NMBU

The theoretical background to PPP

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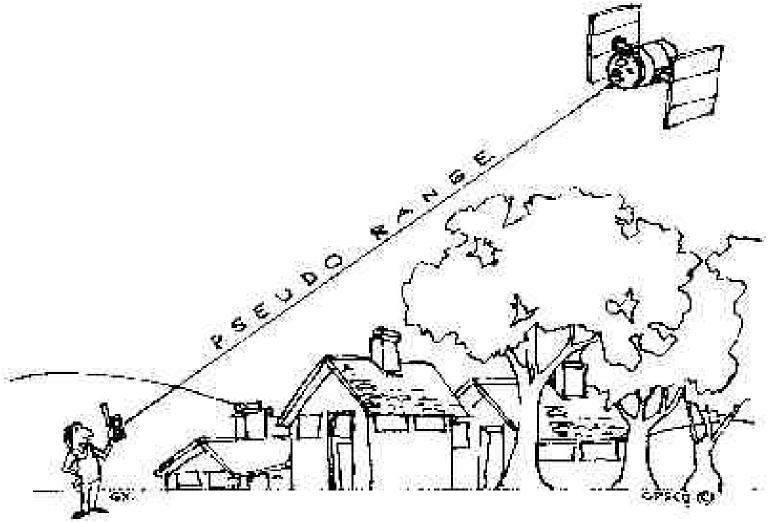
Outline

- Positioning using GNSS
- Drawbacks and advantages of Precise Point Positioning - PPP
- Long convergence time – even with ambiguity resolution techniques
- Traditional PPP - a “float solution”
- Ambiguity resolution methods in PPP: PPP-AR
- Interoperability
- Possible measures to reduce convergence time in PPP / PPP-AR
- Linear combinations of observations or raw methods?

Positioning with GNSS



Satellite coordinates and clock corrections (x^s,y^s,z^s,dt^s) are received from satellites
GNSS observations are range measurements (pseudoranges) between satellites and receiver : code and carrier phase
Estimating receiver coordinates and clock correction (x_r,y_r,z_r,dt_r) and quality numbers in a estimation process.

Positioning with GPS (case)

SPS (Standard Positioning Service)

- civilian use of GPS
- single-frequency, C/A-code, broadcasted navigation message
- accuracy approximately ca. 3-5 m (95 %) horizontally, factor ~2 weaker vertically

Main error sources in SPS

- accuracy of range measurements (pseudorange code)
- accuracy of satellite coordinates (broadcast ephemerides)
- accuracy of satellite clock corrections
- models for atmospheric errors
    - ionosphere (Klobuchar broadcasted model)
    - troposphere (standard troposphere model, e.g. Hopfield).
- multipath
- ”loading”-effects : solid earth tide, ocean and atmospheric loading

Measures to improve accuracy visa SPS

- Satellite Based Augmentation System (satellite based differential system)
    - EGNOS / WAAS / MSAS /…..

- Differential systems based on terrestrial reference stations
    - data from a single reference receiver
    - data from a network of permanent reference stations e.g. the Norwegian CPOS systems

- Precise Point Positioning (PPP)
    - high accuracy positioning using observations from a single GNSS receiver
    - very suited for post-processing purposes
    - current issues:
        - “real-time” PPP
        - carrier phase integer ambiguity resolution : PPP-AR
        - reduction in “convergence time”
    - Norwegian initiatives: ABSPOS (NMBU) and TerraPos (Terratec AS)

Precise Point Positioning - PPP

Pros:

- Use observations from one satellite receiver only
- Improved flexibility
- Reduced costs
- Computationally efficient
- Global coverage with consistent and high accuracy in global reference frame (4D)
  - Static mode : centimeter
  - Kinematic mode : sub-decimetre

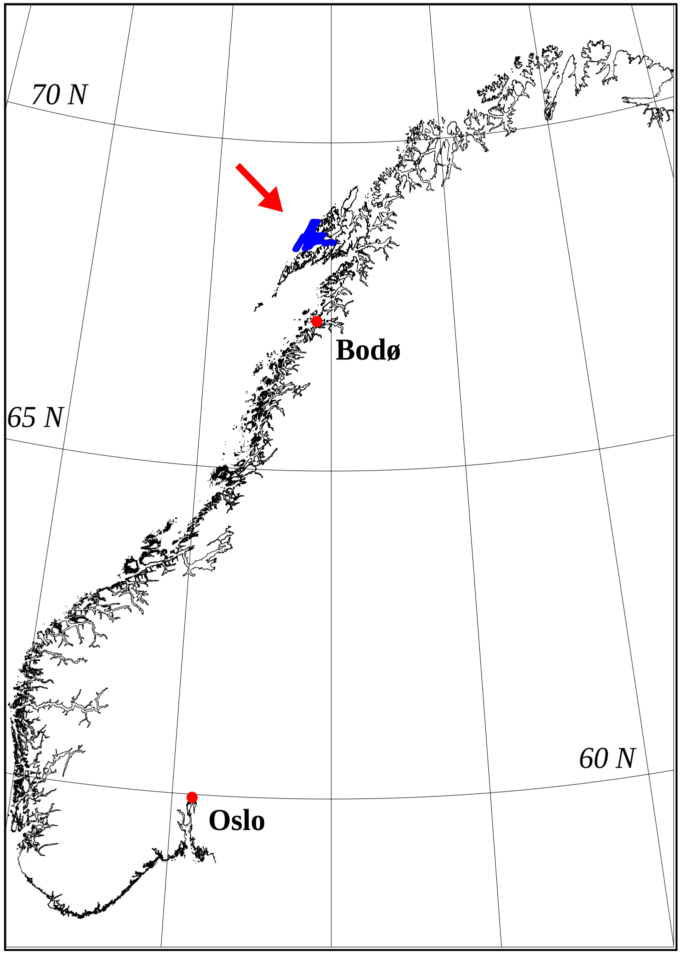
Cons:

- Traditionally a post-processing technique
  - real-time services are now available
- Long convergence time due to “float solution”
  - Ambiguity resolution techniques have been demonstrated, but ……..
- For use in local networks : Transformation in space & time
  - Should be standardized

Precise Point Positioning - PPP

- First developed for fast and efficient processing of large global networks (Zumberge et al. 1997)
- Further developed to process kinematic observations from remote areas

The "Vesterålfjord project" with Statens Kartverk-Sjø, 2002

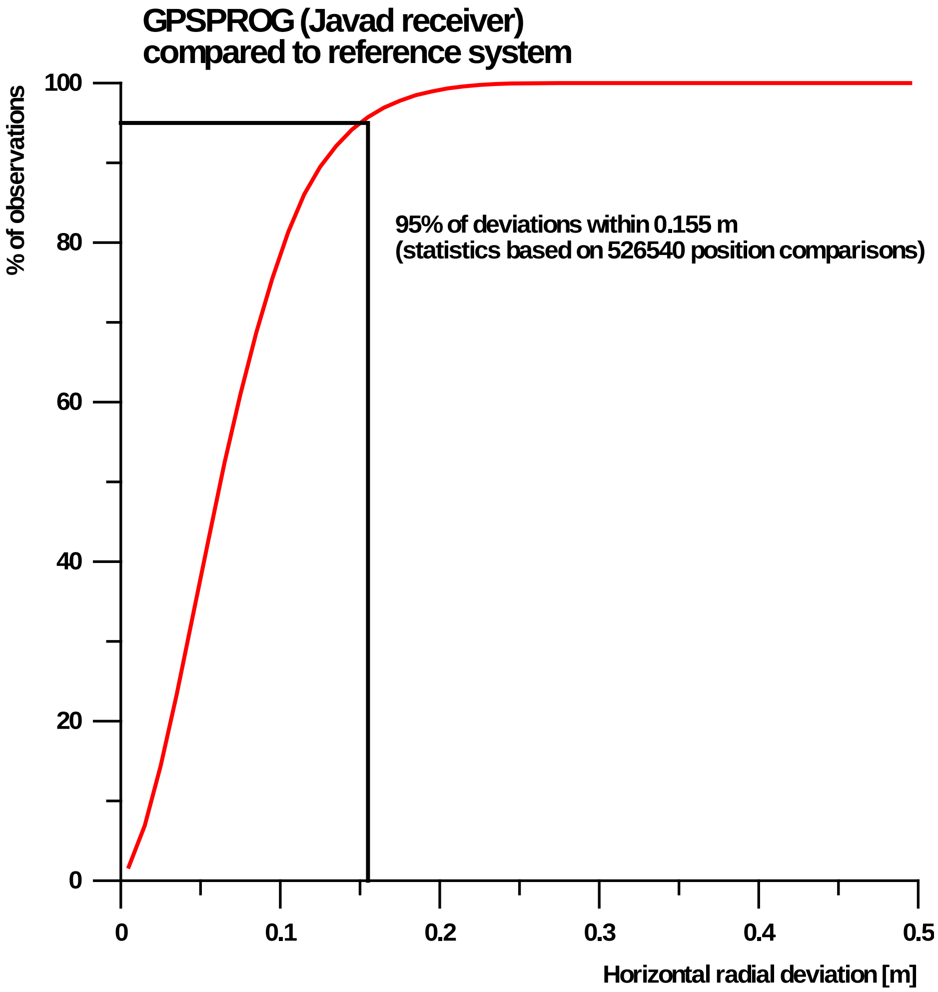


- 68° N / 14° E
- March 2002
- Reference receiver in "Bodø"
  - ~ 150 km from area

The “Vesterålfjord project” with Statens Kartverk-Sjø, 2002

M/S H.U. Sverdrup II

The “Vesterålfjord project” with Statens Kartverk-Sjø, 2002

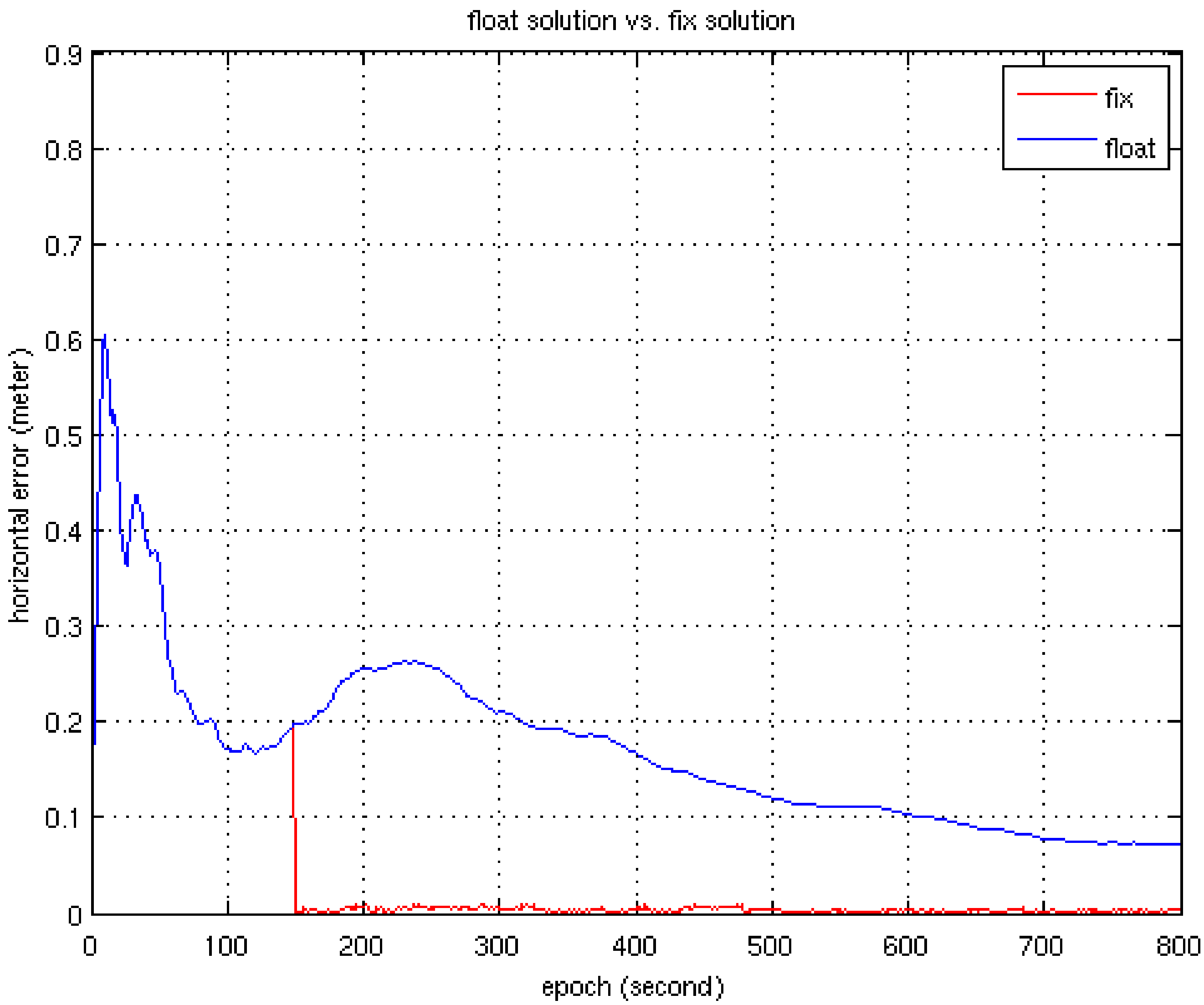


Precise Point Positioning - PPP

- First developed for fast and efficient processing of large global networks (Zumberge et al. 1997)
- Further developed to process kinematic observations from remote areas
- A very popular technique when having long timeseries of continues observations, e.g.
  - Geodetic and geodynamic applications
  - Georeferencing of kinematic sensors
    - Seafloor mapping
    - LIDAR
    - Aerial photogrammetry
    - ..

“Not yet an alternative to differential RTK systems”

Example: differential processing of a short baseline



Quality measures of PPP

- Accuracy
- Convergence time

  time required to reach a specific level of accuracy, does not deviate beyond this level after reaching it.

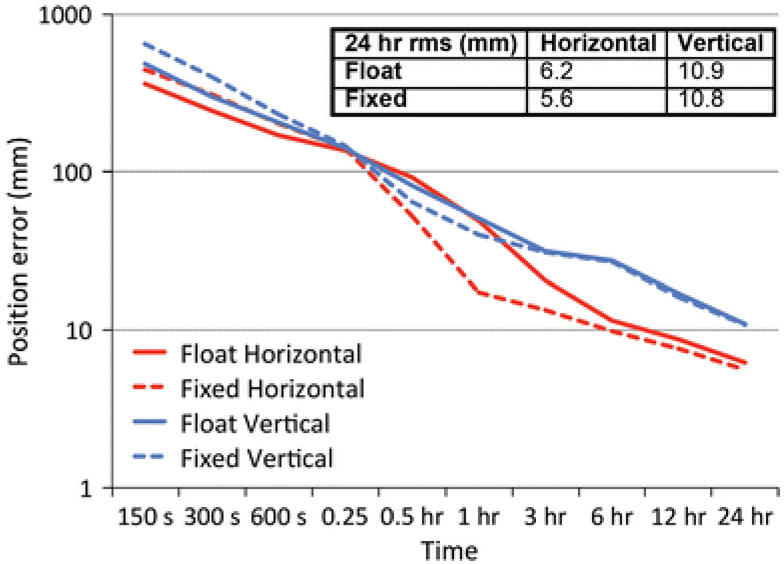
Recommended convergence time for standard static PPP solution (float solution) to converge.

Horizontal Accuracy (cm)	Recommended convergence period
20	35 min
10	50 min
5	60 min
2	9 h
1	23 h
0.5	24 h

From: Seepersad and Bisnath (2013), Integrity monitoring in Precise Point Positioning, Proceedings of ION GNSS 2013

Quality measures of PPP – with Ambiguity resolution

Processing of GPS-observations from 300 globally distributed IGS sites
1 week of data in 2012.
GPS-only



24 hr rms (mm)	Horizontal	Vertical
Float	6.2	10.9
Fixed	5.6	10.8

From: Choy, Bisnath, Rizos, (2016), Uncovering common misconceptions in GNSS Precise Point Positioning, GPS Solution-"online first".

Paradox

- When processing dual frequency observations from GPS-only, applying ambiguity resolution techniques does not significantly reduce convergence time.
- Possible reasons:
  - Immature PPP ambiguity resolution techniques? (speculation).
  - Current PPP ambiguity resolution techniques makes use of linear combinations were ionospheric effect is eliminated. For short baseline RTK, the ionosphere is almost eliminated and the underlying constraint makes fast ambiguity fixing possible.
  - Current PPP ambiguity resolution techniques makes use of the Melbourne-Wübbena linear combination to estimate the wide-lane ambiguity, which heavily depend on the quality of the code-observations.

This “paradox” will be addressed later in the presentation…………………

Correction model for traditional PPP

- Precise satellite coordinates, earth orientation parameters and satellite clock corrections, available from e.g. International GNSS Service (IGS)
  - well defined formats
  - ftp-servers
  - free of charge
- Use dual frequency code **and** carrier phase observations
- Site dependant effects
  - Solid earth tide
  - Ocean loading
  - (Atmospheric loading)
  - Receiver antenna calibration
- Satellite dependant effects
  - Satellite antenna calibration
  - Phase wind-up
- ……..

Take into account compatibility issues :

- **IERS and IGS conventions**

PPP processing options

- Geodetic GNSS software (e.g. GIPSY, Bernese, NAPEOS, ….)
- Dedicated post-processing PPP-software , e.g.
  - P3 (University of Calgary)
  - ABSPOS (NMBU)
  - TerraPos (Terratec AS)
  - Aplanix
  - ….
- Online post-processing tools, e.g.
  - gAGE (UPC – Spain)
  - GAPS (Canada)
  - magicPPP (Spain)
  - NRCan PPP (Canada)
  - ….
- Real-Time services, e.g.
  - Real-time IGS
  - Global Differential GPS / Real-time GIPSY (USA)
  - Private companies: Fugro, Trimble,…

Some compatibility issues

- Apply antenna calibration information as used in network solution
  e.g IGS antex-file ftp://ftp.igs.org/pub/station/general/igs08.atx

- Raw observations must comply with network standard
  E.g. IGS uses P1 and P2 code observables in the estimation of their products

PPP receiver observes P1 & P2	:	OK
PPP receiver observes C1 & P2	:	Apply P1-C1 DCB correction: $P1 = C1 + DCB_{P1\text{-}C1}$

Utility for RINEX 2-files by Jim Ray :
http://www.nrl.navy.mil/ssdd/sites/www.nrl.navy.mil.ssdd/files/files/cc2noncc.f
Bias-file from CODE :
http://aiuws.unibe.ch/ionosphere/p1c1.f

- IERS conventions
  https://www.iers.org/IERS/EN/DataProducts/Conventions/conventions.html

IGS products

GPS Satellite Ephemerides / Satellite & Station Clocks

Type		Accuracy	Latency	Updates	Sample Interval
Broadcast	orbits	~100 cm	real time	--	daily
	Sat. clocks	~5 ns RMS ~2.5 ns SDev			
Ultra-Rapid (predicted half)	orbits	~5 cm	real time	at 03, 09, 15, 21 UTC	15 min
	Sat. clocks	~3 ns RMS ~1.5 ns SDev			
Ultra-Rapid (observed half)	orbits	~3 cm	3 - 9 hours	at 03, 09, 15, 21 UTC	15 min
	Sat. clocks	~150 ps RMS ~50 ps SDev			
Rapid	orbits	~2.5 cm	17 - 41 hours	at 17 UTC daily	15 min
	Sat. & Stn. clocks	~75 ps RMS ~25 ps SDev			5 min
Final	orbits	~2.5 cm	12 - 18 days	every Thursday	15 min
	Sat. & Stn. clocks	~75 ps RMS ~20 ps SDev			Sat.: 30s Stn.: 5 min

http://www.igs.org/products

Observation equations dual frequency GPS

Equations as given in most introductory level textbooks:

$$P_i = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_i^2} I_1 + \varepsilon_{P_i}$$

$$L_i = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_i^2} I_1 - \lambda_i N_i + \varepsilon_{L_i}$$

Equations as given in "advanced" level textbooks:

$$P_i = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_i^2} I_1 + b_{P_i}^r - b_{P_i}^s + \varepsilon_{P_i}$$

$$L_i = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_i^2} I_1 + b_{L_i}^r - b_{L_i}^s - \lambda_i N_i + \varepsilon_{L_i}$$

i : index for frequency
P_i: Code observation
ρ : geometric distance
c : speed of light in vacum
dt^r: receiver clock error
dt^s: satellite clock error
T : tropospheric delay
f_i : frequency
I_1 : first order ionospheric term on L1
ε_{P_i}: noise and unmodeled effects on code

L_i : phase observation scaled to meter
N_i : carrier phase integer ambiguity
λ_i : wavelength on frequency i
ε_{L_i}: noise and unmodeled effects on phase

$b_{P_i}^r$: receiver code hardware bias
$b_{P_i}^s$: satellite code hardware bias

$b_{L_i}^r$: receiver phase hardware bias
$b_{L_i}^s$: satellite phase hardware bias

Observation equations GPS

Equations as given in most introductory level textbooks:

$$P_i = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_i^2}\, I_1\, \varepsilon_{P_i}$$

$$L_i = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_i^2}\, I_1 - \lambda_i N_i + \varepsilon_{L_i}$$

Equations as given in "advanced" level textbooks:

$$P_i = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_i^2}\, I_1 + b^r_{P_i} - b^s_{P_i} + \varepsilon_{P_i}$$

$$L_i = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_i^2}\, I_1 + b^r_{L_i} - b^s_{L_i} - \lambda_i N_i + \varepsilon_{L_i}$$

i : *index for frequency*
P_i: *Code observation*
ρ : *geometric distance*
c : *speed of light in vacum*
dt^r: *receiver clock error*
dt^s: *satellite clock error*
T : *tropospheric delay*
f_i : *frequency*
I_1 : *first order ionospheric term on L1*
ε_{P_i}: *noise and unmodeled effects on code*

L_i : *carrier phase observation*
λ_i : *wavelength on frequency i*
ε_{P_i}: *noise and unmodeled effects on phase*

$b^r_{P_i}$: *receiver code hardware bias*
$b^s_{P_i}$: *satellite code hardware bias*

$b^r_{L_i}$: *receiver phase hardware bias*
$b^s_{L_i}$: *satellite phase hardware bias*

Linear combinations of observables

"trick to achieve benefits"

In a dual frequency system, for carrier phase measured in cycles, φ_i , the general form is:

$$\phi_{LC} = m_1 \ \varphi_1 + m_2 \ \varphi_2 \ , \qquad m_1 \ and \ m_2 \ are \ arbitrary \ numbers$$

Frequency of linear combination

$$f_{LC} = \ m_1 \ f_1 + \ m_2 \ f_2$$

Wavelength of linear combination

$$\lambda_{LC} = \frac{c}{f_{LC}} = \frac{c}{m_1 \ f_1 + \ m_2 \ f_2}$$

Increased noise factor $\sqrt{{m_1}^2 + {m_2}^2}$, assuming independent observations of equal precision.

Ionospheric amplification factor

$$V_I = \frac{m_1 \ f_2 + \ m_2 \ f_1}{m_1 \ f_1 + \ m_2 \ f_1}$$

Linear combinations of carrier phase observations when scaled to unit of meter: $L_1 = \varphi_1 \lambda_1 \quad and \; L_1 = \varphi_1 \lambda_1$

$$L_{LC} = \alpha L_1 + \beta L_2$$

$$\alpha = \frac{m_1 f_1}{(m_1 f_1 + m_2 f_2)}$$

$$\beta = \frac{m_2 f_2}{(m_1 f_1 + m_2 f_2)}$$

Linear combinations used in PPP and PPP-AR

Signal	Observations in cycles coefficients		Observations in meters coefficients		Wavelength λ_{LC} (cm)	Ionospheric amplification factor V_I	Typical noise (cm)
	m_1	m_2	α	β			
L_1	1	0	1	0	19.0	0.779	0.30
L_2	0	1	0	1	24.4	1.283	0.39
L_{WL}	1	-1	4.529	-3.529	86.2	1	1.93
L_{NL}	1	1	0.562	0.438	10.7	-1	0.24
L_{IF}	77	-60	2.546	-1.546	0.6	0	0.97

L_{WL} is the phase wide-lane linear combination
L_{NL} is the phase narrow-lane linear combination
L_{IF} is the phase ionospheric-free linear combination (first order effect eliminated)

Similarly, linear combinations of code observations can be made with:

$$P_{LC} = \alpha P_1 + \beta P_2$$

Signal	Observations in cycles coefficients		Observations in meters coefficients		Wavelength λ_{LC} (cm)	Ionospheric amplification factor V_I	Typical noise (cm)
	m_1	m_2	α	β			
P_1	-	-	1	0	-	-0.779	30
P_2	-	-	0	1	-	-1.283	30
P_{NL}	1	1	0.526	0.438	-	1	21
P_{IF}	-	-	2.546	-1.546	-	0	89

P_{NL} is the code narrow-lane linear combination
P_{IF} is the code ionospheric-free linear combination (first order effect eliminated)

First-order ionospheric effect:

- Eliminated in ionospheric-free combination
- Same magnitude, but opposite sign on phase and code
- Same magnitude, but opposite sign for wide-lane and narrow-lane

➔ When subtracting phase wide-lane and code narrow-lane, the first order ionospheric effect is eliminated.

Satellite and receiver biases propagate into the linear combination

$$P_{IF} = \alpha_{IF} P_1 + \beta_{IF} P_2$$
$$= \rho + \left(cdt^r + b^r_{P_{IF}}\right) - \left(cdt^s + b^s_{P_{IF}}\right) + T + \varepsilon_{P_{IF}}$$

$$L_{IF} = \alpha_{IF} L_1 + \beta_{IF} L_2$$
$$= \rho + \left(cdt^r + b^r_{L_{IF}}\right) - \left(cdt^s + b^s_{L_{IF}}\right) + T - \lambda_{IF} N_{IF} + \varepsilon_{L_{IF}}$$

$cdt^r_{P_{IF}} = cdt^r + b^r_{P_{IF}}$	-	"receiver code clock error"
$cdt^s_{P_{IF}} = cdt^s + b^s_{P_{IF}}$	-	"satellite code clock error"
$cdt^r_{L_{IF}} = cdt^r + b^r_{L_{IF}}$	-	"receiver phase clock error"
$cdt^s_{L_{IF}} = cdt^s + b^s_{L_{IF}}$	-	"satellite phase clock error"

$$b^r_{P_{IF}} = \alpha_{IF} b^r_{P_1} + \beta_{IF} b^r_{P_2}$$
$$b^s_{P_{IF}} = \alpha_{IF} b^s_{P_1} + \beta_{IF} b^s_{P_2}$$
$$b^r_{L_{IF}} = \alpha_{IF} b^r_{L_1} + \beta_{IF} b^r_{L_2}$$
$$b^s_{L_{IF}} = \alpha_{IF} b^s_{L_1} + \beta_{IF} b^s_{L_2}$$

Phase wide-lane- and code narrow-lane

The phase wide-lane and code narrow-lane combinations are often used in ambiguity resolution. These linear combinations will also be contaminated by the raw satellite- and receiver biases.

$$
\begin{aligned}
L_{WL} &= \alpha_{WL} L_1 + \beta_{WL} L_2 \\
&= \rho + (dt^r - dt^s) + T + I_{L_{WL}} - \color{red}{\lambda_{WL} N_{WL}} \\
&\quad + \left(\alpha_{WL} b^r_{L_1} + \beta_{WL} b^r_{L_2}\right) - \left(\alpha_{WL} b^s_{L_1} + \beta_{WL} b^s_{L_2}\right) + \varepsilon_{L_{WL}}
\end{aligned}
$$

$$
\begin{aligned}
P_{NL} &= \alpha_{NL} P_1 + \beta_{NL} P_2 \\
&= \rho + (dt^r - dt^s) + T + I_{L_{WL}} \\
&\quad + \left(\alpha_{NL} b^r_{P_1} + \beta_{NL} b^r_{P_2}\right) - \left(\alpha_{NL} b^s_{P_1} + \beta_{NL} b^s_{P_2}\right) + \varepsilon_{P_{NL}}
\end{aligned}
$$

Melbourne-Wübbena combination for estimating wide-lane ambiguity

Melbourne-Wübbena linear combination of phase and code is both geometry- and ionospheric free.

$$MW = L_{WL} - P_{NL} = -\lambda_{WL} N_{WL} + (b^{r}_{MW} - b^{s}_{MW}) + \varepsilon_{MW}$$

$$b^{r}_{MW} = \left(\alpha_{WL} b^{r}_{L_1} + \beta_{WL} b^{r}_{L_2}\right) - \left(\alpha_{NL} b^{r}_{P_1} + \beta_{NL} b^{r}_{P_2}\right)$$
$$b^{s}_{MW} = \left(\alpha_{WL} b^{s}_{L_1} + \beta_{WL} b^{s}_{L_2}\right) - \left(\alpha_{NL} b^{s}_{P_1} + \beta_{NL} b^{s}_{P_2}\right)$$

When applying a double-difference technique, receiver (b^{r}_{MW}) and satellite (b^{s}_{MW}) biases are canceled through double differencing, and the wide-lane ambiguity can be identified as integer values.

The wide-lane is however not optimal for final processing due to amplified noise and ionospheric effect.

Wide- / narrow-lane methods for ambiguity resolution

Wide- / narrow-lane methods for ambiguity resolution

1. Estimate integer wide-lane ambiguities using the Melbourne—Wübbena combination

2. Introduce the integer wide-lane ambiguity in the ionospheric-free linear combination

$$N_{IF} = \frac{c}{f_1 + f_2} N_1 + \frac{c\, f_2}{(f_1^2 - f_2^2)} N_{WL}$$

For GPS f_1 and f_2 frequencies, an alternative form is

$$N_{IF} = 17N_1 + 60N_{WL}$$

The N_{WL} ambiguity is introduced as a known quantity and the N_1 ambiguity is solved in the estimation process using the ionospheric-free observables.

Ordinary PPP

External information, from e.g. IGS:
- Precise satellite coordinates : x^s, y^s, z^s
- Precise satellite clock corrections : $cdt^s_{P_{IF}} = cdt^s + b^s_{P_{IF}}$

The satellite coordinates and satellite clock corrections are estimated using the ionospheric-free combinations of code and phase.

On the user side, the same ionospheric-free linear combinations are used:

$$
\begin{aligned}
P_{IF} &= \rho + cdt^r_{P_{IF}} - cdt^s_{P_{IF}} + T + \varepsilon_{P_{IF}} \\
L_{IF} &= \rho + cdt^r_{P_{IF}} - cdt^s_{P_{IF}} + T - \lambda_{IF} N_{IF} \\
&\qquad + \left(b^r_{L_{IF}} - b^r_{P_{IF}}\right) - \left(b^s_{L_{IF}} - b^s_{P_{IF}}\right) + \varepsilon_{L_{IF}}
\end{aligned}
$$

"Phase ambiguity" parameter: $-\lambda_{IF} N_{IF} + \left(b^r_{L_{IF}} - b^r_{P_{IF}}\right) - \left(b^s_{L_{IF}} - b^s_{P_{IF}}\right)$

Phase ambiguity parameter in ordinary PPP

Remember that the satellite code clocks as available from e.g. IGS are contaminated by the satellite code biases.

$$cdt^s_{P_{IF}} = cdt^s + b^s_{P_{IF}}$$

When the contaminated satellite code clocks are used to correct the phase observations, the satellite code biases are introduced in the observation equation for the phase.

Similarly, when estimating one common receiver code clock parameter, the receiver code bias are introduced in the observation equation for phase.

Biased phase ambiguity parameter in traditional PPP:

$$A = -\lambda_{IF} N_{IF} + \left(b^r_{L_{IF}} - b^r_{P_{IF}}\right) - \left(b^s_{L_{IF}} - b^s_{P_{IF}}\right)$$

The ambiguity parameter is non-integer due to **both code- and phase biases** ➔ float solution.

PPP-AR : PPP Ambiguity Resolution
- the "naive" approach

If external precise satellite clock corrections were augmented with corresponding satellite biases for each signal, the ambiguity parameters could be estimated as integers.

Full external information in the form of "absolute" PPP-AR parameters, from e.g. IGS:

Precise satellite coordinates : x^s, y^s, z^s
Precise satellite clock corrections : cdt^s
Precise biases for each frequency and signal : $b^s_{P_1}, b^s_{P_2}, b^s_{L_1}, b^s_{L_2}$

Question:
Using observations from a global network of reference stations, is it possible to estimate the full set of "absolute" parameters?

Answer: No!

Why?
Datum-defect (see e.g. Teunissen and Khodabandeh, 2015)

Pragmatic approaches to PPP-AR
- estimate “relative” PPP-AR parameters

“Relative” parameters with properties that enable the identification of integer phase ambiguities parameters.

- Decupled clock model (Collins 2008)
- Integer recovery clock model (Laurichesse et al. 2008)
- Fractional Cycle Bias/Uncalibrated Hardware Delay (Ge et al. 2008, Geng et al. 2009)

The “relative” parameters from these methods are equivalent in the form that it is possible to convert/transform from one set of parameters to another.

Current agencies that provide “relative” PPP-AR parameters

Agency	PPP-AR method
NRC Natural Resources Canada	**Decupled clock model**
CNES Centre Nationals d’Etude Spatiales	**Integer recovery clock model**
GFZ Deutsches GeoforschungsZentrum	**Fractional cycle- / Uncalibrated hardware delay model**
SOPAC Schripps Orbit and Permanent Array Center	**Fractional cycle- / Uncalibrated hardware delay model**

The decoupled clock model

Starting point :

The code and phase observations are not sufficiently synchronized, the timing of the observations must be considered separately.

$$P_{IF} = \rho + cdt^{r}_{P_{IF}} - cdt^{s}_{P_{IF}} + T + \varepsilon_{P_{IF}}$$
$$L_{IF} = \rho + cdt^{r}_{L_{IF}} - cdt^{s}_{L_{IF}} + T - \lambda_{IF}(17N_1 + 60N_{WL}) + \varepsilon_{L_{IF}}$$

$$MW = L_{WL} - P_{NL} = -\lambda_{WL} N_{WL} + (b^{r}_{MW} - b^{s}_{MW}) + \varepsilon_{MW}$$

Satellite decoupled clock parameters supplied to the users (estimated in network solution by agency):

$$cdt^{s}_{P_{IF}}, cdt^{s}_{L_{IF}}, b^{s}_{MW}$$

Receiver decoupled clock parameters (estimated PPP-AR solution by user):

$$cdt^{r}_{P_{IF}}, cdt^{r}_{L_{IF}}, b^{r}_{MW}$$

Decupled clock parameters

Parameters in a network solution with *m* receivers and *n* common satellites:

- Number of observations: *3mn*

- Number of unknowns: *4m + 3n + 2mn*
    - Troposphere: *m*
    - Receivers decoupled clocks: *3m*
    - Satellite decoupled clocks: *3n*
    - Phase ambiguities: *2mn*

The troposphere parameters will typically vary slowly over time while the ambiguity parameters will be constants (given no cycle slips). The decoupled clock parameters will have some sort of dynamical behavior.

The original decoupled clock model will however have a datum defect and be singular (as for the “naive” PPP-AR approach).

The decoupled clock model
- handling the datum defect in the network solution

Constrain some of the parameters ("define datum").

One receiver is defined to be reference receiver in the network

Receiver	Defined datum	Constrain
Ref. receiver	Clock datum for network	$cdt^r_{P_{IF}} = cdt^r_{L_{IF}} = b^r_{MW} = 0$
	Ambiguity datum ref.receiver	N_1 and N_{WL} for all satellites set to arbitrary integers
Non ref. receivers	Ambiguity datum for non ref. receivers	Chose an arbitrary ref. satellite N_1 and N_{WL} for ref. satellite set to arbitrary integers

The remaining unknown parameters now become estimable. N_1 and N_{WL} ambiguities can be estimated as integer-values.

The decoupled clock model

- handling the datum defect in the PPP-AR user solution

Needs access to precise satellite coordinates and satellite decoupled clocks (downloaded from post-processing or streamed in real-time).

$$cdt^{s}_{P_{IF}}, cdt^{s}_{L_{IF}}, b^{s}_{MW}$$

Receiver	Defined datum	Constrain
PPP-AR user	Clock datum for network	Retained from reference network.
	Ambiguity datum ref.receiver	Chose an arbitrary ref. satellite N_1 and N_{WL} for ref. satellite set to arbitrary integers

The remaining unknown parameters now become estimable. N_1 and N_{WL} ambiguities can be estimated as integer-values.

Constrain / datum definition

- interpretation

- Integer differences in ambiguity datum, will be absorbed in the estimated receiver decoupled clock parameters.

- PPP-AR is a relative technique

- The single receiver PPP-AR ambiguities are double differenced ambiguities and thus not undifferenced ambiguities

- The PPP-AR equations are a rewriting of the classical double difference equations.

See e.g. Teunissen&Khodabandeh, 2015, Journal of Geodesy 89:217-240.

What we learned at school……….

The following quote from most classic text-books on GNSS is therefore still valid (??) :

> "*It is only possible to isolate the integer values for phase ambiguities when using double-differences.*"

Interoperability

For the different PPP-AR techniques, the equations are essentially the same for the user.
Numerous authors have also shown that the different techniques contain the same information – as they are all rewritings of the double-difference approach.

There is therefore a one-to-one transformation relationship between the different set of PPP-AR parameters.

E.g. when transforming from the "Fractional cycle- / Uncalibrated hardware delay" parameters to the "Decoupled clock" parameters

$$\begin{bmatrix} dt^s_{P_{IF}} \\ dt^s_{L_{IF}} \\ b^s_{WL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{\lambda_{NL}} & \frac{1}{\lambda_{NL}} & -\frac{\lambda_1}{\lambda_2-\lambda_1} \\ -\frac{1}{\lambda_{NL}} & \frac{1}{\lambda_{NL}} & -\frac{\lambda_2}{\lambda_2-\lambda_1} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & \frac{\lambda_1}{\lambda_2-\lambda_1} \\ 0 & -1 & \frac{\lambda_2}{\lambda_2-\lambda_1} \end{bmatrix} \begin{bmatrix} dt^s_{P_{IF}} \\ a^s_1 \\ a^s_{WL} \end{bmatrix} + \begin{bmatrix} 0 \\ z_1 \\ z_{WL} \end{bmatrix}$$

Interoperability

Transformation between sets of PPP-AR parameters allows interoperability

- **The PPP-AR user**
  - transform parameters to comply with his software implementation (e.g. Decoupled Clocks-, Integer Recovery Clock- or Fractional cycle- / Uncalibrated hardware delay model).

- **The network server**
  - Transform and make available parameters in several formats
  - Make available parameters in a standardized format
    - Initiative by RTCM Special Committee 104 SSR working group, State Space Representation

Proposed RTCM SSR model

Based on the idea that the phase bias is inherent to each frequency. One common clock and one phase bias per phase observable is identified and broadcast.

- State Space Representation
- Allows multiple frequencies and signals

Parameter	RTCM SSR message	Quantity
GPS/GLONASS orbits/clocks	1060/1066	D, dt_P
GPS code biases	1059/1065	b_P
GPS phase biases	1265	b_L

The proposed RTCM format has been implemented in the CNES real-time analysis center software since 15.09.2015 and is available on the IGS CLK93 real-time data stream.

Interoperability – does it work ?

Recent research on PPP-AR interoperability using transformed parameters:

- Seepersad&Bisnath, 2016, Clarifying the ambiguities, GPS World, http://gpsworld.com/clarifying-the-ambiguities/

PPP-AR user software based on the Decoupled Clock model

Processing with

- Decoupled Clock parameters, from Natural Resources Canada
- Transformed Integer Recovery clock parameters, from CNES
- Transformed Uncalibrated Phase Bias parameters from SOPAC

Interoperability – does it work ?

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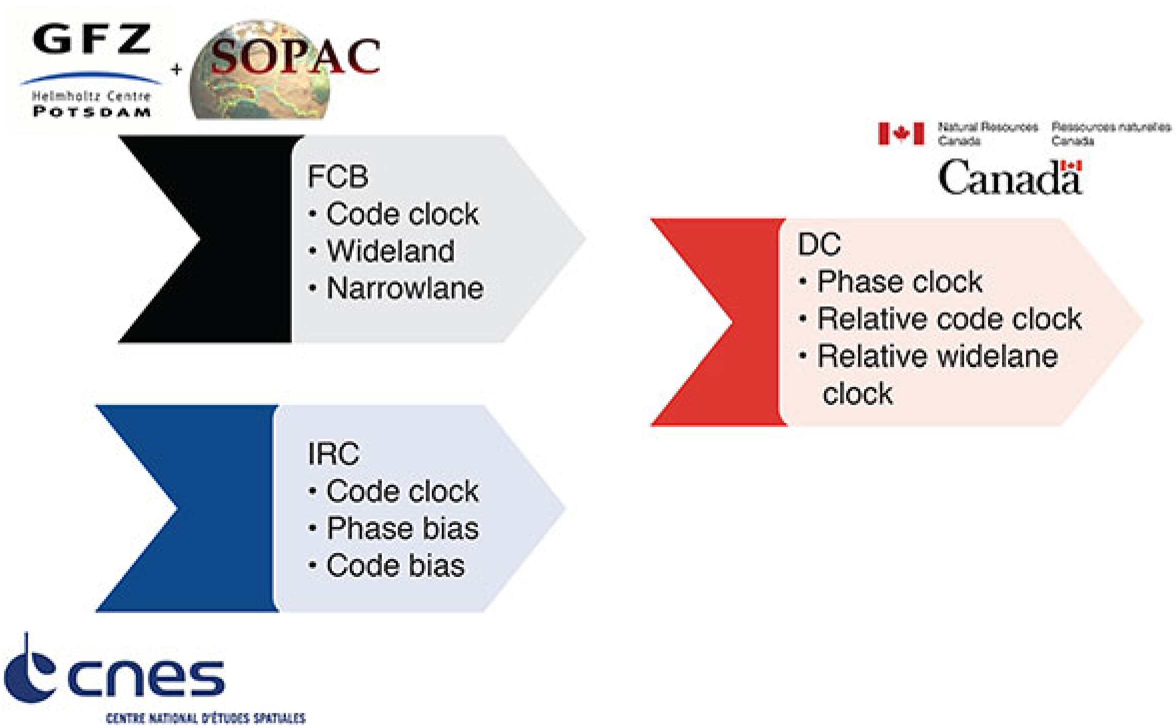

Interoperability – does it work ?

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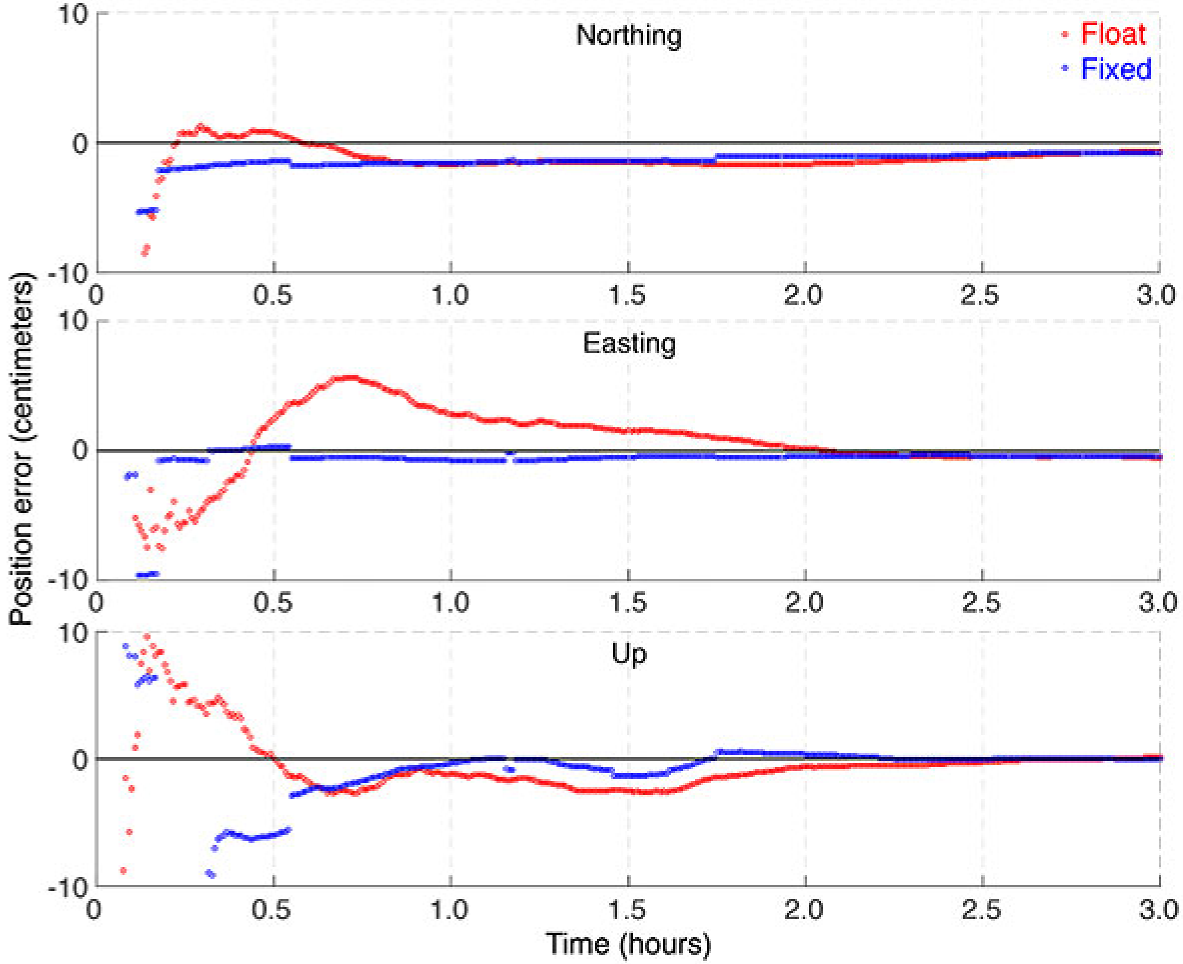
PPP processing with original Decoupled Clock Parameters

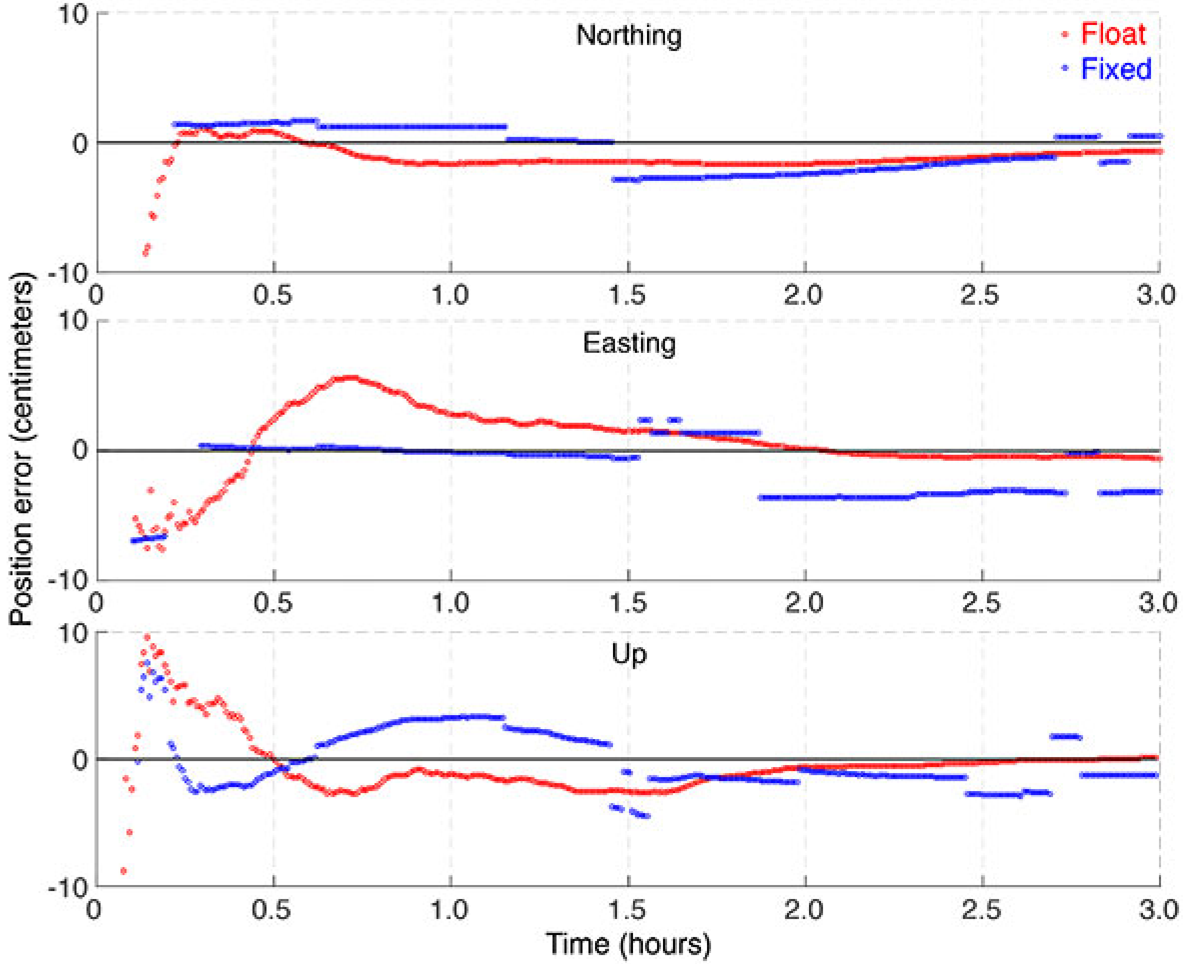


Interoperability – does it work ?

Recent research on PPP-AR interoperability using transformed parameters:

- Seepersad&Bisnath, 2016, Clarifying the ambiguities, GPS World, http://gpsworld.com/clarifying-the-ambiguities/

PPP processing with transformed Integer Recovery Clock parameters



Interoperability – does it work ?

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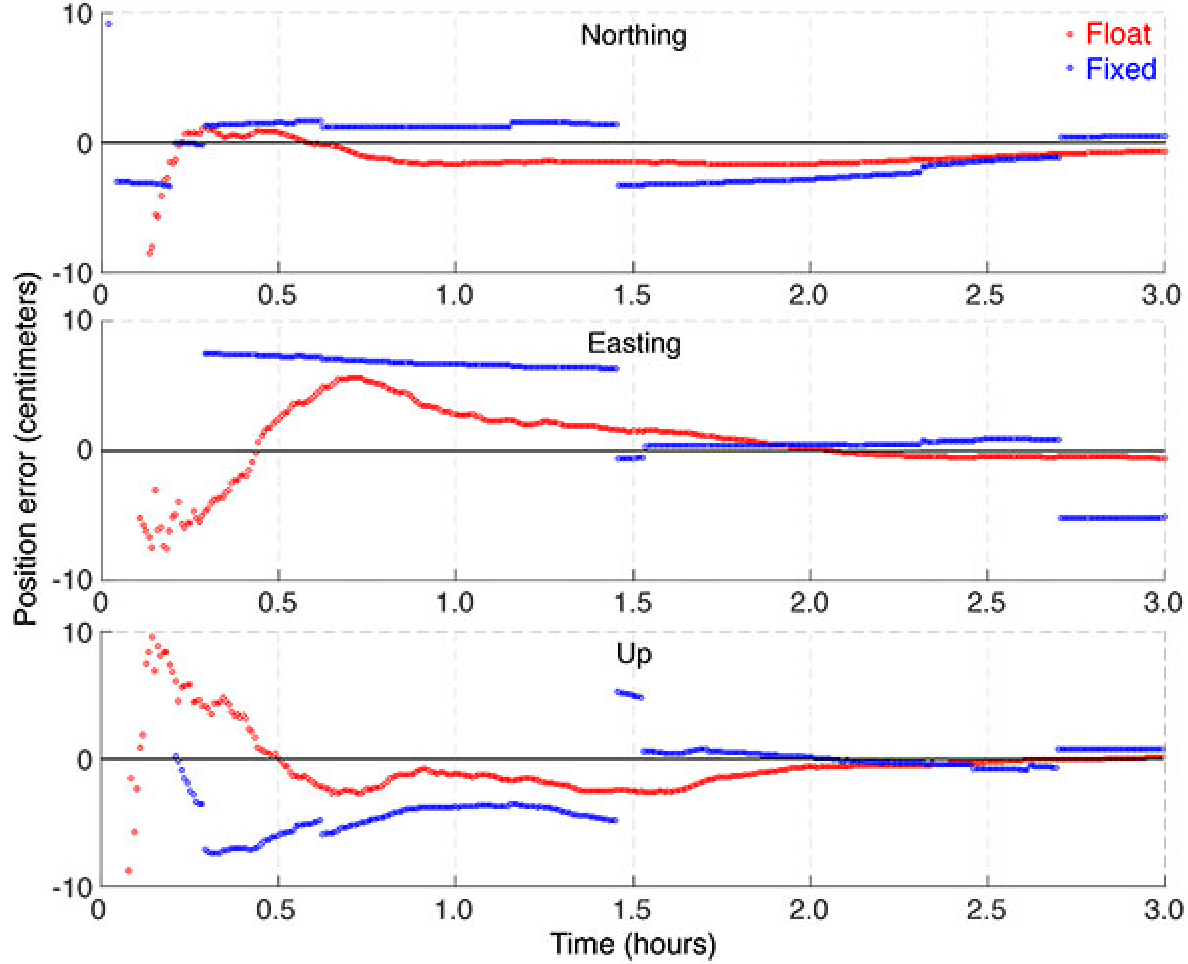


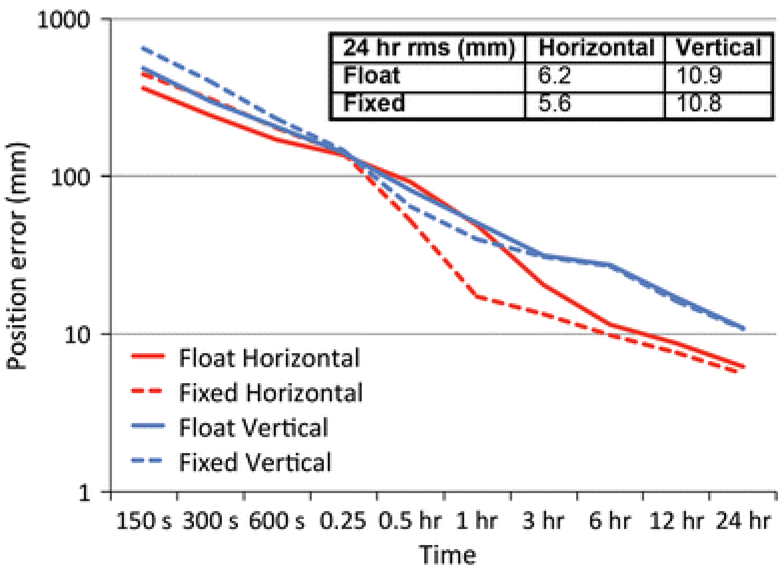
PPP processing with transformed Fractional Cycle Bias parameters

Interoperability – possible problems

- Unclear definitions and conventions
- Modelling and processing not standardized at service provider and PPP-AR user

The long convergence time for PPP using dual frequency GPS observations

From: Choy, Bisnath, Rizos, (2016), Uncovering common misconceptions in GNSS Precise Point Positioning, GPS Solution-”online first”.



Possible measures:

- Ionospheric models
- Triple frequency GNSS
- Multi-system GNSS
- Calibrating at known points
- Modelling receiver clock

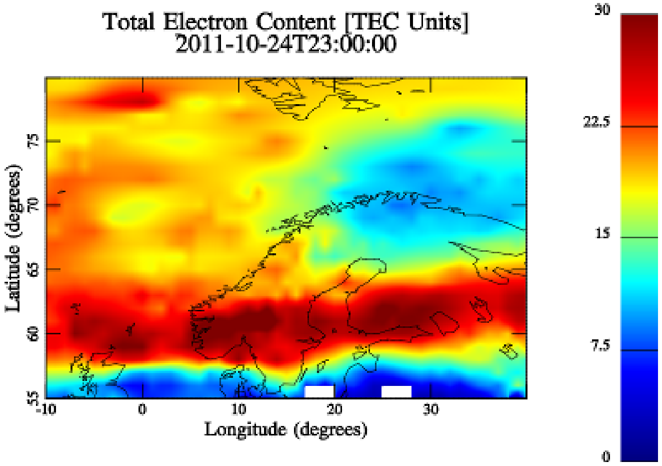
Enhancing PPP-AR
- ionospheric modelling / models

Model and estimate the ionosphere in the PPP solution

- Constrain and extrapolate over gaps
- Rapid re-convergence
- Sensitive to
  - Rapid ionospheric fluctuations
  - Length of gap

External ionospheric models, e.g.

- Klobuchar model
- Global Ionospheric Maps (GIMSs) from e.g. IGS
  - accuracy 2-8 TECU / 0.3-1.3 meter
- Regional/local ionospheric models, e.g. from
- Norwegian Mapping Authority
  - accuracy ?? TECU



Enhancing PPP-AR

- ionospheric modelling / models

- “difficult” when using ionospheric-free linear combinations
  - argument for the “raw method”

- PPP will then depend on a regional/local network of reference stations, i.e. not any more a global “absolute” technique.

- …..
- …..

Enhancing PPP-AR

- triple frequency GNSS

- Enhances traditional PPP (e.g Heng and Bock 2013, Tegedor and Ovstedal 2013)
- Enhances PPP-AR (e.g. Lauricheese 2015)
- Aspects
  - Phase anomaly L_5, e.g. SVN62/PRN25
  - Inter-frequency biases (handled in PPP-AR)
    - but differences between brands of receivers
  - No L5 antenna calibration available
  - When using linear combinations of observables
    - Which combinations?
    - The combinations introduces correlations between observations

Enhancing PPP

- multi system GNSS

Adds observational strength relative to GPS only (Li et al. 2015, Tegedor et al. 2014)

Intersystem biases
- differences between brands of receivers

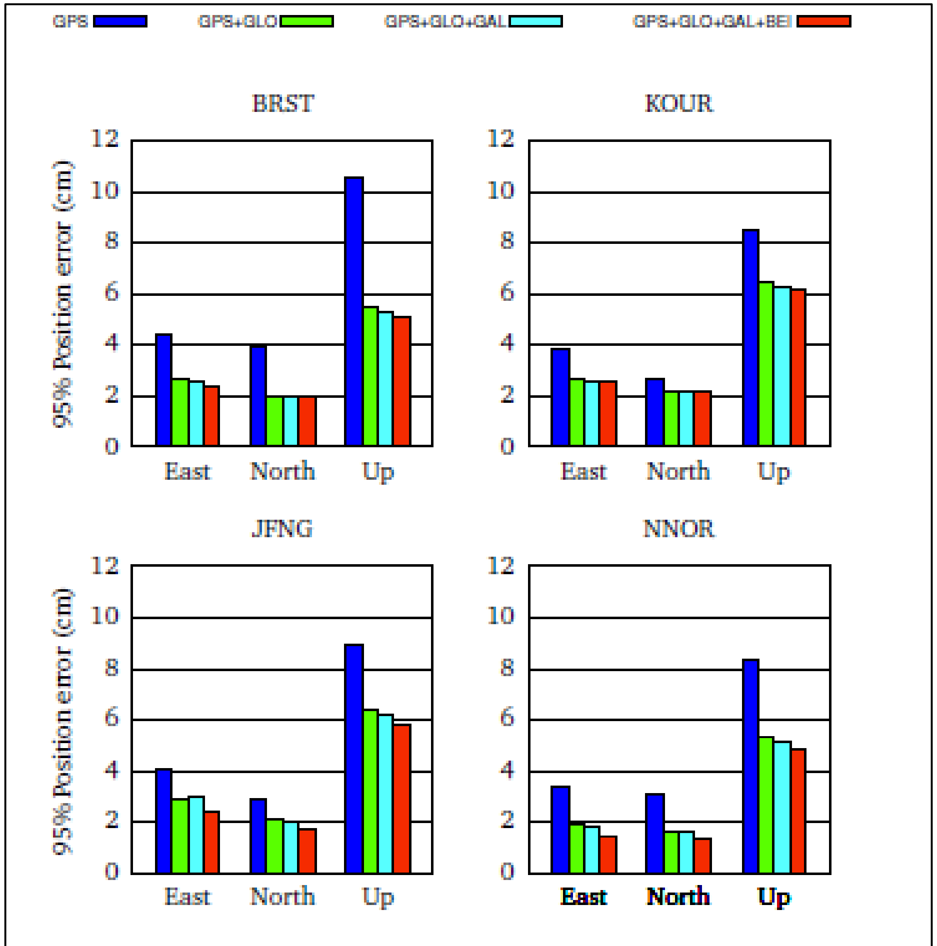
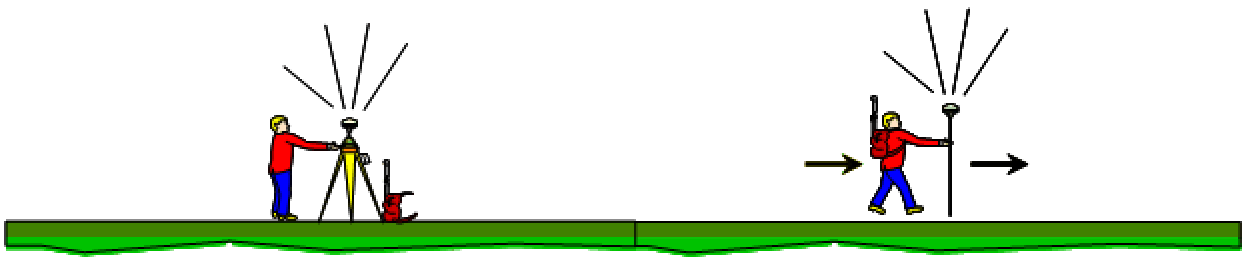


Fig. 6.6: Multi-constellation kinematic positioning statistics for several stations in MGEX.

From Tegedor et al. 2014

Enhancing PPP-AR

- calibrating at known point



Like in the old days of “kinematic GPS” : Initial initialization

- not always possible……………

Enhancing PPP-AR

- modelling receiver clocks ??

- Require high-stability receiver clocks
  - Might become available for reasonable price ?
- Stochastic modelling and constraint improves estimated parameters

  E.g. Wang&Rothacher (2013)

Enhancing PPP-AR

- ???????

Suggestions ??

Linear combinations or raw observations ?

-” theoretically equivalent but has practical aspects”

- Advantages
  - Ionospheric effects eliminated
  - Make signal with longer wavelength, enhances ambiguity resolution
  - Many clever techniques for e.g. ambiguity resolution
  - Proven to work
- Drawbacks
  - Not possible to constrain ionosphere in ambiguity resolution
    - Probably main reason for the long convergence time in PPP-AR
  - Techniques based on the Melbourne-Wübbena method is very dependent on quality of code observations – difficult in real-world scenario.
  - Difficult in a multi-frequency system
    - Which combinations to choose?
    - Introduces correlations between observations (when more than to frequencies)
    - More difficult convention for state space representation of corrections
  - Increased noise

The raw method

- e.g. triple-frequency GPS

$$L_1 = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_i^2} I_1 + b_{L_1}^r - b_{L_1}^s - \lambda_1 N_1 + \varepsilon_{L_1}$$

$$L_2 = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_2^2} I_1 + b_{L_2}^r - b_{L_2}^s - \lambda_2 N_2 + \varepsilon_{L_2}$$

$$L_5 = \rho + c\,(dt^r - dt^s) + T - \frac{f_1^2}{f_5^2} I_1 + b_{L_5}^r - b_{L_5}^s - \lambda_5 N_5 + \varepsilon_{L_5}$$

$$P_1 = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_1^2} I_1 + b_{P_1}^r - b_{P_1}^s + \varepsilon_{P_1}$$

$$P_2 = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_2^2} I_1 + b_{P_2}^r - b_{P_2}^s + \varepsilon_{P_2}$$

$$C_5 = \rho + c\,(dt^r - dt^s) + T + \frac{f_1^2}{f_5^2} I_1 + b_{C_5}^r - b_{C_5}^s + \varepsilon_{P_5}$$

One extra ionospheric unknown for each epoch

- Can be used to constrain ionosphere ➔ geometrical strength
- Can be pre-eliminated (if number of unknowns is of concern……….)
- Must introduce constraint between adjacent I_1 estimates to avid singularity (and improve geometrical strength)

The end