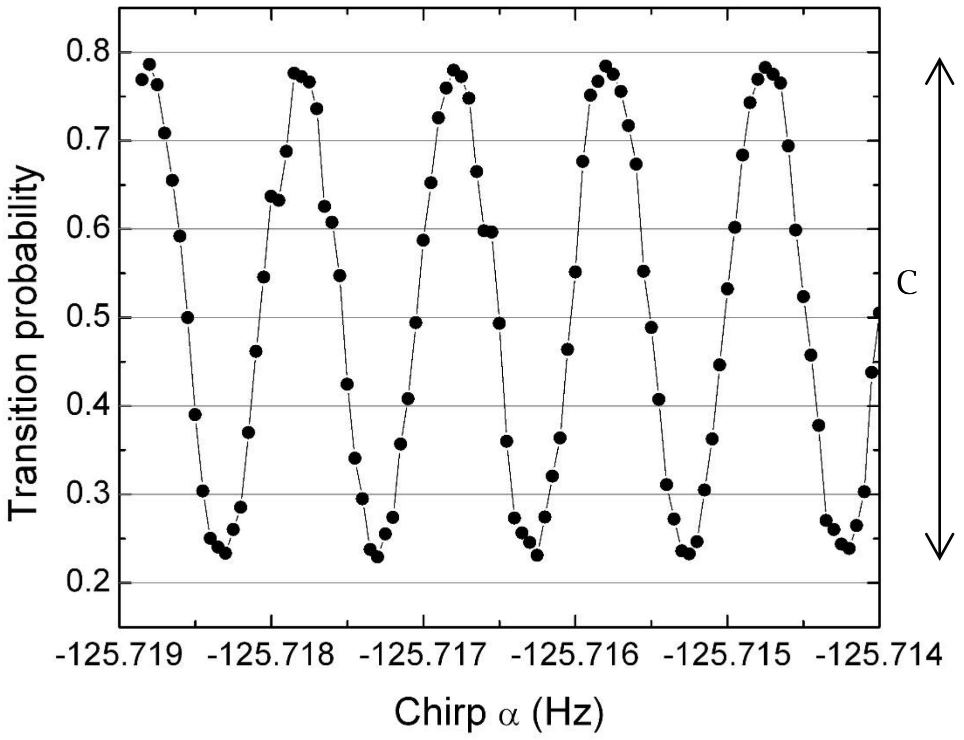
- Free fall → Doppler shift of the resonance condition of the Raman transition

$$\omega_1 - \omega_2 = \omega_{\mathrm{e}} - \omega_{\mathrm{f}} + \vec{k}_{\mathrm{eff}}\vec{v}(t) + \frac{\hbar k_{\mathrm{eff}}^2}{2m}$$

- Ramping of the frequency difference to stay on resonance : $\omega = \omega_0 + \alpha t$



$$\triangle\Phi = -\vec{k}_{\mathrm{eff}}\vec{g}T^2 + \alpha T^2$$



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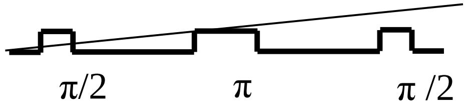

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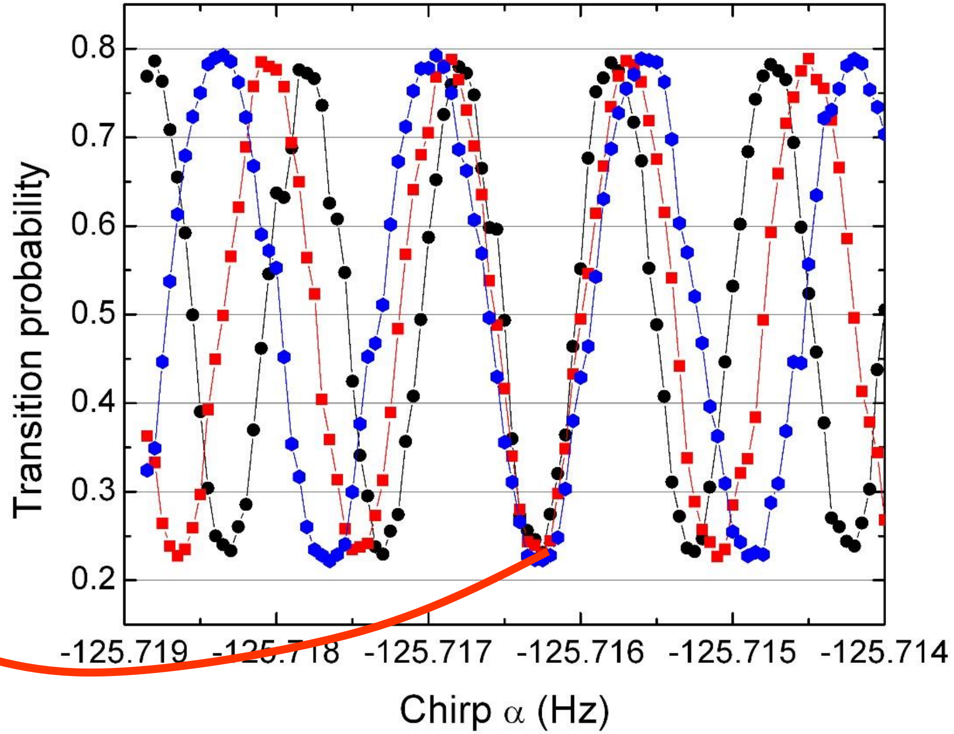

- Dark fringe : independent of T

$$\boxed{g = \frac{\alpha_0}{k_{\text{eff}}}}$$

The g measurement is a frequency measurement