Unifying Height Systems Using Chronometric Leveling

Study funded via SFB 1464 TerraQ - Project-ID 43461778

NKG 2022 General Assembly

05 - 08 September | Copenhagen, Denmark

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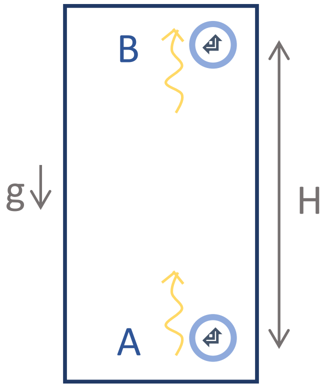
Gravitational redshift and geodetic applications

Using **Einstein's Equivalence principle** together with the relativistic Doppler shift equation, the fractional frequency change at the two earthbound clocks is

$$\frac{\Delta f}{f} = \frac{-gH}{c^2} = \frac{C_A - C_B}{c^2} = \frac{V_B - V_A}{c^2}$$

where $C_A = H_A * g$ and $C_B = H_B * g$ represent the geopotential numbers at A and B



From **gravitational time dilation** derived from the metric tensor equation, the fractional frequency change at the two earthbound clocks is

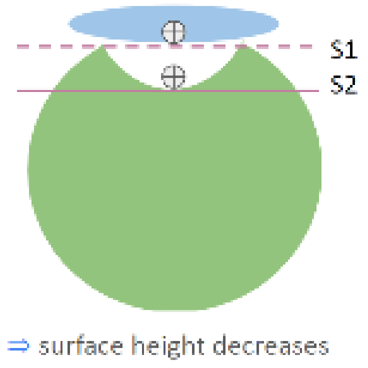
$$\frac{\Delta\tau}{\tau} = \frac{\Delta f}{f} = \frac{V_B - V_A}{c^2}$$

where V_A and V_B represent the geopotential values at A and B

- Clock performance with fractional uncertainties of 1 part in 10^{18} corresponds to a gravity potential variation of **0.1 m²/s²** or a height difference of **1 cm**
- Terrestrial clock networks can be used for the detection of time-variable gravity signals
- Unification of regional height reference systems through chronometric leveling approach

Detection of time-variable gravity signals using terrestrial clock networks

Mass redistribution causes geopotential variation



$$\delta U = \sum(1+k_n'-h_n')*\delta V_{2,n} = \delta V - d*g$$

δV_2 ➡ applied load potential

$[h_n', l_n', k_n']$ ➡ load Love numbers

- GRACE-like missions give $\sum(1+k_n'')*\delta V_{2,n} = \delta V$
- GNSS observe vertical deformations (d) $\sum h_n'*\delta V_{2,n} = d*g$
- Clocks observe the integral effect of both mass variation and surface deformation

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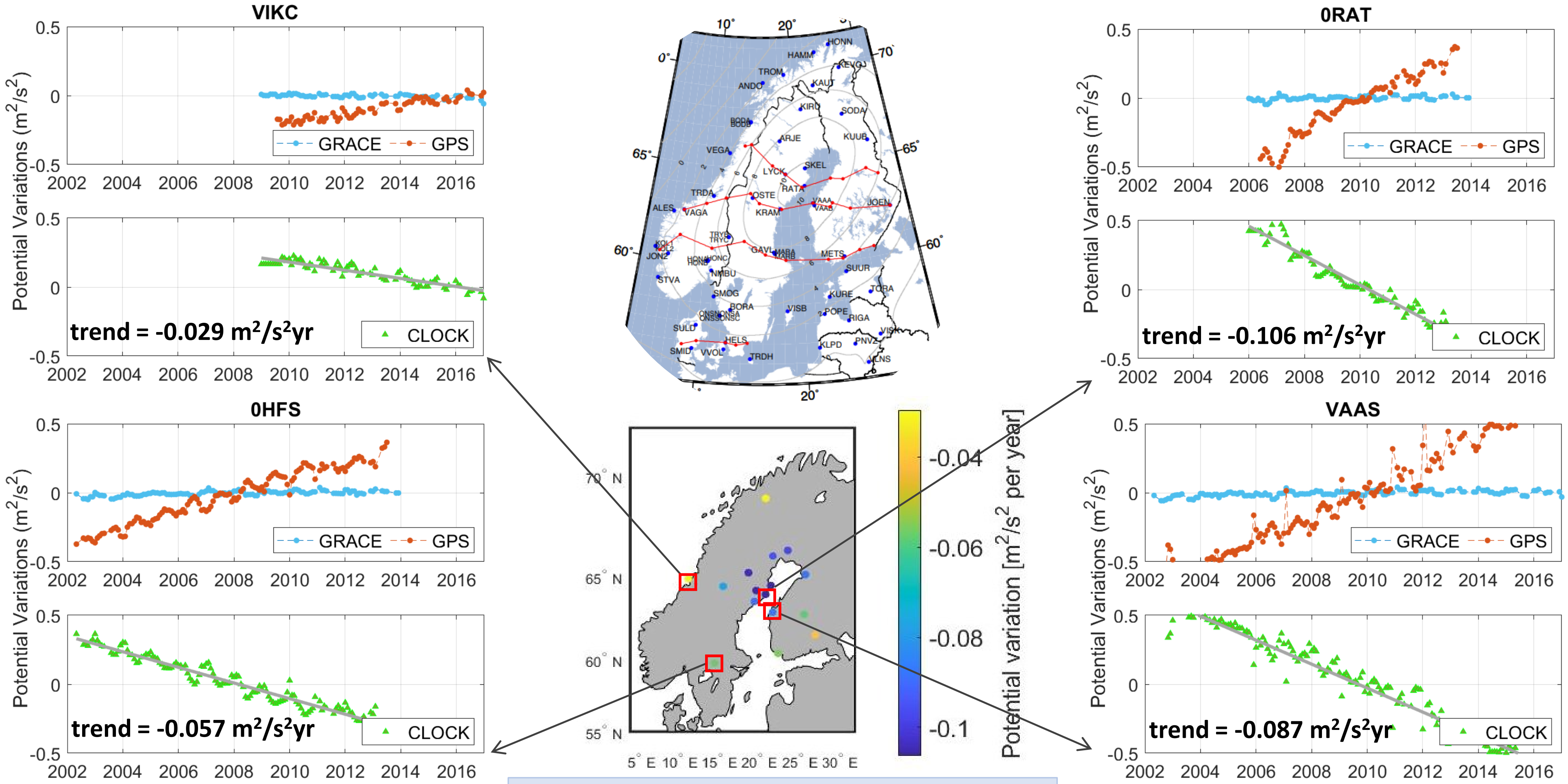
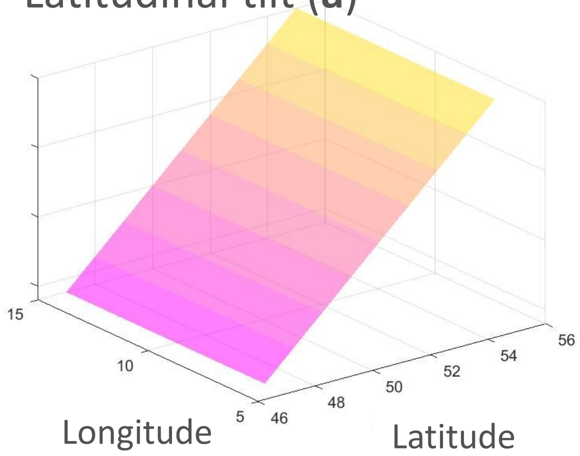
Fennoscandian land uplift – Detection of GIA signals



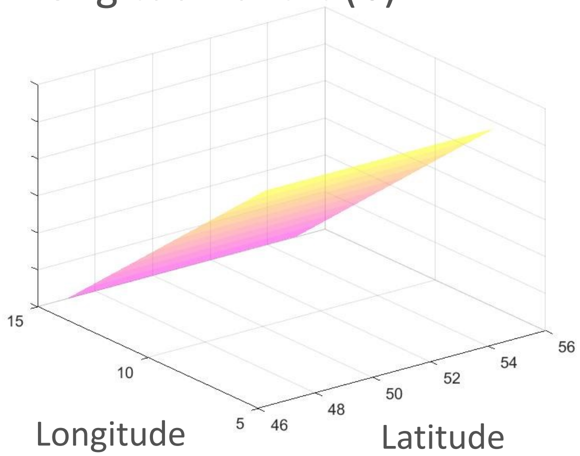
Clock measurement accuracy – 0.1 m^2/s^2

Height system unification –
Realization of more realistic biases between local height systems

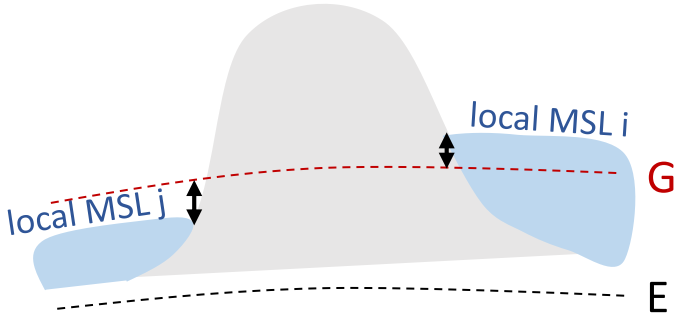
Latitudinal tilt (**a**)



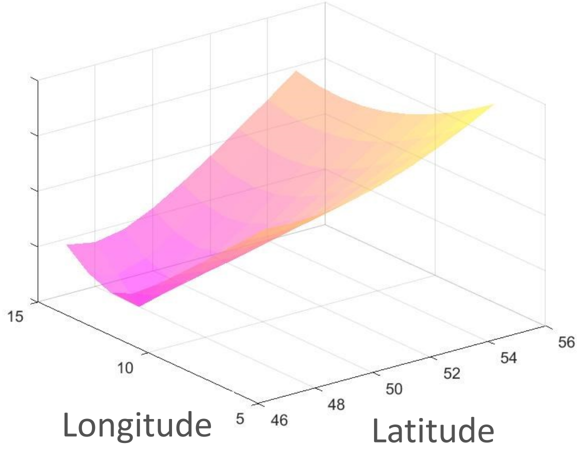
Longitudinal tilt (**b**)



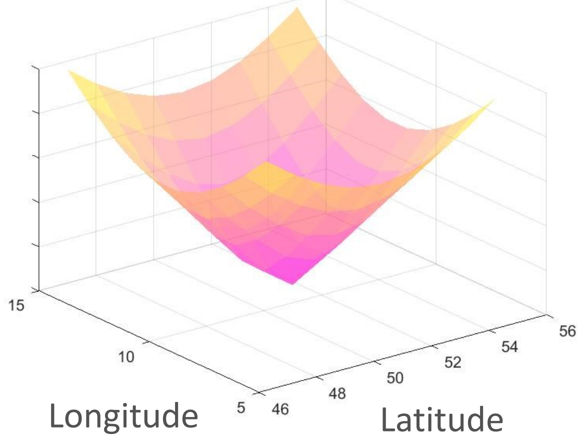
Offset(**c**)



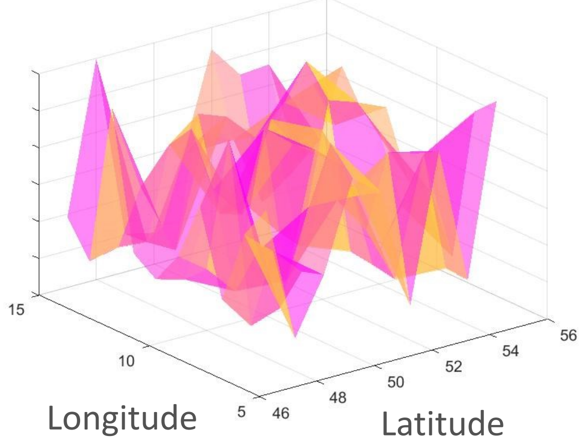
Tide gauge tilt (**t**)



Noisy levelling point tilt (**n**)



Mountain tilt (**m**)



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Determination of realistic biases between local height systems - Test cases

The observation equation is built considering possible realistic tilts and offsets as

$$\mathbf{H^L_i = H^U_i + biases\ (offsets+tilts)}$$

$\mathbf{H^L_i}$ ➡ Height of the leveling point in the local system

$\mathbf{H^U_i}$ ➡ Height of the leveling point in the unified system

- **Tilts along a few leveling lines - error increases with distance**

  $H^L_i = H^U_i + a^L\Delta X_i + b^L\Delta Y_i + c^L + \mathbf{n^L\Delta SN_i}$

- **Concentric variation with distance from tide gauge**

  $H^L_i = H^U_i + a^L\Delta X_i + b^L\Delta Y_i + c^L + \mathbf{t^L\Delta ST_i}$

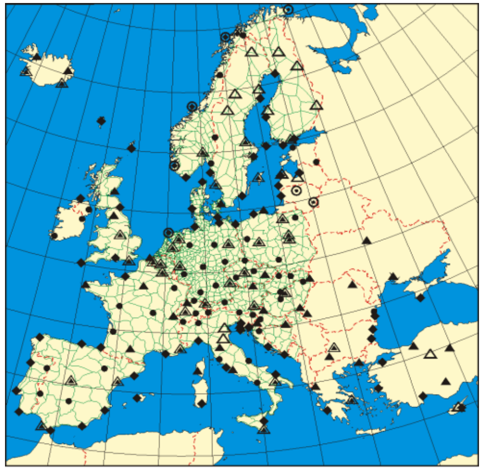
- **Mountain effect - fixed systematic noise per 500 m of height**

  $H^L_i = H^U_i + a^L\Delta X_i + b^L\Delta Y_i + c^L + \mathbf{m^L(H^L_i/500)}$

- **Extended model (including a^L, b^L, c^L, t^L, m^L)**

  $H^L_i = H^U_i + a^L\Delta X_i + b^L\Delta Y_i + c^L + \mathbf{t^L\Delta ST_i + m^L(H^L_i/500)}$

Chronometric leveling



A priori system – EUVN/2000

(European Vertical Reference Network)

Classification

Noise added to local height values H_i^L
(offset, tilts & random noise)

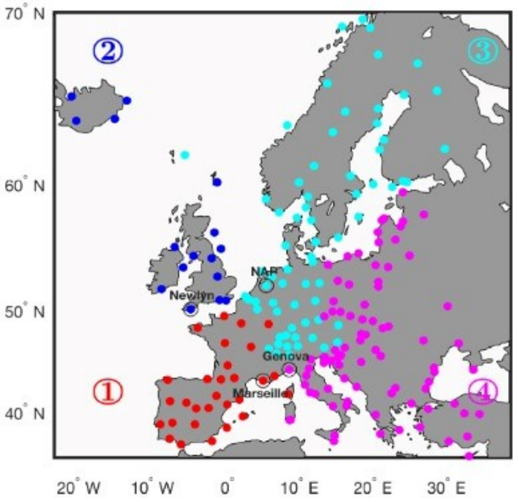
Random noise added to clock observations

Reunification

(Clock-based adjustment)

Comparison

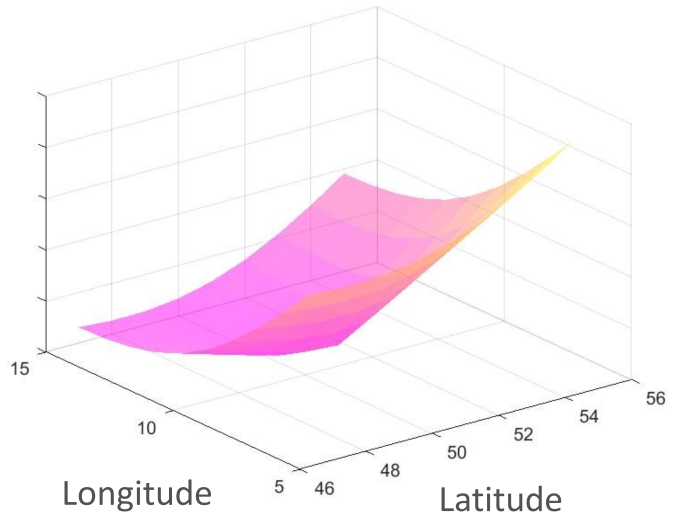
(reunified system with the a priori system)

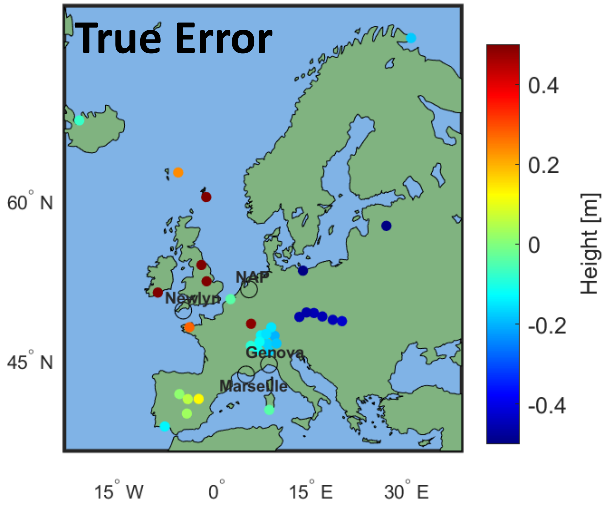


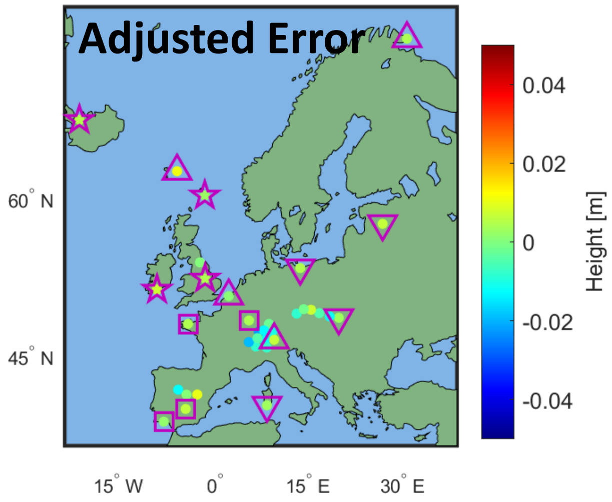
$$H_i^L = \frac{c_i^U}{\gamma_i} + biases$$

$$\Delta W_{ij} = W_i^U - W_j^U = -(C_i^U - C_j^U)$$

1. Tilts along a few leveling lines - error increases with distance





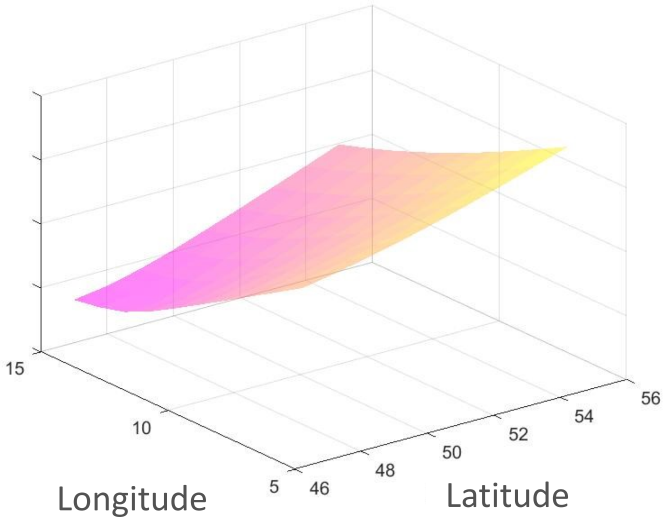


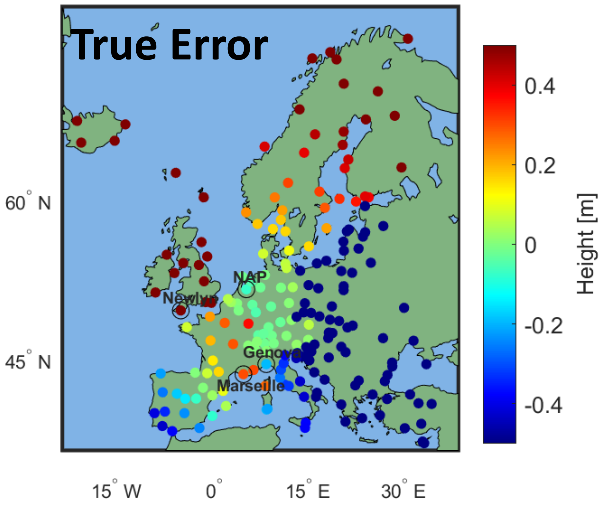
4 clocks in each system

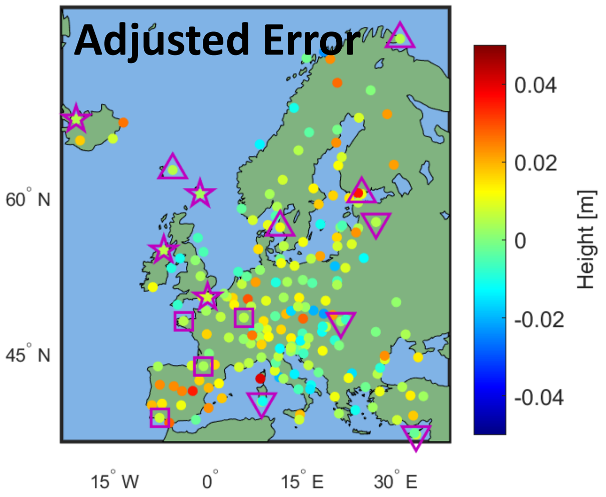
- 3 external
- 1 random

Biases	G1		G2		G3		G4	
	True	Esti.	True	Esti.	True	Esti.	True	Esti.
n^L(m/100km)	**0.02**	**0.016**	**0.02**	**0.009**	**0.02**	**0.023**	**0.02**	**0.023**
a^L (m/100km)	0.03	0.029	-0.02	-0.019	0.015	0.014	-0.03	-0.029
b^L(m/100km)	0.02	0.021	0.03	0.031	-0.015	-0.014	-0.002	-0.022
c^L (m)	-0.18	-0.182	0.25	0.233			0.08	0.079
RMS (m)		**0.007**		**0.005**		**0.011**		**0.006**

2. Concentric variation with distance from tide gauge







4 clocks in each system

- 3 at the corners based on levelling point distribution
- 1 at center

Biases	G1		G2		G3		G4	
	True	Esti.	True	Esti.	True	Esti.	True	Esti.
t^L (m/100km)	**-0.02**	**-0.021**	**0.035**	**0.034**	**0.03**	**0.034**	**-0.03**	**-0.031**
a^L (m/100km)	0.03	0.032	-0.02	-.0197	0.015	0.016	-0.03	-0.03
b^L (m/100km)	0.02	0.019	0.03	0.029	-0.015	-0.016	-0.02	-0.019
c^L (m)	-0.18	-0.184	0.25	0.266			0.08	0.088
RMS (m)	**0.017**		**0.010**		**0.0137**		**0.011**	

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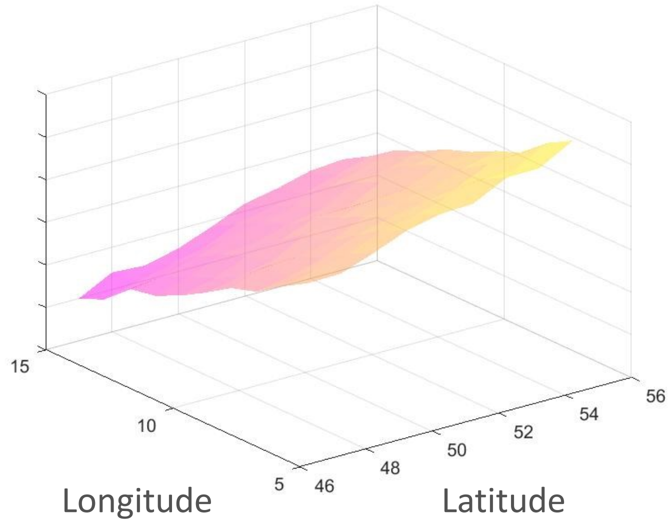
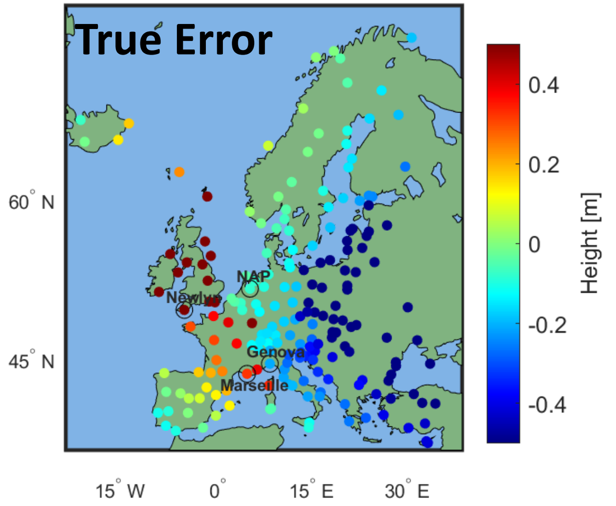
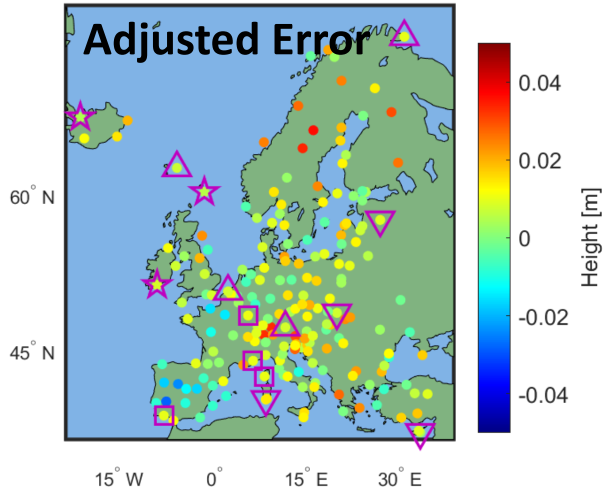
3. Mountain effect - fixed systematic noise per 500 m of height







4 clocks in each system

- 3 at the corners
- 1 randomly at the elevated point

Biases	G1		G2		G3		G4	
	True	Esti.	True	Esti.	True	Esti.	True	Esti.
m^L(m)	**0.015**	**0.014**			**0.02**	**0.01**	**0.015**	**0.014**
a^L (m/100km)	0.03	0.03	-0.02	-0.021	0.015	0.014	-0.03	-0.03
b^L (m/100km)	0.02	0.019	0.03	0.029	-0.015	-0.014	-0.02	-0.019
c^L (m)	-0.18	-0.167	0.25	0.249			0.08	0.07
RMS (m)		**0.012**		**0.013**		**0.015**		**0.013**

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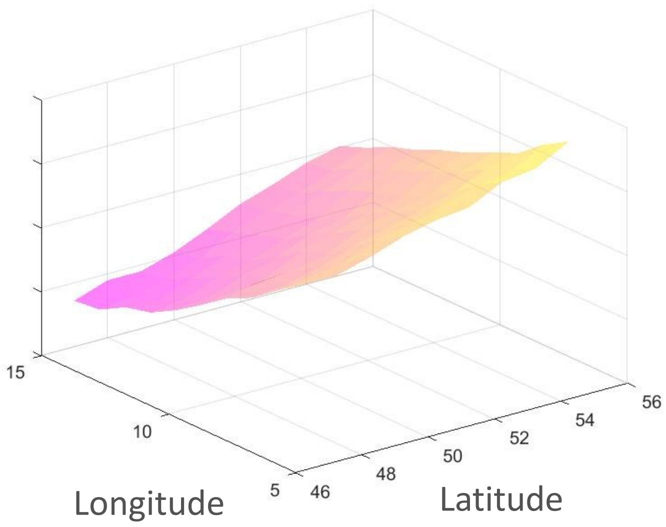
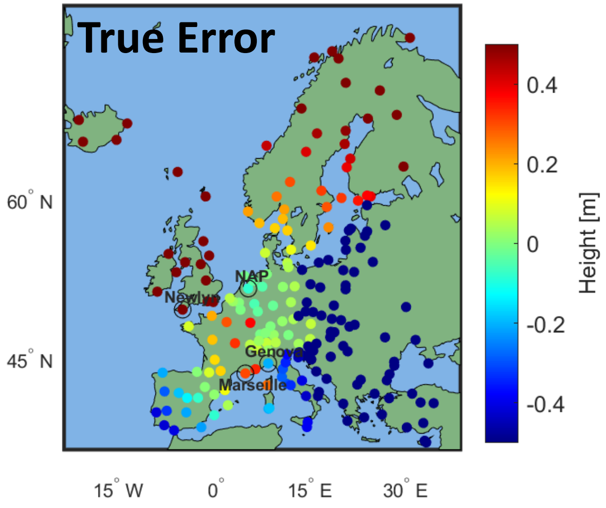
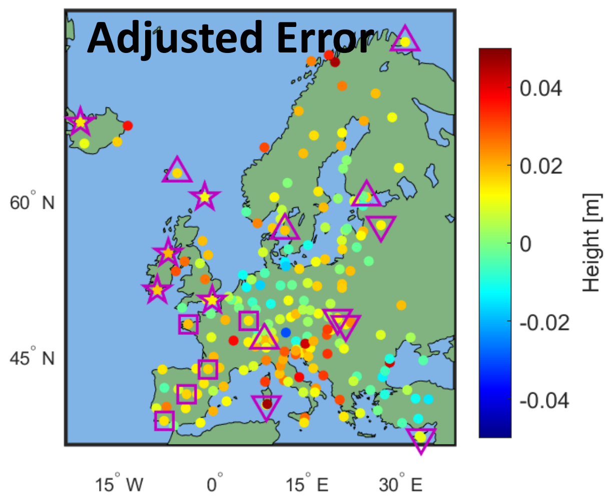
4. Extended model (including a^L, b^L, c^L, t^L, m^L)







5 clocks in each system

- 3 at the corners
- 1 at the center
- 1 randomly at the elevated point

Biases	G1		G2		G3		G4	
	True	Esti.	True	Esti.	True	Esti.	True	Esti.
m^L(m)	0.015	0.012			0.02	0.011	0.015	0.019
t^L(m/100km)	-0.02	-0.021	0.035	0.038	0.03	0.029	-0.025	-0.029
a^L (m/100km)	0.03	0.030	-0.02	-0.024	0.015	0.014	-0.03	-0.029
b^L(m/100km)	0.02	0.019	0.03	0.03	-0.015	-0.014	-0.02	-0.019
c^L (m)	-0.18	-0.176	0.25	0.228			0.08	0.053
RMS (m)		**0.017**		**0.019**		**0.015**		**0.018**

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DHHN92 vs. DHHN2016

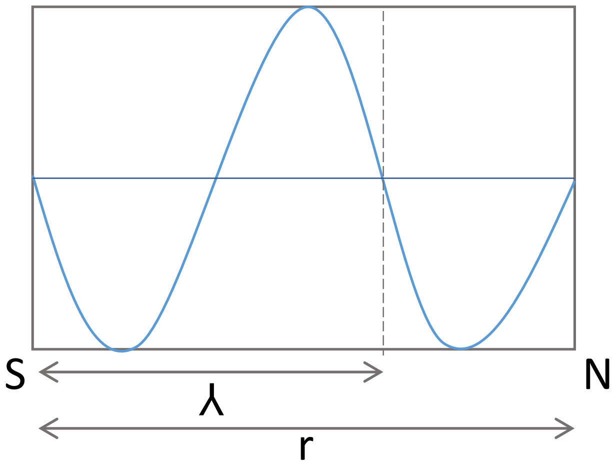
- Test whether clock networks can be used for detecting temporal elevation changes in case of new height system realization (example Germany)?

$$\frac{\Delta f}{f} = \frac{g\Delta H}{c^2}$$

$H^{16}_i = H^{92}_i + \Delta H_i$

Simple case- when only a periodic N-S variation is assumed:

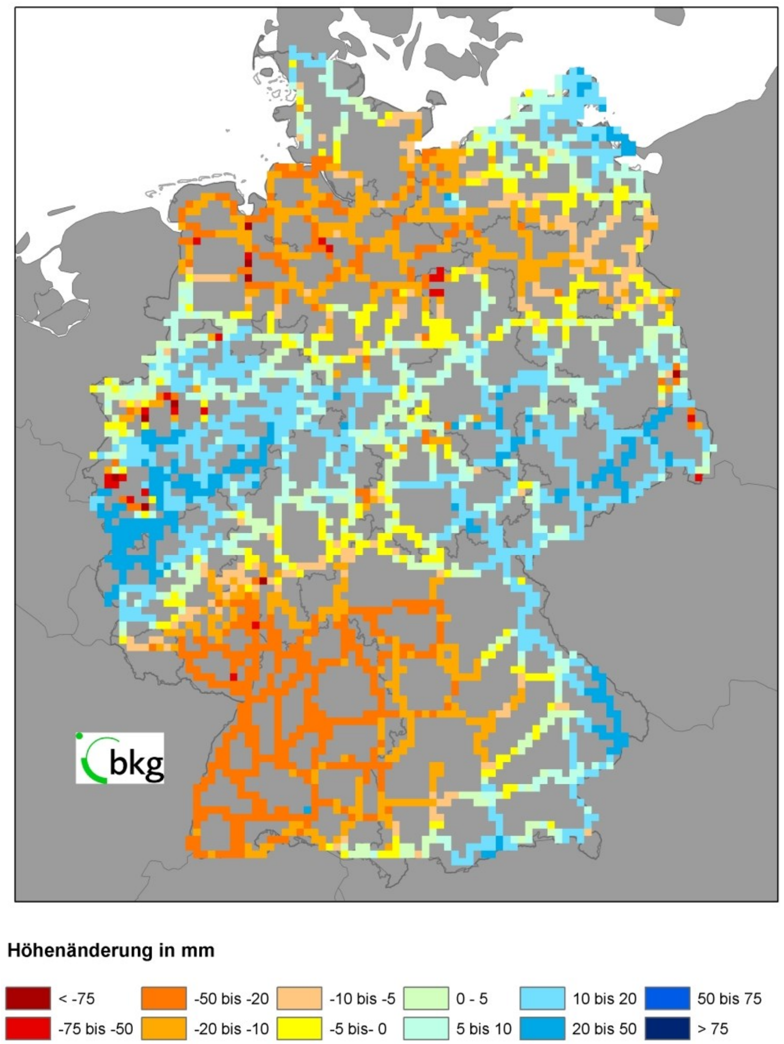
$\Delta H_i = A \cos(k\, r_i + \emptyset)$



$A \sim 3$ cm

$\lambda \sim \frac{2}{3} r$

$\emptyset \sim \frac{\Pi}{2}$

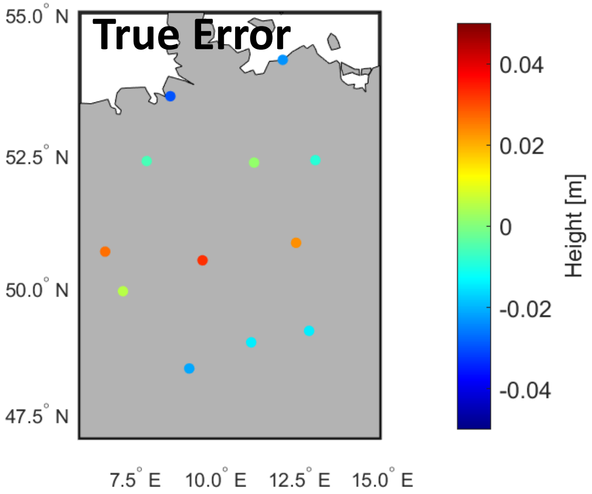


DHHN92 vs. DHHN2016 – Clock based adjustment

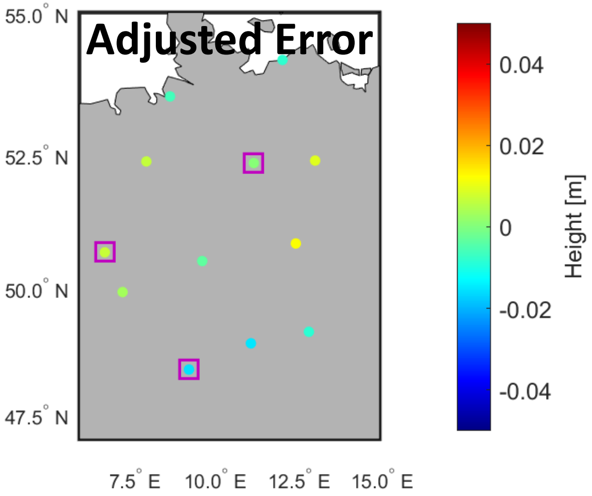
- $C^{92}_i = C^{16}_i - \Delta C_i$ (observation equation replacing height with geopotential number)
- $\Delta W_{ji}^{16} - \Delta W_{ji}^{92} = \Delta C_j - \Delta C_i$ (assuming we have clock observations at the two epochs)
- Repeated regression by changing wavelength:

$$\mathbf{C^{92}_i = C^{16}_i - B\cos(k\,x_i) - C\sin(k\,x_i)}$$

$$\mathbf{\Delta W_{ji}^{16} - \Delta W_{ji}^{92} = B\,(\cos(k\,x_j) - \cos(k\,x_i)) + C\,(\sin(k\,x_j) - \sin(k\,x_i))}$$



Estimated parameters of the periodic function			
Wave length (m)	Phase (rad)	Amplitude (m)	RMS (m)
5.930667e+05	1.5128	0.0366	0.0090



- Good results when clocks are distributed based on the wave function or at all 13 points

Summary

- Terrestrial clock networks can be well used for the detection of time-variable gravity signals
- The deformation part can be obtained from GNSS measurements
- Clock networks can validate GRACE results
- Terrestrial clock networks can effectively be used for estimating realistic biases between regional height systems
- Hence chronometric leveling can be used for realizing a unified height reference system with an accuracy of approx. 1 cm
- The accuracy in the bias estimation highly depends upon the number of clocks and their spatial distribution in each local system
- Temporal elevation changes between two height systems can be well determined with clock observations at the two epochs

Thank You!

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