

Investigations of the requirements for a 5 mm geoid model – a project status report

Lars E. Sjöberg¹, Jonas Ågren²

¹ Royal Institute of Technology (KTH), Stockholm, Sweden

² Lantmäteriet, Gävle, Sweden

Background

- Along with future improvements of GNSS height determination, GNSS users will ask for increasingly better geoid models. It is not unlikely that a geoid standard uncertainty of 5 mm will more or less be required in a couple of years.
- What is required to obtain this standard uncertainty for the geoid model? Is it even possible? What should be done now to reach the goal in the future? Should we start to collect more gravity? Heights? Methodology?
- To answer such questions, the project **Investigations of the requirements for a future 5 mm (quasi)geoid model** was started in 2011 within the Nordic Geodetic Commission (NKG).

Initial project members

- J Ågren (Sweden)
- M Bilker-Koivula (Finland)
- R Forsberg (Denmark)
- O Omang (Norway)
- L E Sjöberg (Sweden; chair)
- G Strykowski (Denmark)

Expected deliveries

- A clear definition of “a 5 mm geoid”
- A treatment of theoretical obstacles and limitations
- Recommendations for improvements in existing methods
- Recommendations on what data are required

So far, mainly 1.) and 4.) have been dealt with using Sweden as initial example.

What is a 5 mm geoid?

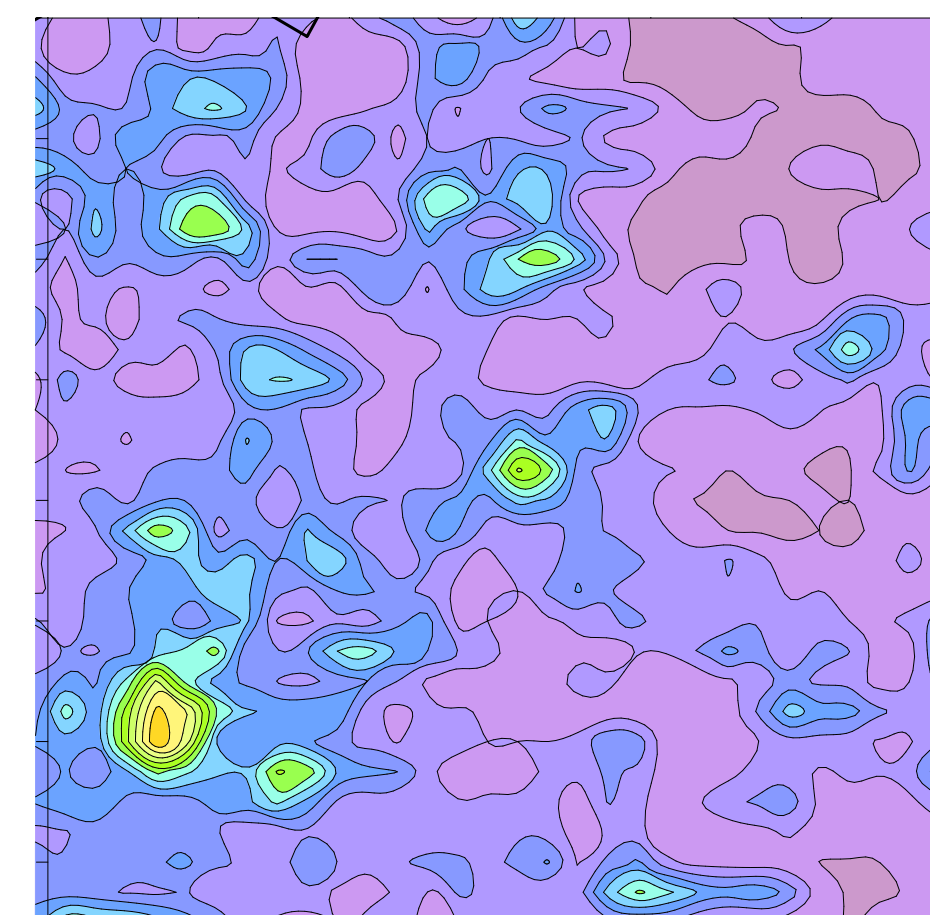
In the project we answer this question as follows:

- 5 mm is the overall relative accuracy for a regional gravimetric quasigeoid model.
- Since most Nordic and European height systems use normal heights, we limit the study to the quasigeoid.
- With “accuracy” we mean standard uncertainty.
- It is assumed that no corrector surface is computed and applied based on GNSS/levelling.

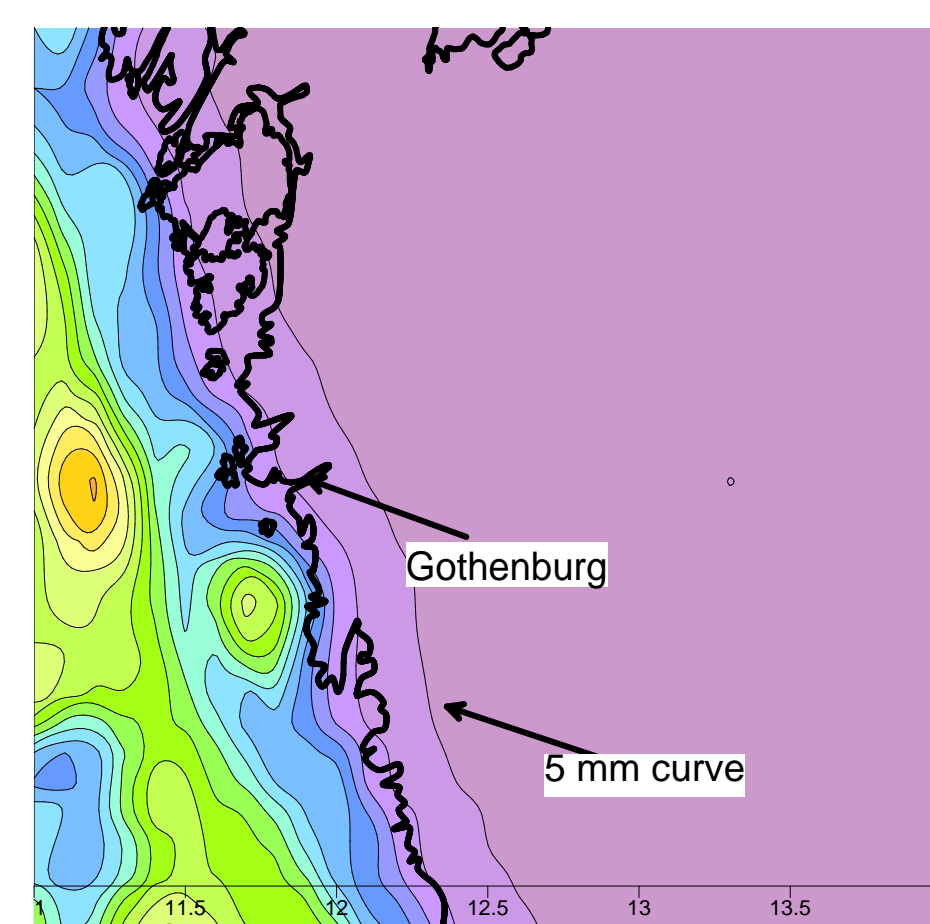
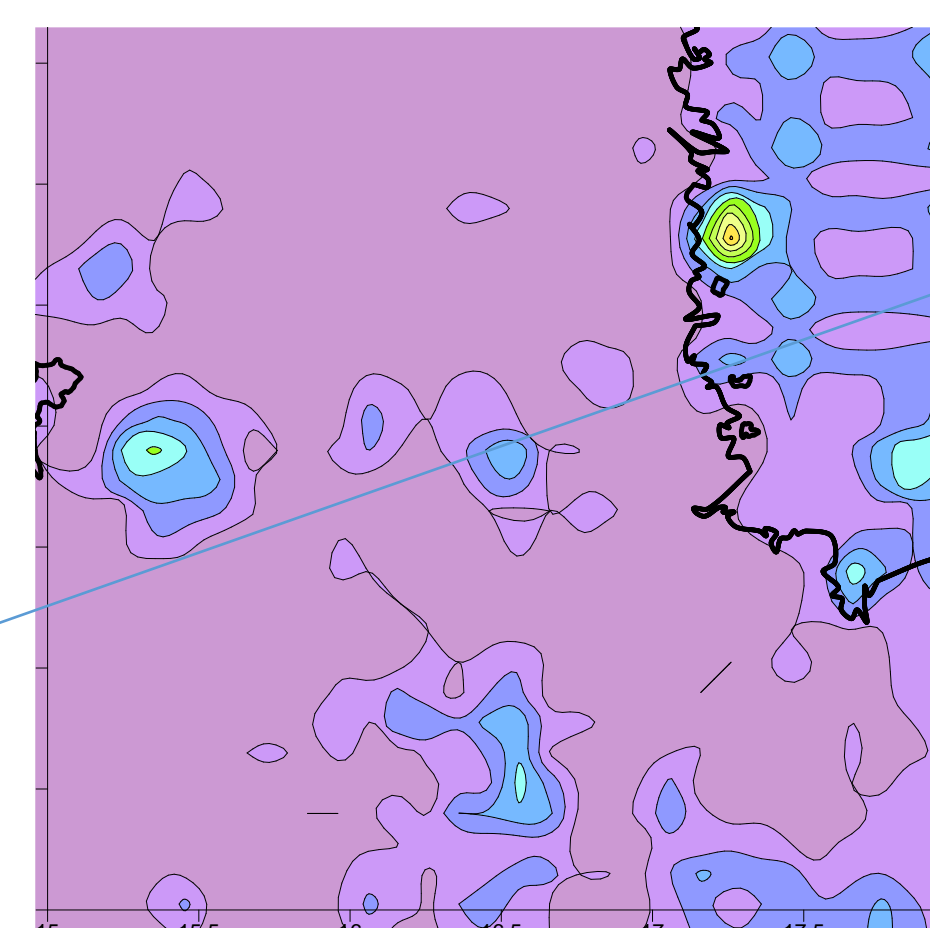
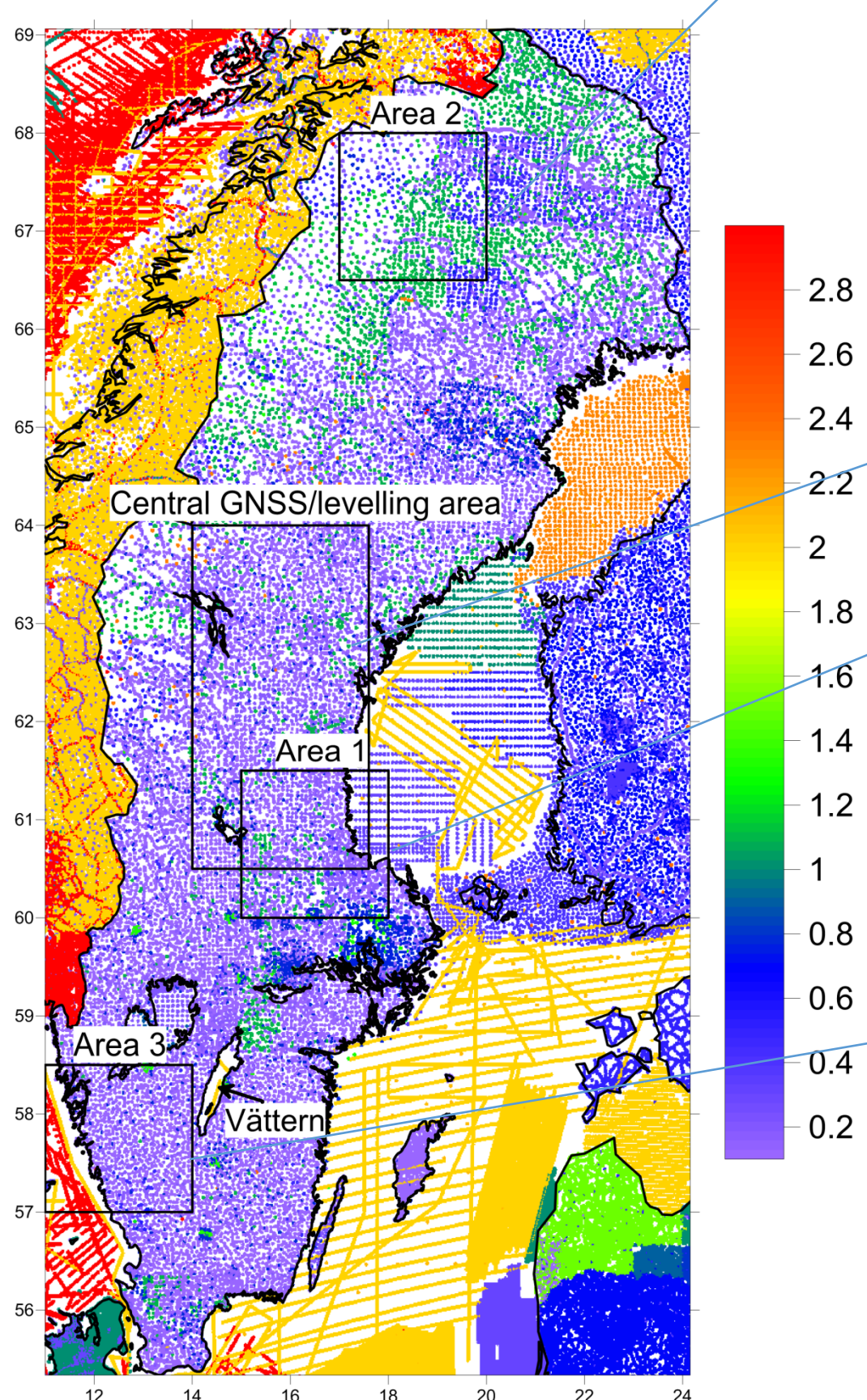
Example: Error propagation by least squares collocation over Sweden

- Empirical signal covariance functions were computed for five different test areas in Sweden. A similar analysis was made almost 30 years ago by Forsberg (1986) for the whole Nordic region. Here improved gravity data, a modern EGM (GOCO03S) and a dense 3'' x 3'' DEM are used.
- The error covariance matrix was computed as the sum of
 - an uncorrelated (diagonal) noise part based on the individual standard errors below
 - a correlated noise part following the reciprocal distance model of Moritz (1980). The correlation length varied between 0.25 and 1 degrees.
- The signal and correlated error covariance functions was assumed to be homogeneous and isotropic.

Propagated height anomaly standard errors (m)



Standard errors assumed for the gravity anomalies (from Lantmäteriet's database in Sweden) (mGal)

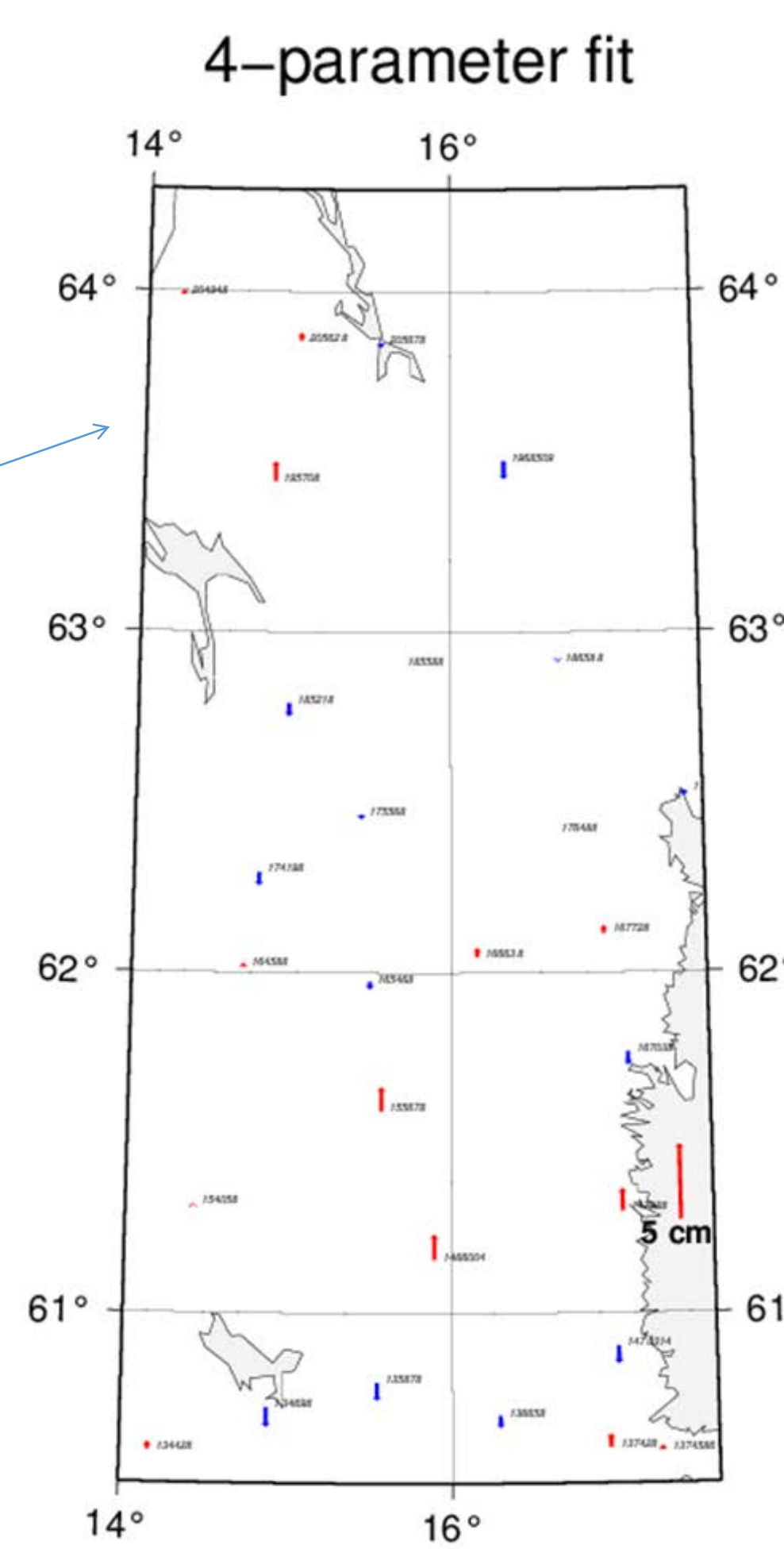


Example : Evaluation of a gravimetric model using Swedish GNSS/levelling

- The gravimetric quasigeoid model was computed by the Least Squares Modification of Stokes' formula with Additive corrections (LSMSA or KTH-method; see Sjöberg 1991, 2003,...) applied in more or less the same way as in Ågren et al. (2009).
 - Least squares (stochastic) kernel modification (Sjöberg 1991).
 - Additive corrections for downward (analytical) continuation, atmosphere and the ellipsoidal correction.
 - Surface gravity anomalies gridded using a remove-interpolate-restore technique. EGM and RTM effects removed/restored. RTM effect computed using TC (Forsberg).
- The following observations were used:
 - Gravity observations from the updated Swedish database (2012) and from the NKG gravity database (2004 version)
 - The GOCO03S EGM up to the degree 200 (Mayer-Gürr et al. 2012)
 - DEM with 3''x3'' resolution.

Statistics for the GNSS/levelling residuals without a fit and after 1- and 4-parameter fits. Unit: m.

Area	Fit	# pts	Min	Max	Mean	StdDev	RMS
Whole Sweden	No fit	197	-0.699	-0.564	-0.641	0.021	0.641
	1-par.	197	-0.058	0.076	0.000	0.021	0.021
	4-par.	197	-0.055	0.083	0.000	0.020	0.020
Central area	No fit	28	-0.651	-0.608	-0.632	0.012	0.632
	1-par.	28	-0.018	0.024	0.000	0.012	0.012
	4-par.	28	-0.013	0.017	0.000	0.009	0.009



$$\sigma_{\text{quasigeoid}} = \sqrt{RMS_{4\text{-par. fit}}^2 - \sigma_{\text{GNSS}}^2 - \sigma_{\text{levelling}}^2} = \sqrt{RMS_{4\text{-par. fit}}^2 - \sigma_{\text{GNSS/lev}}^2}$$

Over long distances (whole of Sweden):

- Standard error for GNSS ≈ 10 mm
- Standard error for levelling $\approx 10\text{-}15$ mm
- \Rightarrow Standard error for the gravimetric quasigeoid model $\approx 10\text{-}15$ mm

In the central area (smooth part with good gravity):

- Standard error for GNSS ≈ 5 mm
- Standard error for levelling ≈ 5 mm
- \Rightarrow Standard error for the gravimetric quasigeoid model ≈ 5 mm

Summary

General questions about the project details were raised in two circular letters. In two specific studies it was shown that

- the average 5 km resolution and quality of the gravity data in Sweden are sufficient for the task provided that the data are updated for systematic errors and data gaps. Gravity data in the surrounding areas also need to be improved, e.g. in the Baltic Sea.
- systematic errors in DEMs are not a problem over Sweden, where a high resolution DEM with high quality is available, at least not as long as the same DEM heights are used both in the remove and restore phases of the topographic corrections.
- some methodological improvements may be needed.

Recommendations

- The above and further studies should be extended to the rest of member countries to reach a conclusive goal of the project.
- The need for methodological improvements should also be further investigated. Various limitations of the error propagations should also be dealt with.
- However, it is questionable whether this study is suitable as a NKG project. The more theoretical and methodological questions are very difficult and time consuming. External funding would be required for academic researchers to work deeply on this.
- One alternative would be to continue as a PhD project, but this would also require funding.

Some conclusions from the error propagations

It is possible to compute a gravimetric quasigeoid model with 5 mm standard uncertainty over Sweden in case the following data requirements are fulfilled:

- The gravity anomaly resolution should be at least 5 km. (Has also been confirmed with 1 km gravity data in a few test areas in Sweden, but these results are not presented here.)
- There should be no gravity data gaps in the “5 mm quasigeoid area” or in its vicinity.
- The standard error of the uncorrelated (white) noise should be lower than approximately 0.5 mGal for the gravity anomaly.
- The systematic errors should be as low as possible. For the assumed reciprocal distance function with 0.25 degrees correlation length, the standard error has to be lower than about 0.2 mGal. In reality the covariance function is more complex and not precisely known. To be on the safe side, the correlated standard error should be below 0.1 mGal. To achieve this we need to do everything we can to reduce all kinds of systematic effects

