Optimal design of GNSS interference localisation wireless security network

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Introduction

Smart cities and safe navigation require signal and cyber security.

Jamming and spoofing two known signal interferences. Jamming means to transmit a radio frequency signal into the same band as, or a band nearby to, the satellite navigation band of interest to hide the signal and spoofing is the transmission of a fake GNSS signal (Dempster 2016).

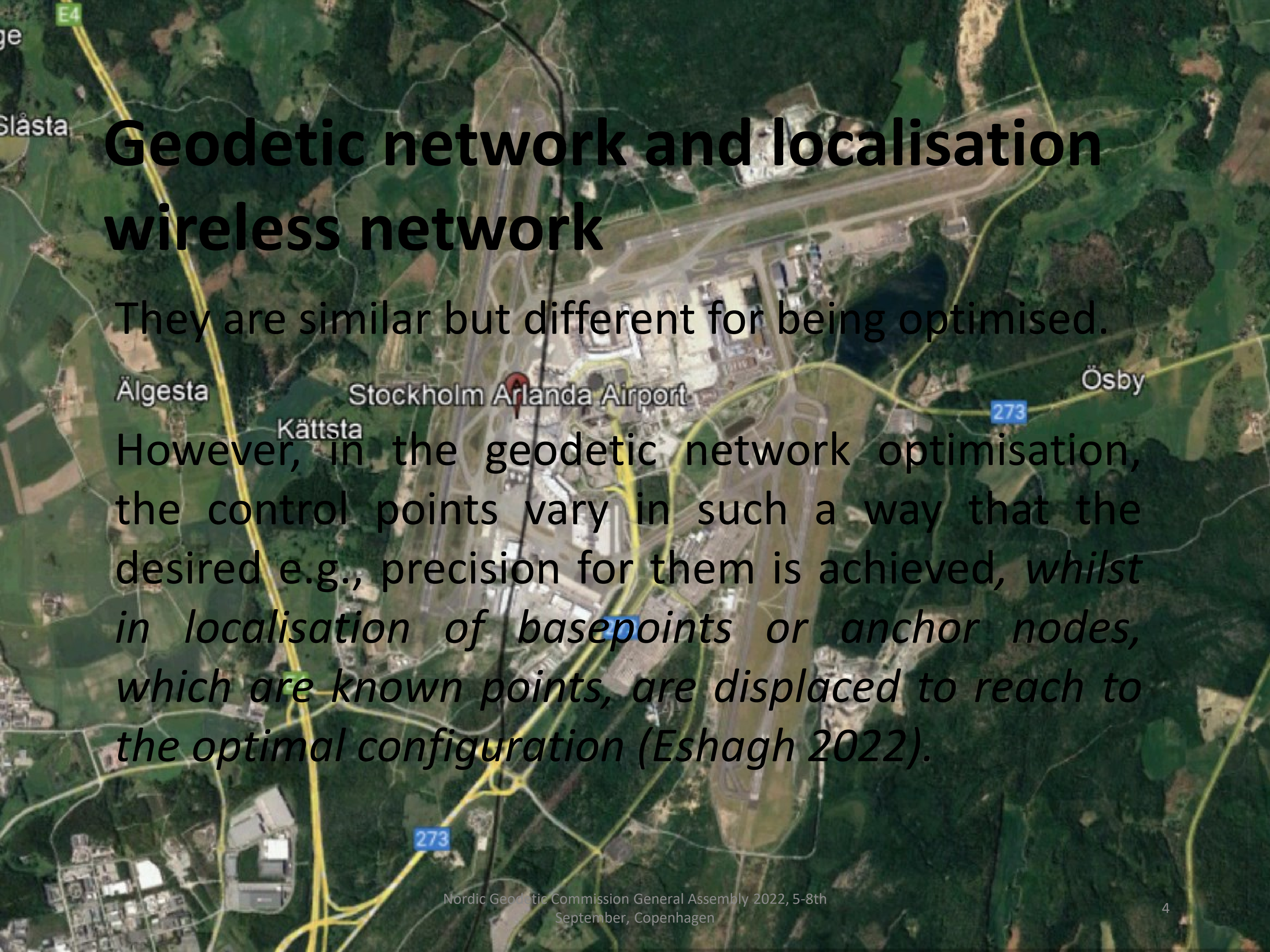
Some real examples

- A faulty TV amplifier jammed the Global Positioning System (GPS) operation at a harbour in Monterey, California for 37 days (Clynch et al. 2003).
- A small jammer, which was used in a delivery van, disrupted the ground-based augmentation system (GBAS) system aiding aircraft approaches at Newark Airport while driving on a nearby highway in 2009 (Hambling 2011 Pullen et al. 2013 and Warburton et al. 2011).
- The Central Radio Management Office of South Korea reported several disruptions from 2010–2012 due to GPS jammers affected (Seo and M. Kim 2013).
- In Italy some from TV signals in the GNSS band, disrupting GPS (Metolla et al. 2008).

Geodetic network and localisation wireless network

They are similar but different for being optimised.

However, in the geodetic network optimisation, the control points vary in such a way that the desired e.g., precision for them is achieved, *whilst in localisation of basepoints or anchor nodes, which are known points, are displaced to reach to the optimal configuration (Eshagh 2022).*



Localisation wireless security network

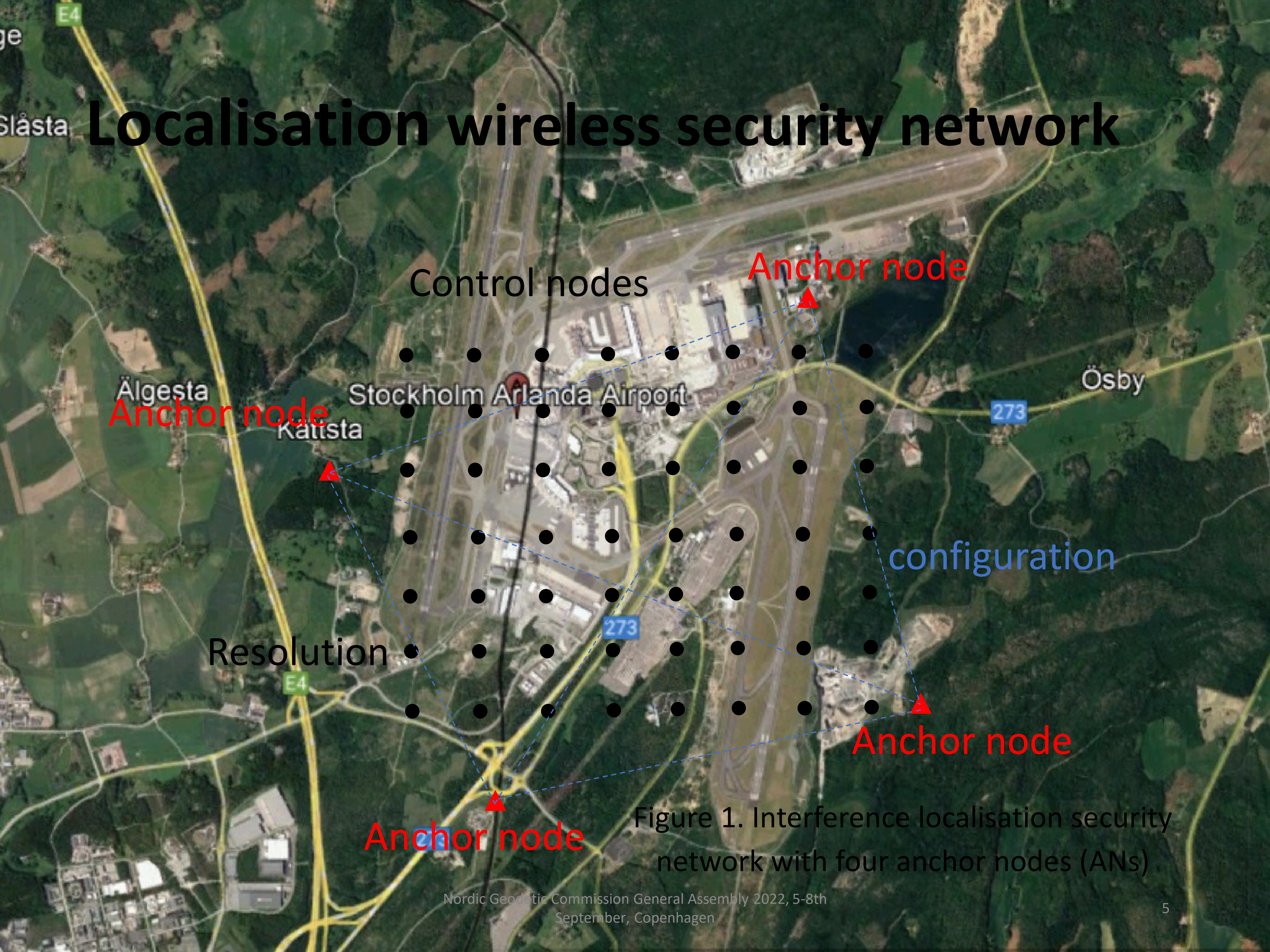
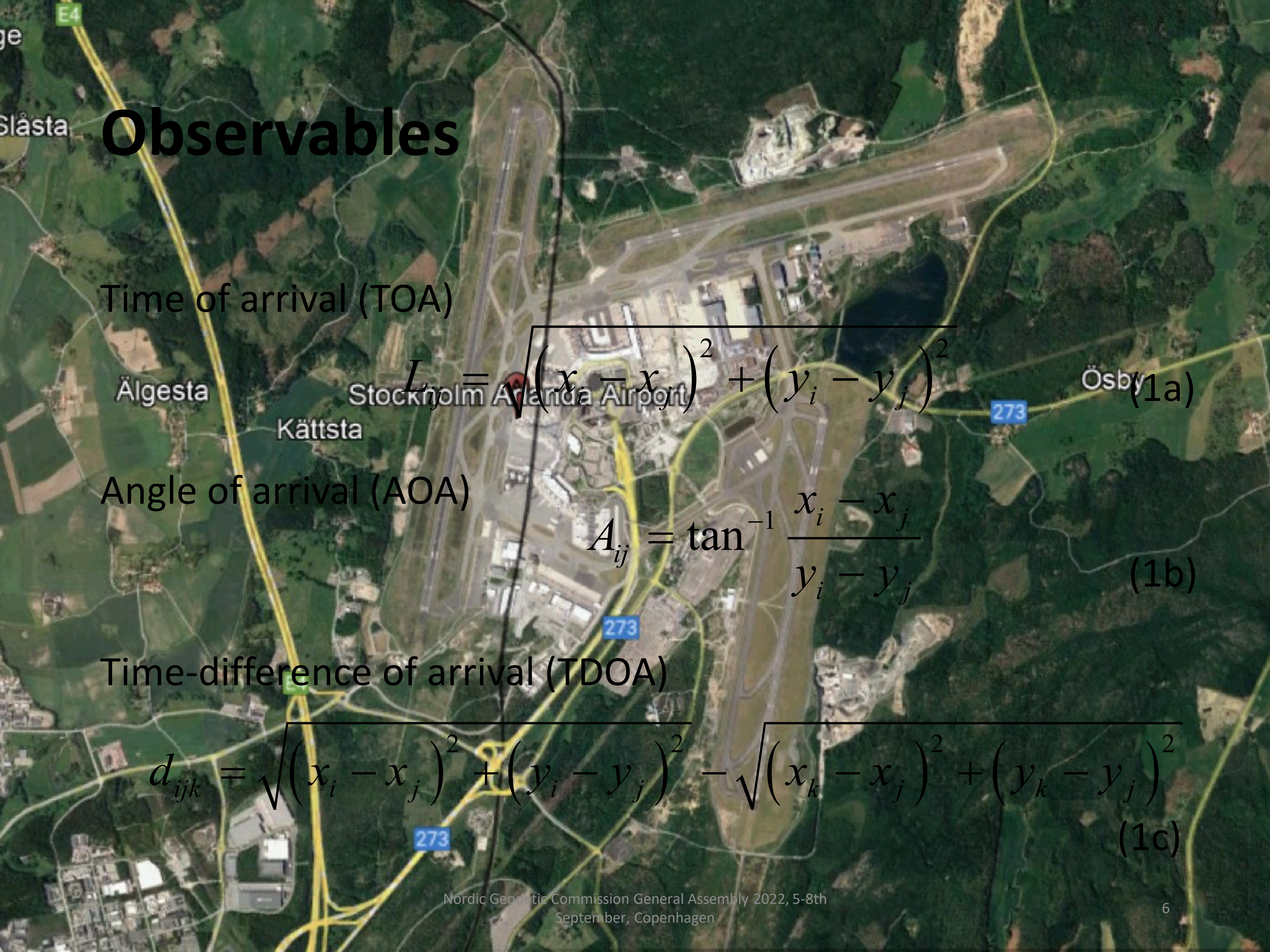


Figure 1. Interference localisation security network with four anchor nodes (ANs)

Observables



Time of arrival (TOA)

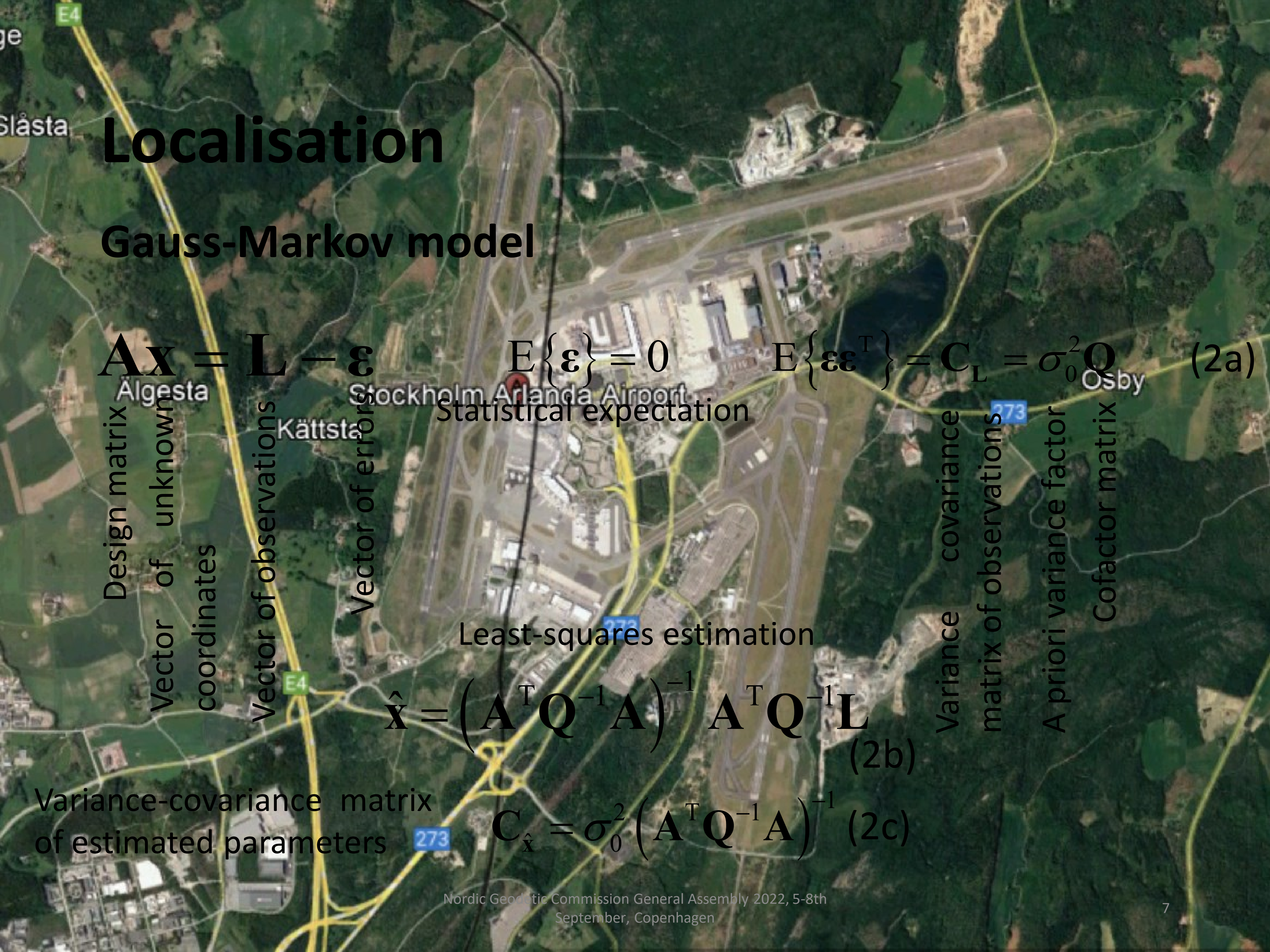
$$L_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \quad (1a)$$

Angle of arrival (AOA)

$$A_{ij} = \tan^{-1} \frac{x_i - x_j}{y_i - y_j} \quad (1b)$$

Time-difference of arrival (TDOA)

$$d_{ijk} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} - \sqrt{\left(x_k - x_j\right)^2 + \left(y_k - y_j\right)^2} \quad (1c)$$



Localisation

Gauss-Markov model

$$\mathbf{Ax} = \mathbf{L} - \boldsymbol{\varepsilon} \qquad \mathrm{E}\{\boldsymbol{\varepsilon}\} = 0 \qquad \mathrm{E}\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\} = \mathbf{C}_{\mathbf{L}} = \sigma_0^2 \mathbf{Q} \qquad \text{(2a)}$$

Design matrix

Vector of unknown coordinates

Vector of observations

Vector of errors

Statistical expectation

Variance covariance matrix of observations

A priori variance factor

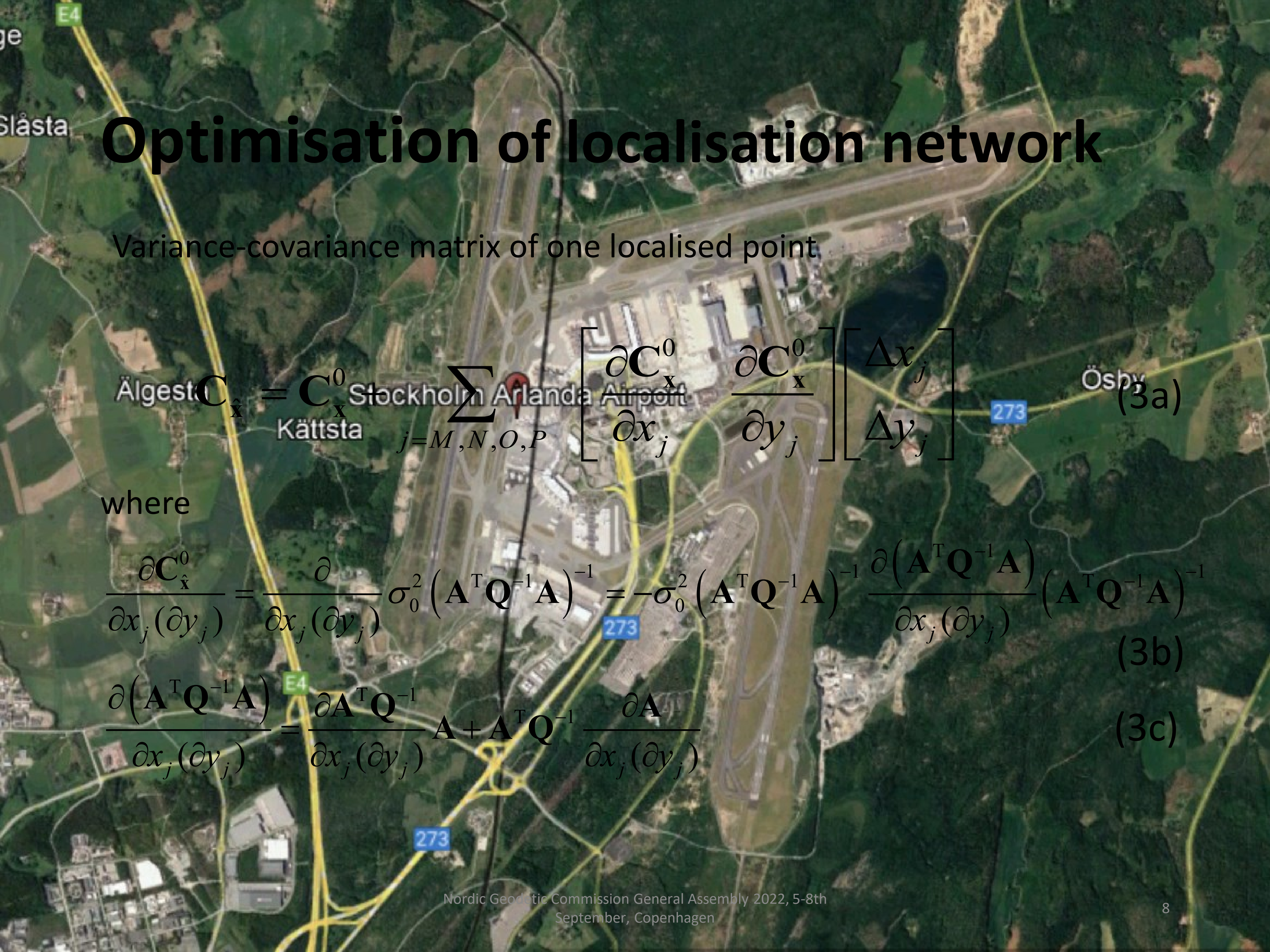
Cofactor matrix

Least-squares estimation

$$\hat{\mathbf{x}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{L} \qquad \text{(2b)}$$

Variance-covariance matrix of estimated parameters

$$\mathbf{C}_{\hat{\mathbf{x}}} = \sigma_0^2\left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1} \qquad \text{(2c)}$$



Optimisation of localisation network

Variance-covariance matrix of one localised point

$$\mathbf{C}_{\hat{\mathbf{x}}} = \mathbf{C}^0_{\hat{\mathbf{x}}} + \sum_{j=M,N,O,P} \begin{bmatrix} \dfrac{\partial \mathbf{C}^0_{\hat{\mathbf{x}}}}{\partial x_j} & \dfrac{\partial \mathbf{C}^0_{\hat{\mathbf{x}}}}{\partial y_j} \end{bmatrix} \begin{bmatrix} \Delta x_j \\ \Delta y_j \end{bmatrix} \quad (3a)$$

where

$$\frac{\partial \mathbf{C}^0_{\hat{\mathbf{x}}}}{\partial x_j (\partial y_j)} = \frac{\partial}{\partial x_j (\partial y_j)} \sigma_0^2 \left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1} = -\sigma_0^2 \left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1} \frac{\partial\left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)}{\partial x_j (\partial y_j)} \left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1} \quad (3b)$$

$$\frac{\partial\left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)}{\partial x_j (\partial y_j)} = \frac{\partial \mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}}{\partial x_j (\partial y_j)} \mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1} \frac{\partial \mathbf{A}}{\partial x_j (\partial y_j)} \quad (3c)$$

An over-determined system of equations can be created for more than one point

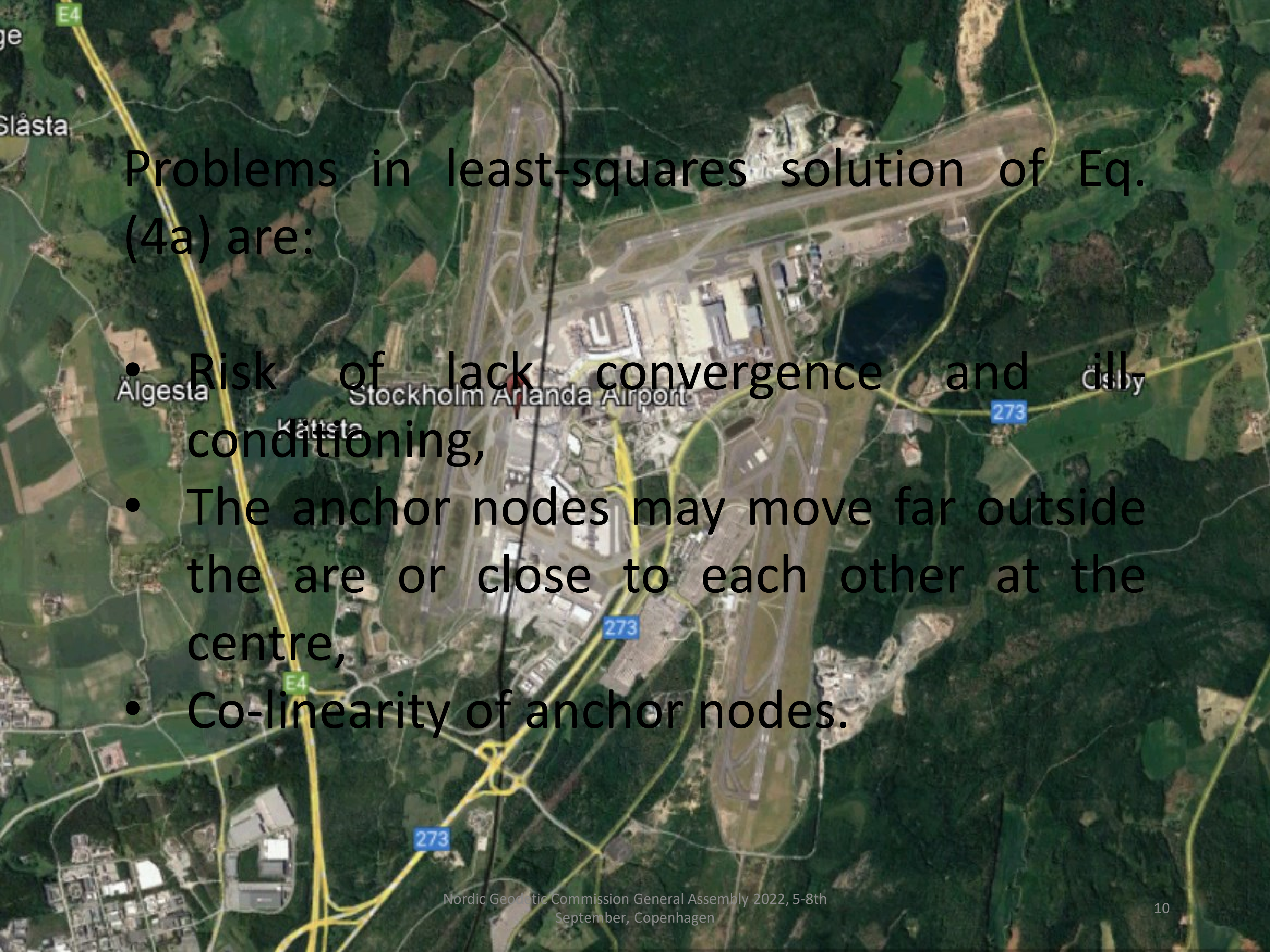
$$\mathbf{B}\Delta\mathbf{x} = \Delta\mathbf{L} - \boldsymbol{\varepsilon}' \qquad (4a)$$

where

$$\Delta\mathbf{L} = \left[\operatorname{vec}\left(\mathbf{C}_{\hat{\mathbf{x}}_1}\right) - \operatorname{vec}\left(\mathbf{C}^0_{\mathbf{x}_1}\right) \quad \operatorname{vec}\left(\mathbf{C}_{\hat{\mathbf{x}}_2}\right) - \operatorname{vec}\left(\mathbf{C}^0_{\mathbf{x}_2}\right) \quad \cdots \quad \operatorname{vec}\left(\mathbf{C}_{\hat{\mathbf{x}}_n}\right) - \operatorname{vec}\left(\mathbf{C}^0_{\mathbf{x}_n}\right) \right]^{\mathrm{T}} \qquad (4b)$$

$$\Delta\mathbf{x} = \left[\Delta x_M \quad \Delta y_M \quad \Delta x_N \quad \Delta y_N \quad \Delta x_O \quad \Delta y_O \quad \Delta x_P \quad \Delta y_P \right]^{\mathrm{T}} \qquad (4c)$$

$$\mathbf{B} = \begin{bmatrix}
\operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial x_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial y_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial x_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial y_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial x_P}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_1}}{\partial y_P}\right) \\
\operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial x_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial y_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial x_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial y_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial x_P}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_2}}{\partial y_P}\right) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial x_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial y_M}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial x_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial y_N}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial y_O}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial x_P}\right) & \operatorname{vec}\left(\frac{\partial \mathbf{C}^0_{\mathbf{x}_n}}{\partial y_P}\right)
\end{bmatrix}_{4n\times 8} \qquad (4d)$$



Problems in least-squares solution of Eq. (4a) are:

- Risk of lack convergence and ill-conditioning,
- The anchor nodes may move far outside the are or close to each other at the centre,
- Co-linearity of anchor nodes.

Limiting the search area for the anchor nodes

Coordinate limits

$$w_j^{\mathrm{L}} \le x_j \le w_j^{\mathrm{U}} \qquad \text{(5a)}$$

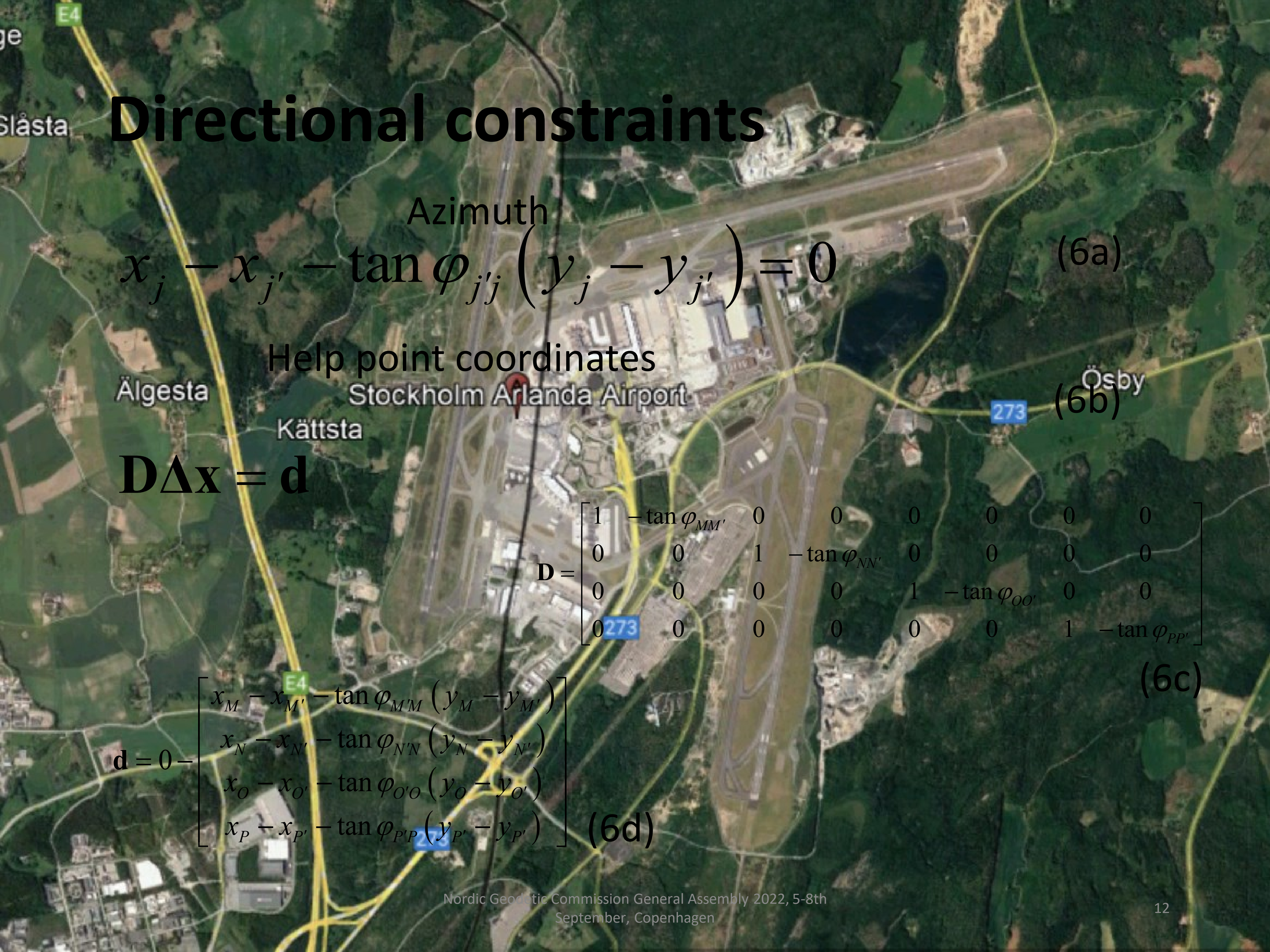
$$v_j^{\mathrm{L}} \le y_j \le v_j^{\mathrm{U}} \qquad \text{(5b)}$$

Coordinate updates limit

$$\mathbf{L}_{\mathrm{b}} \le \Delta\mathbf{x} \le \mathbf{U}_{\mathrm{b}} \qquad \text{(5c)}$$

$$\mathbf{L}_{\mathrm{b}} = \begin{bmatrix} w_M^{\mathrm{L}} - x_M & v_M^{\mathrm{L}} - y_M & w_N^{\mathrm{L}} - x_N & v_N^{\mathrm{L}} - y_N & w_O^{\mathrm{L}} - x_O & v_O^{\mathrm{L}} - y_O & w_P^{\mathrm{L}} - x_P & v_P^{\mathrm{L}} - y_P \end{bmatrix}^{\mathrm{T}} \qquad \text{(5d)}$$

$$\mathbf{U}_{\mathrm{b}} = \begin{bmatrix} w_M^{\mathrm{U}} - x_M & v_M^{\mathrm{U}} - y_M & w_N^{\mathrm{U}} - x_N & v_N^{\mathrm{U}} - y_N & w_O^{\mathrm{U}} - x_O & v_O^{\mathrm{U}} - y_O & w_P^{\mathrm{U}} - x_P & v_P^{\mathrm{U}} - y_P \end{bmatrix}^{\mathrm{T}} \qquad \text{(5e)}$$



Directional constraints

Azimuth

$$x_j - x_{j'} - \tan\varphi_{j'j}\left(y_j - y_{j'}\right) = 0$$ (6a)

Help point coordinates

$$\mathbf{D}\Delta\mathbf{x} = \mathbf{d}$$ (6b)

$$\mathbf{D} = \begin{bmatrix} 1 & -\tan\varphi_{MM'} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\tan\varphi_{NN'} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\tan\varphi_{OO'} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\tan\varphi_{PP'} \end{bmatrix}$$ (6c)

$$\mathbf{d} = 0 - \begin{bmatrix} x_M - x_{M'} - \tan\varphi_{M'M}\left(y_M - y_{M'}\right) \\ x_N - x_{N'} - \tan\varphi_{N'N}\left(y_N - y_{N'}\right) \\ x_O - x_{O'} - \tan\varphi_{O'O}\left(y_O - y_{O'}\right) \\ x_P - x_{P'} - \tan\varphi_{P'P}\left(y_{P'} - y_{P'}\right) \end{bmatrix}$$ (6d)

Optimisation model

$$\min\left(\frac{1}{2}\mathbf{\Delta x}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{B\Delta x}-\mathbf{B}^{\mathrm{T}}\mathbf{\Delta L}\right) \quad (7a)$$

subject to

$$\mathbf{D\Delta x}=\mathbf{d}$$

$$\mathbf{L}_{\mathrm{b}} \leq \mathbf{\Delta x} \leq \mathbf{U}_{\mathrm{b}}$$

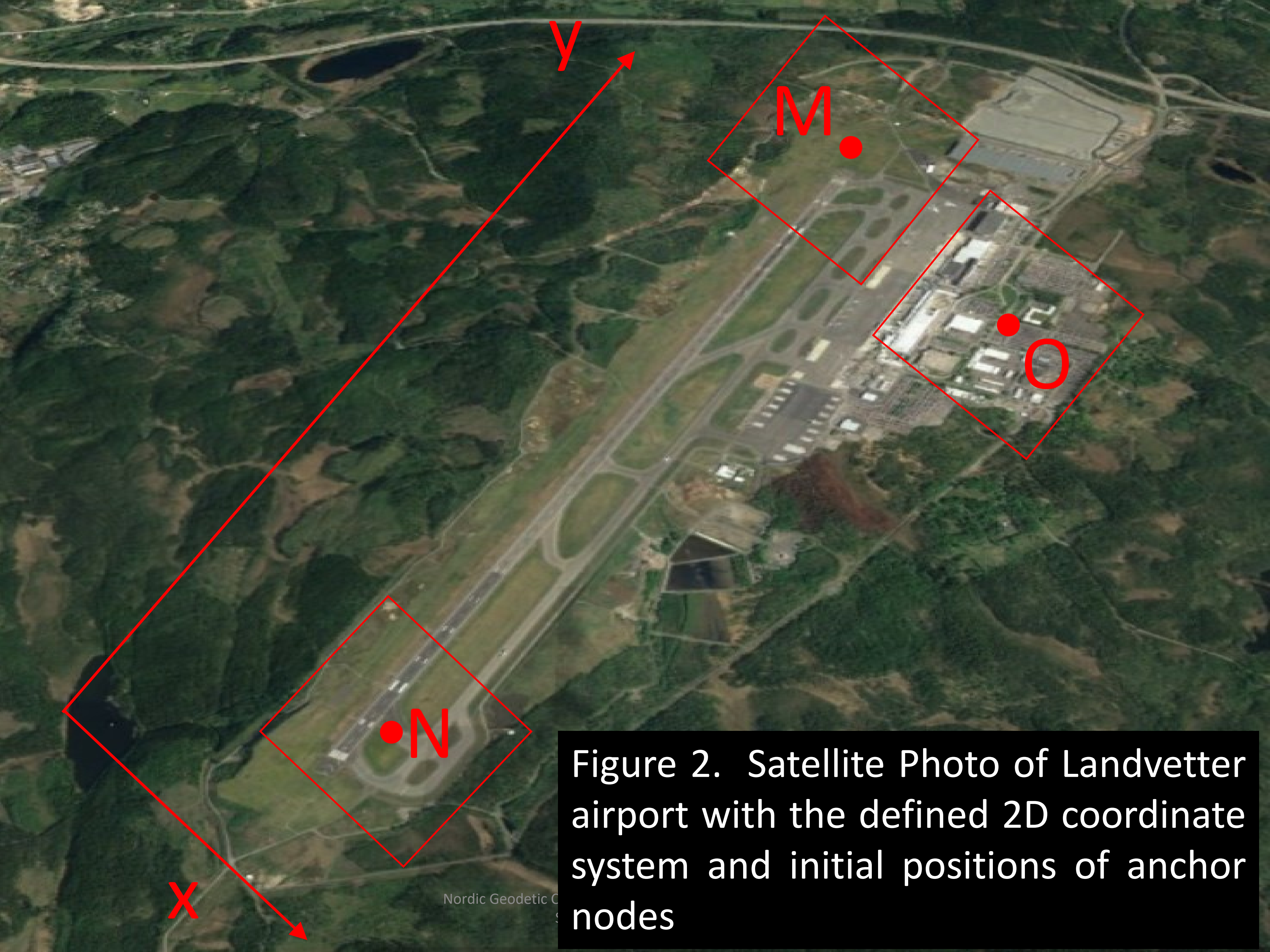


Figure 2. Satellite Photo of Landvetter airport with the defined 2D coordinate system and initial positions of anchor nodes

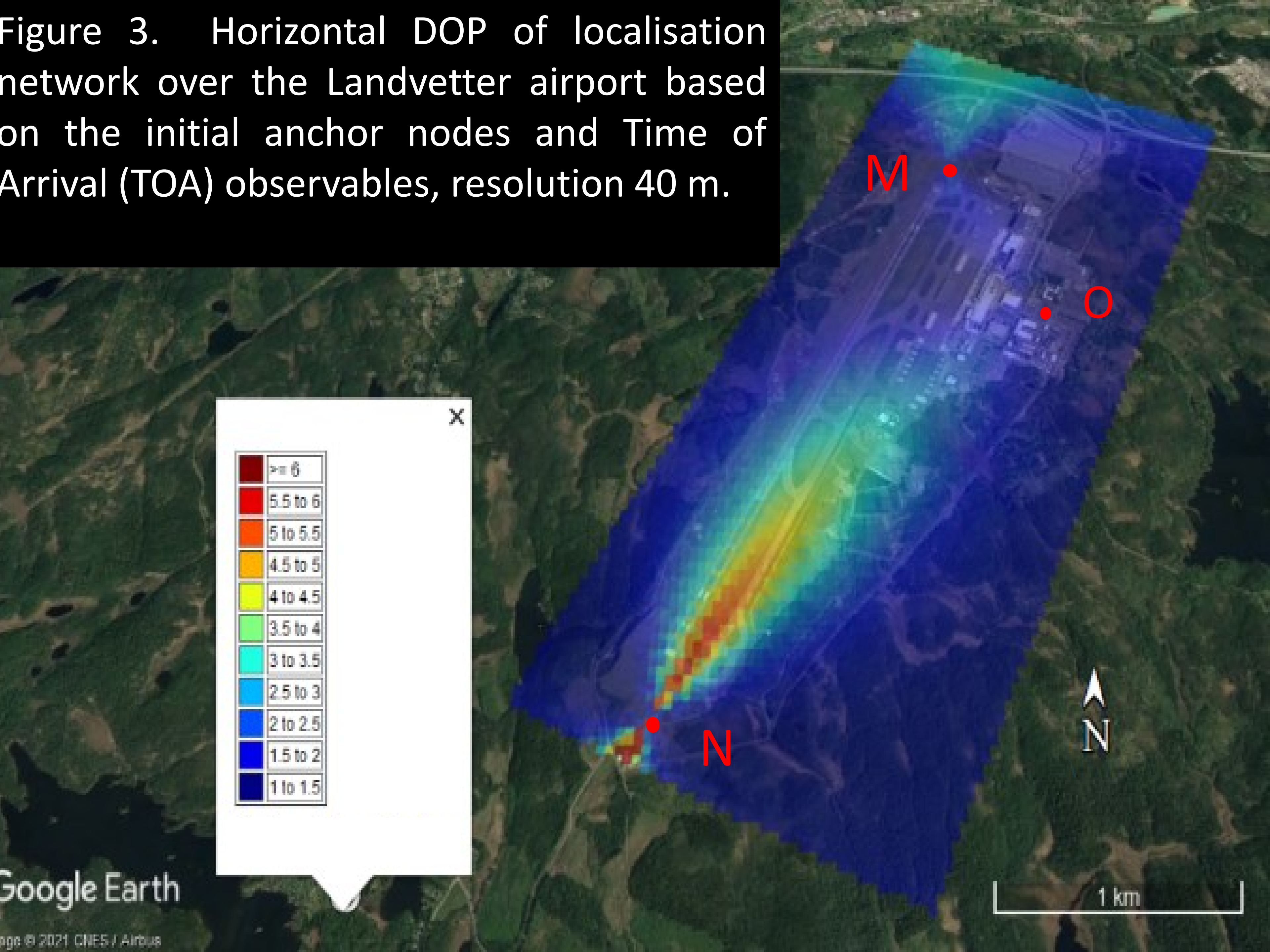


Figure 3. Horizontal DOP of localisation network over the Landvetter airport based on the initial anchor nodes and Time of Arrival (TOA) observables, resolution 40 m.
M
O
N
>= 6
5.5 to 6
5 to 5.5
4.5 to 5
4 to 4.5
3.5 to 4
3 to 3.5
2.5 to 3
2 to 2.5
1.5 to 2
1 to 1.5
N
1 km
Google Earth

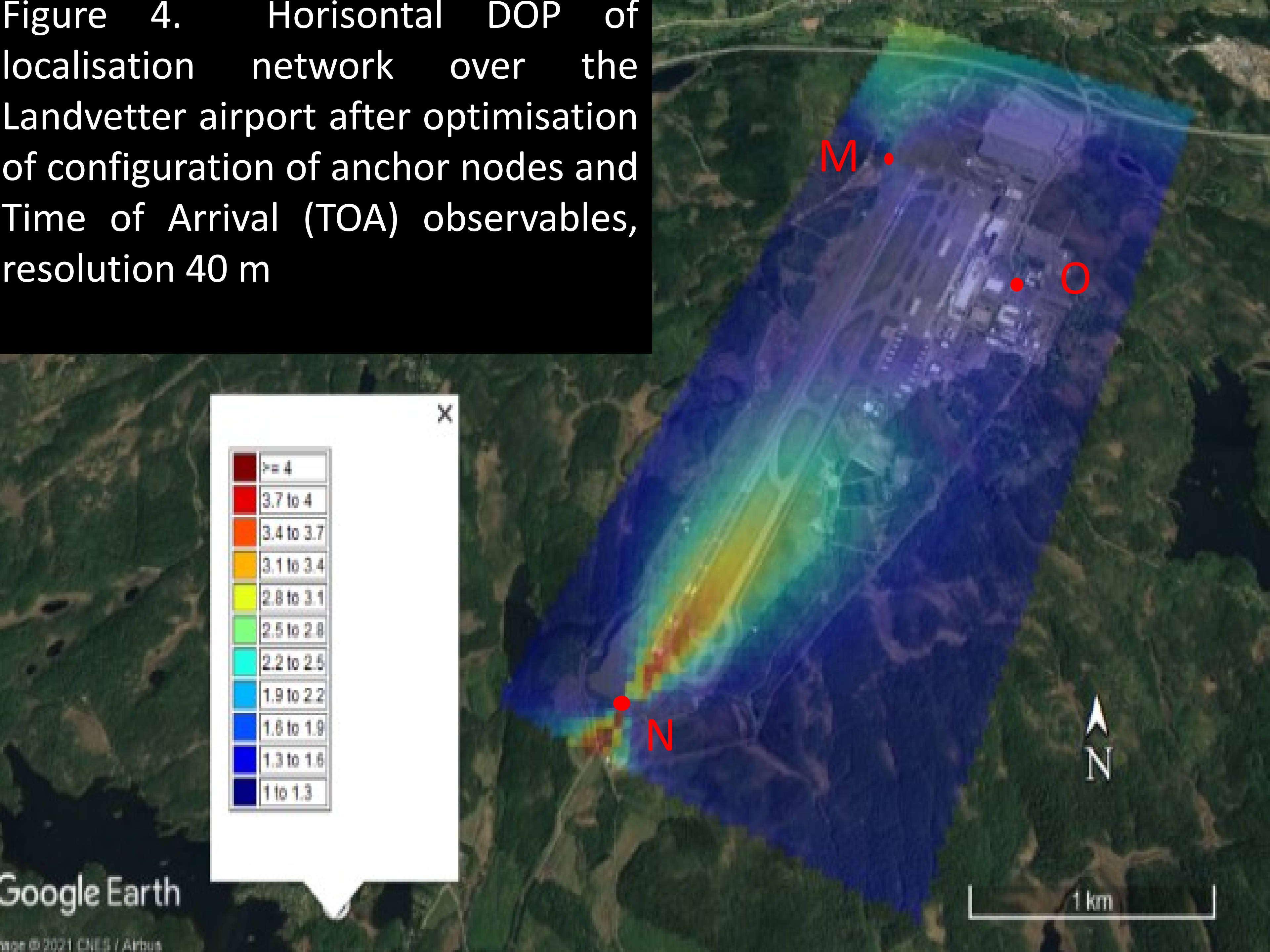


Figure 4. Horisontal DOP of localisation network over the Landvetter airport after optimisation of configuration of anchor nodes and Time of Arrival (TOA) observables, resolution 40 m

Figure 5. Horisontal DOP of localisation network over the Landvetter airport based on the initial anchor nodes and Angle of Arrival (AOA) observables, resolution 40 m

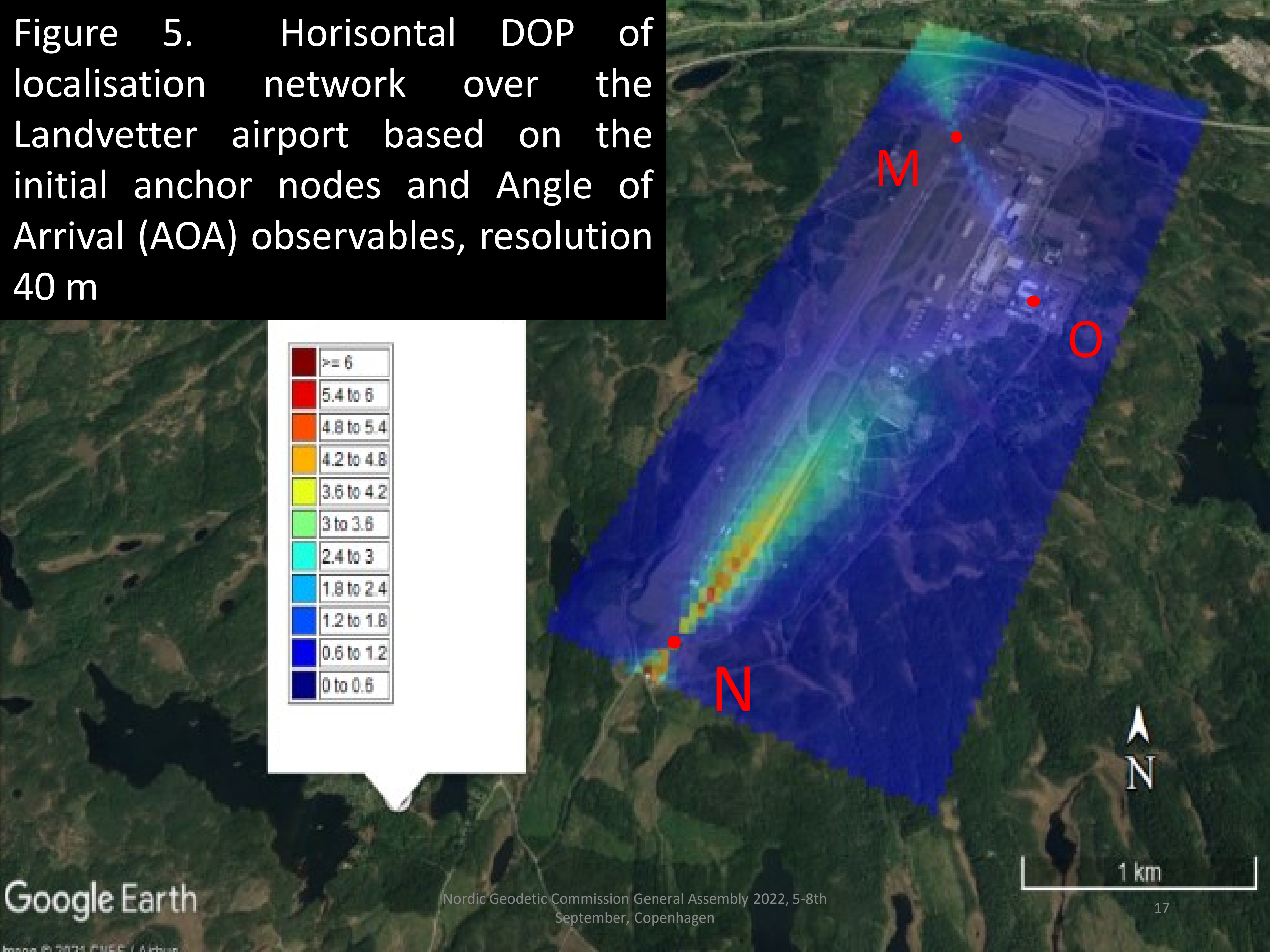
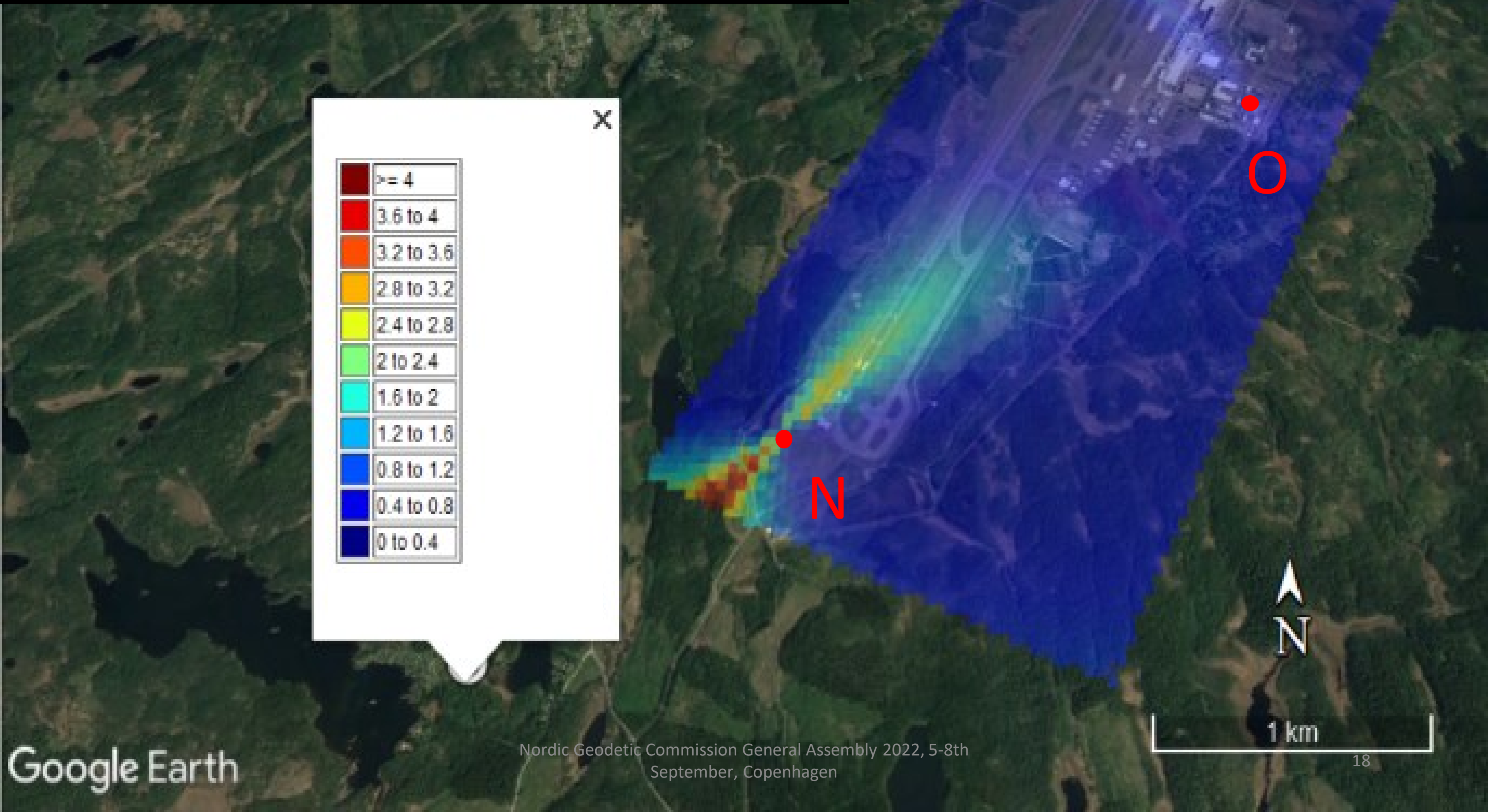


Figure 6. Horizontal DOP of localisation network over the Landvetter airport after optimisation of configuration of anchor nodes and Angle of Arrival (AOA) observables, resolution 40 m



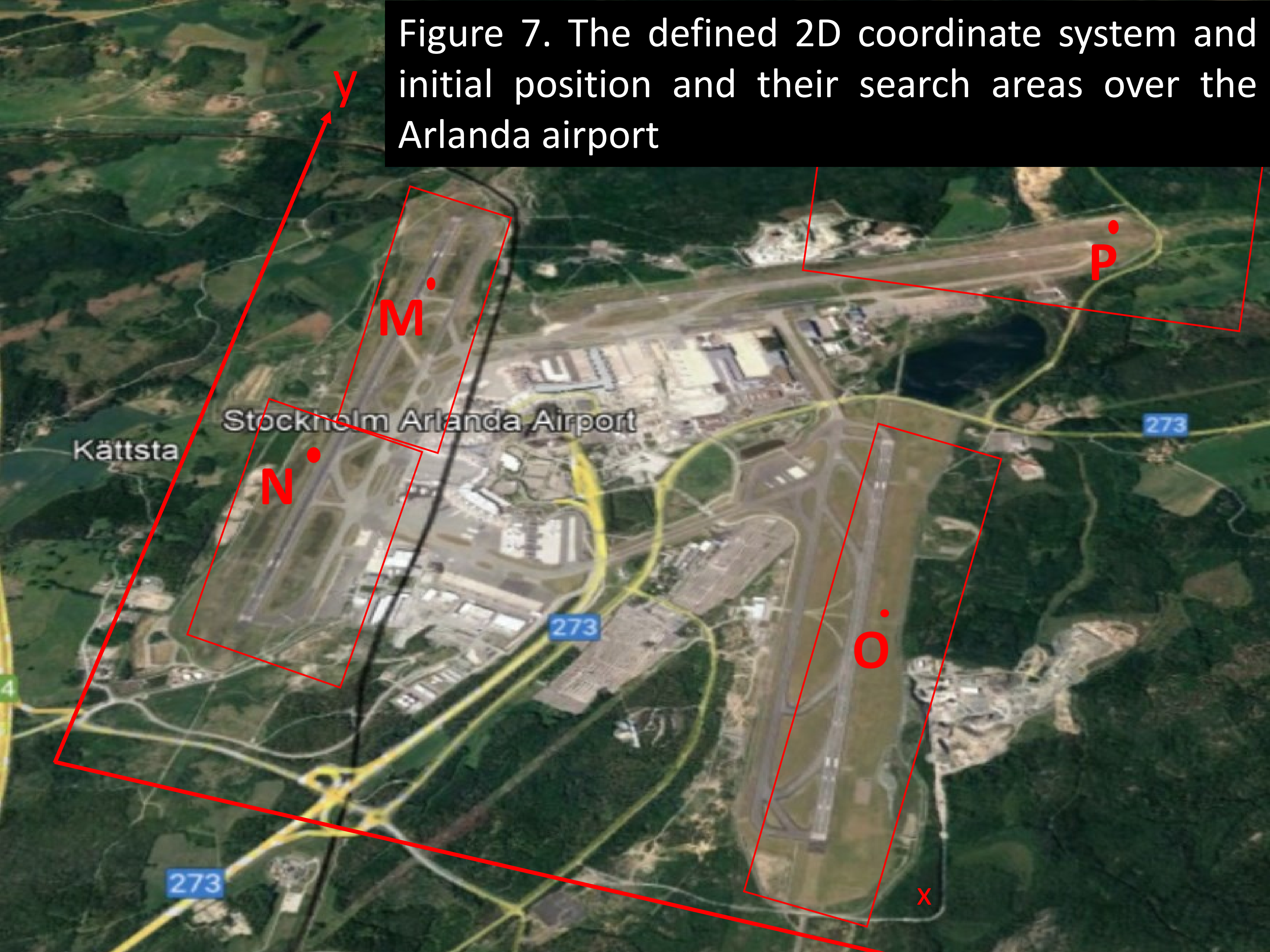


Figure 7. The defined 2D coordinate system and initial position and their search areas over the Arlanda airport

Figure 8. Horizontal DOP network based on initial positions of anchor nodes and Time-difference of arrival (TDOA) observables over the Arlanda airport, resolution 40 m

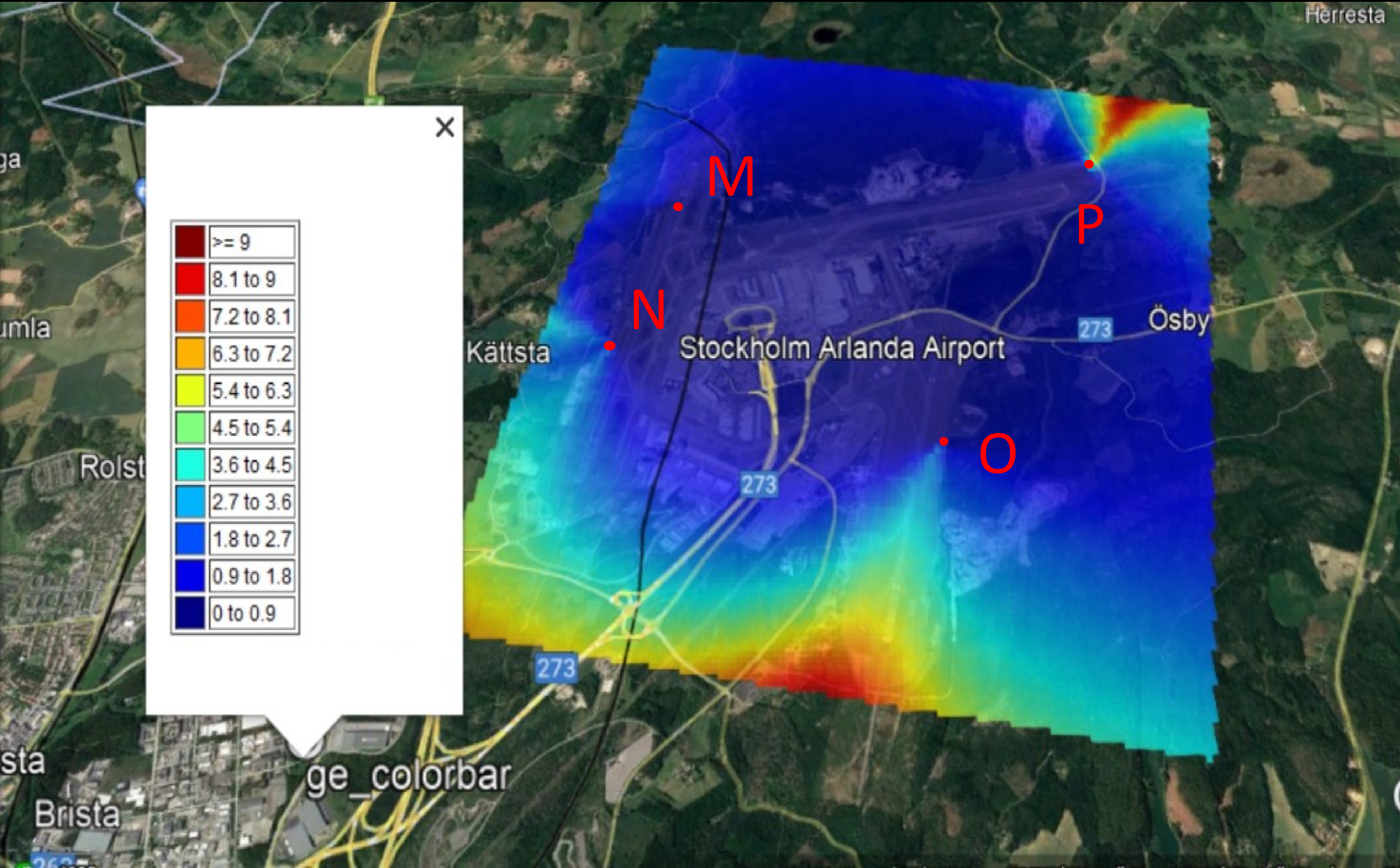


Figure 9. Horizontal DOP network after optimisation of configuration of anchor nodes based on Time-difference of arrival (TDOA) observables over the Arlanda airport, resolution 40 m

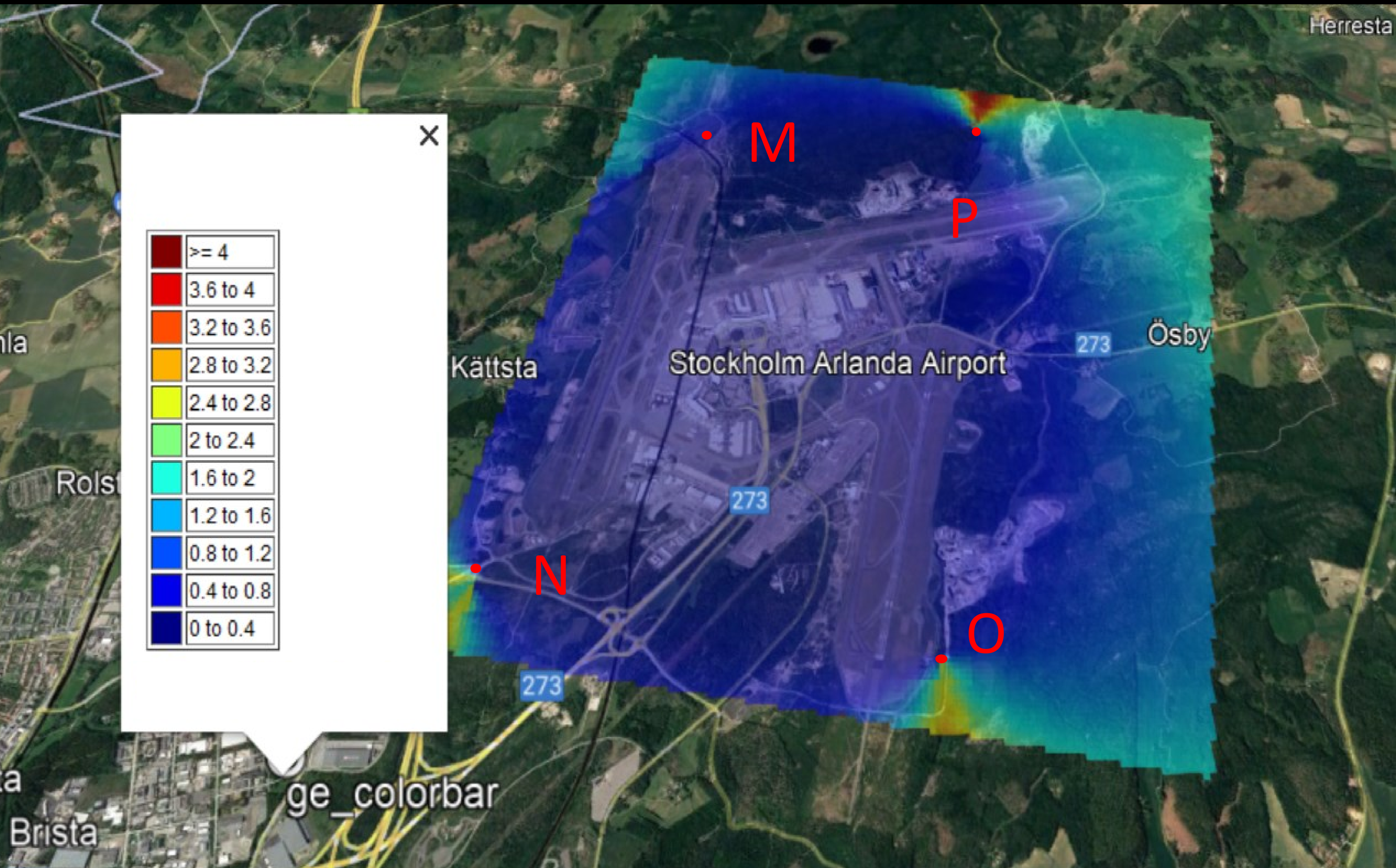
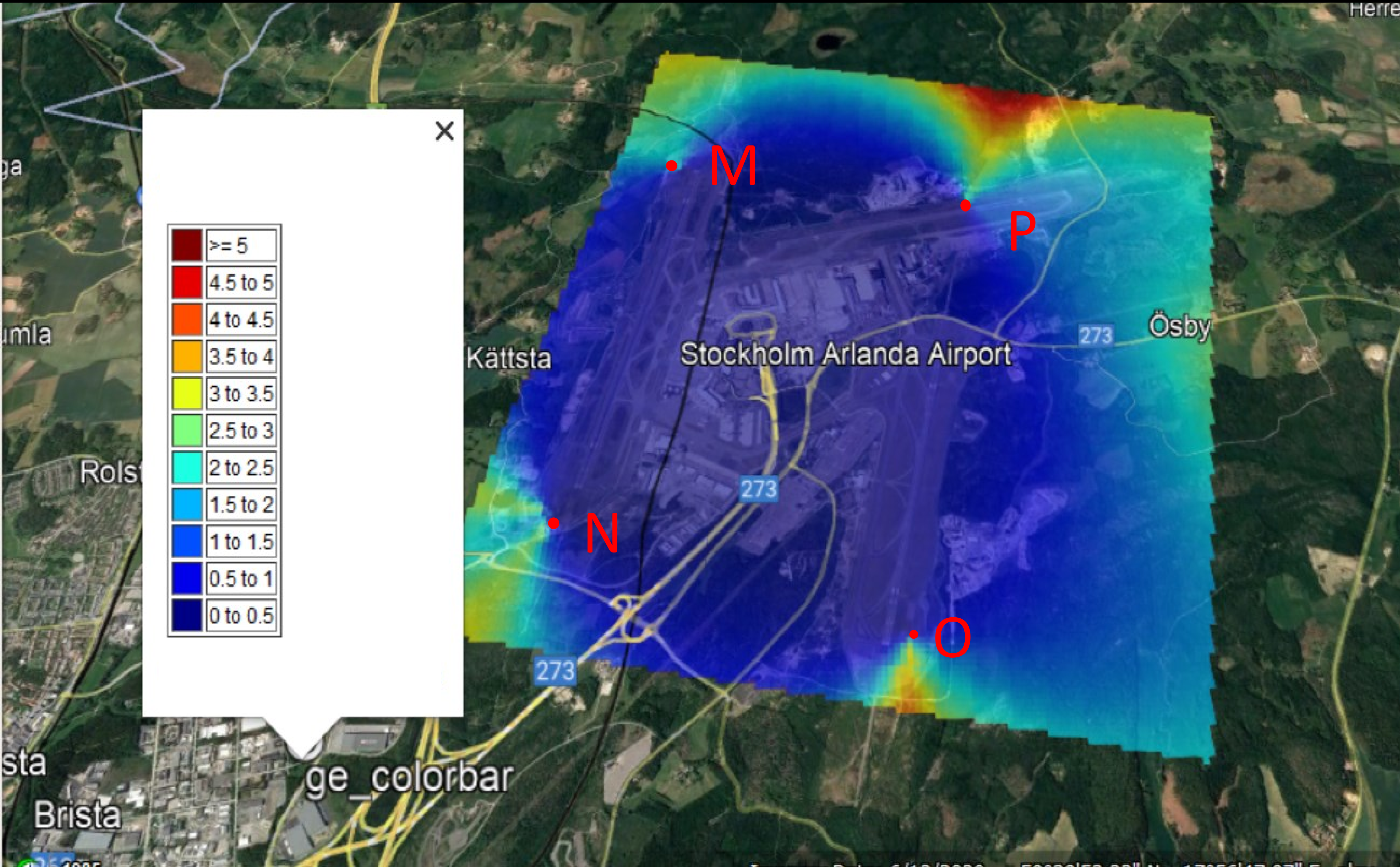
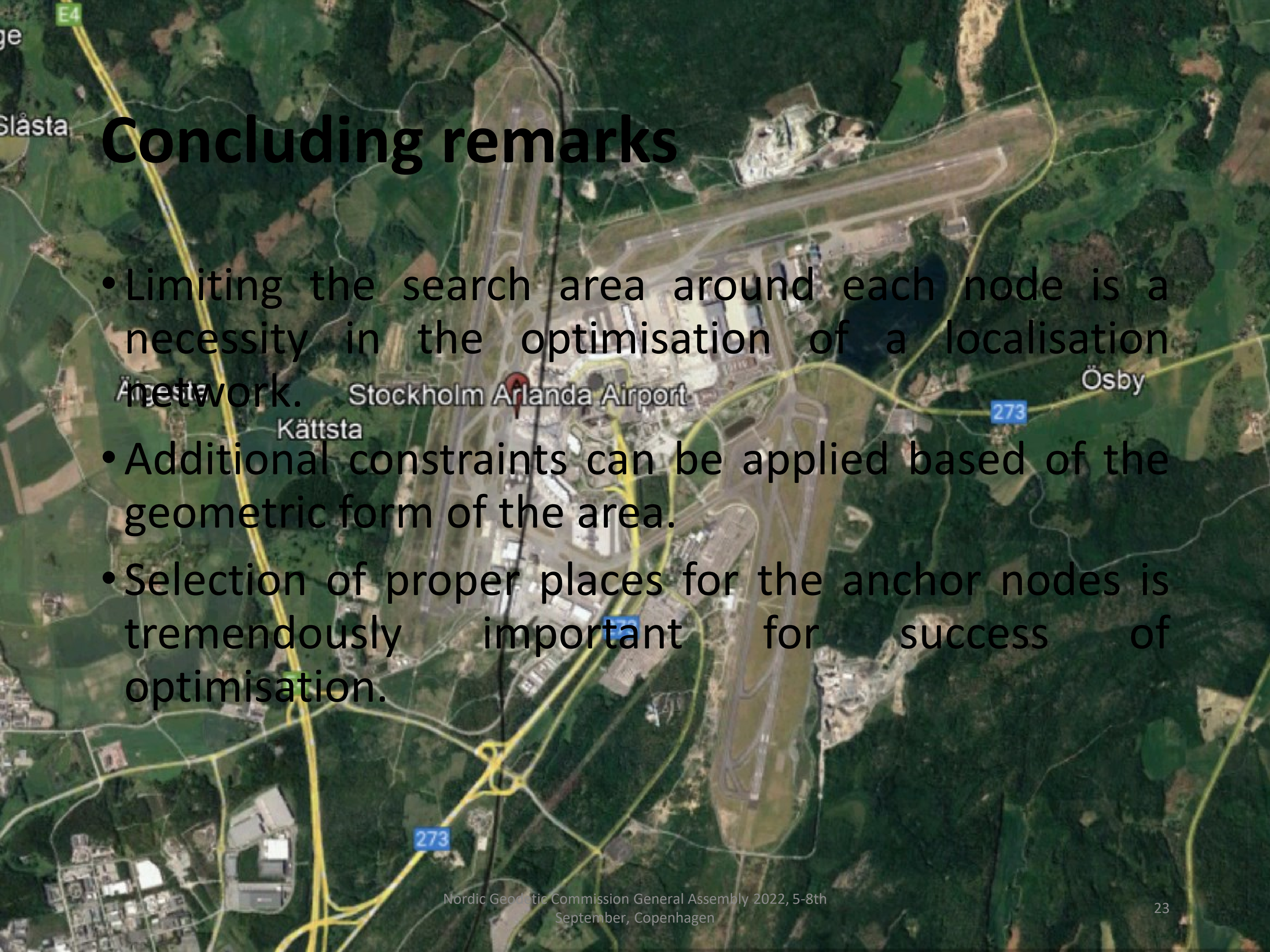


Figure 10. Horizontal DOP network after optimisation of configuration of anchor nodes with directional constraints based on Time-difference of arrival (TDOA) observables over the Arlanda airport, resolution 40 m





Concluding remarks

- Limiting the search area around each node is a necessity in the optimisation of a localisation network.
- Additional constraints can be applied based of the geometric form of the area.
- Selection of proper places for the anchor nodes is tremendously important for success of optimisation.

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Thanks for your attention