



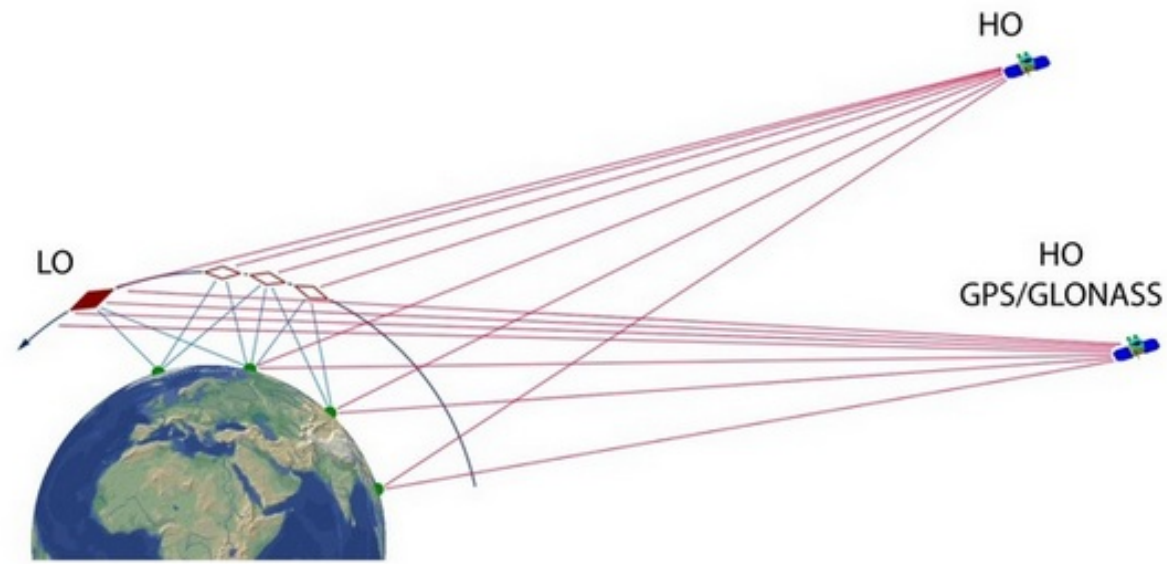
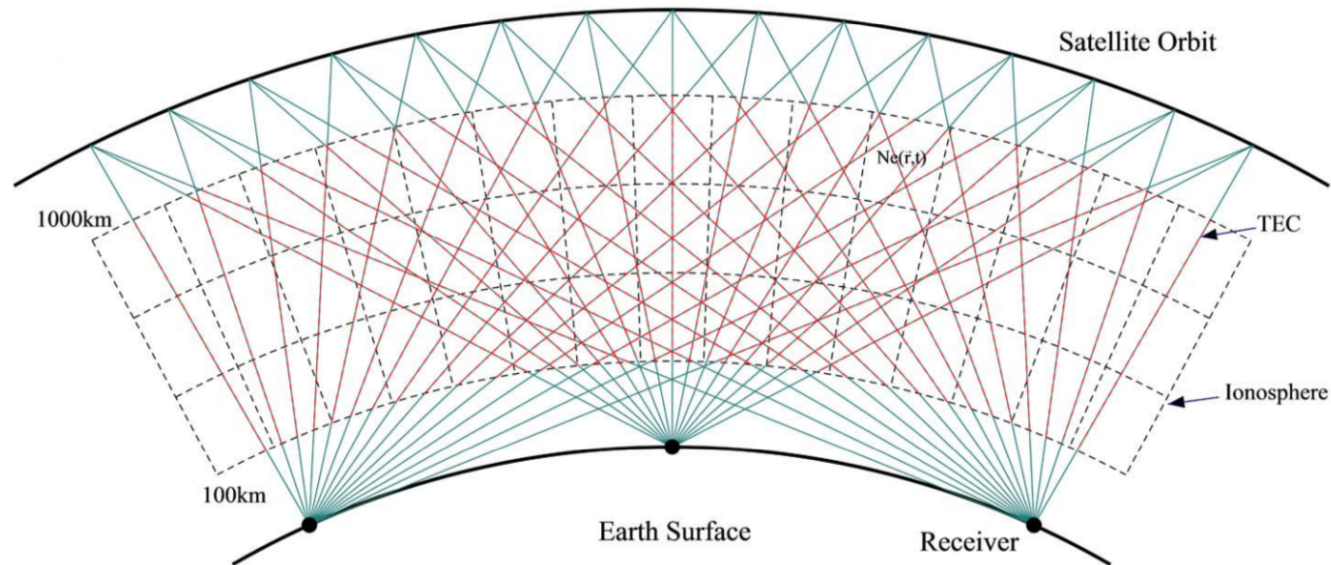
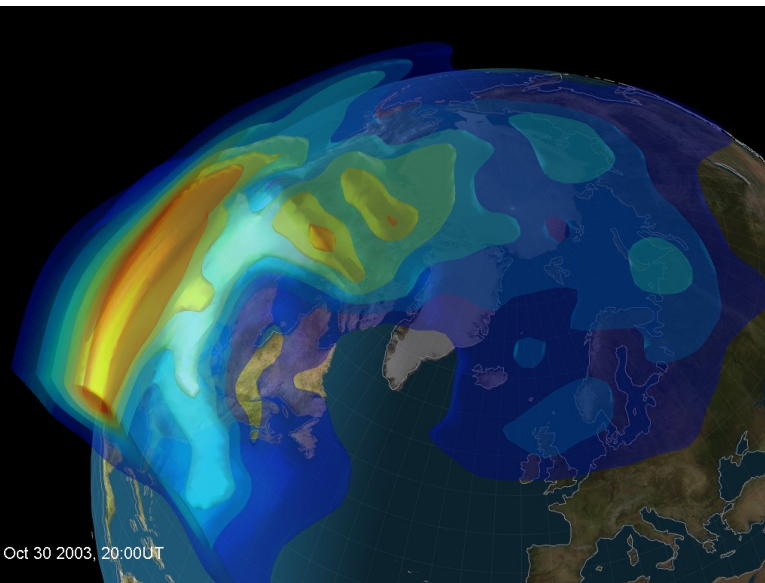
Kartverket

# Ionosphere

Part 3 – Modelling / Estimation

Knut Stanley Jacobsen

# Are we able to model the 3D ionosphere?

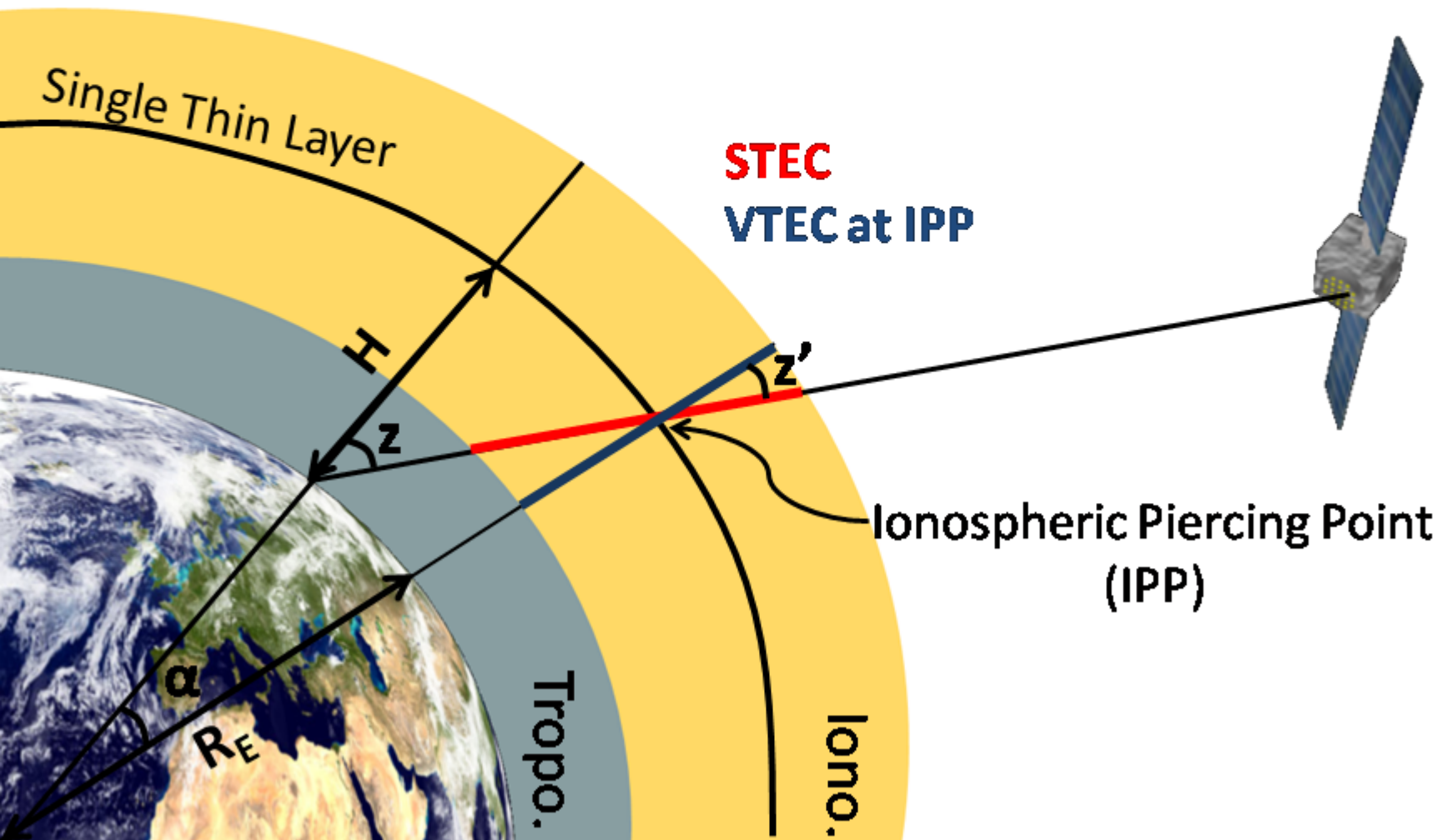


(Also, do we need to?)

# Single Layer Model

A 2D approximation of the 3D ionosphere

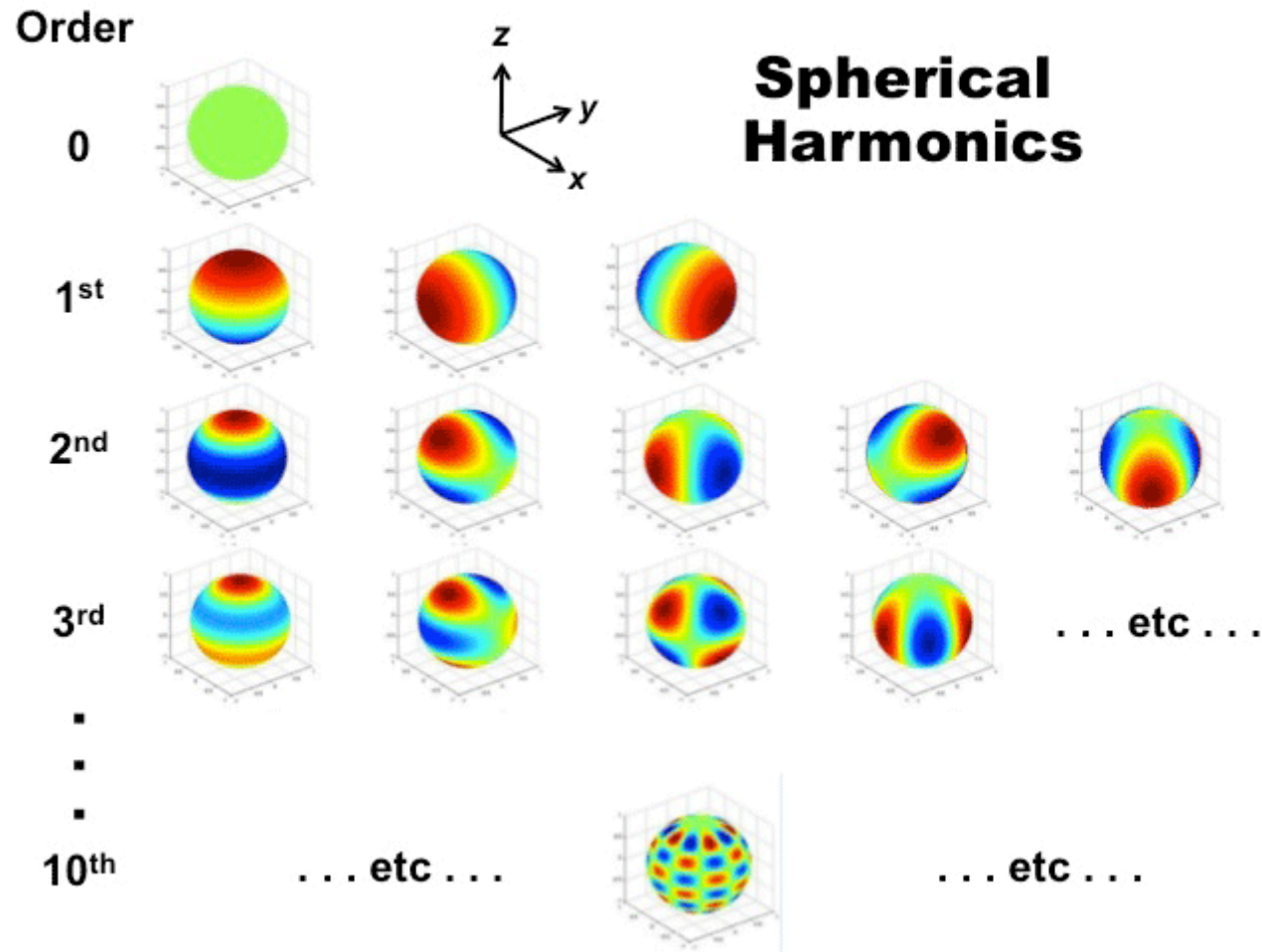
$$VTEC = STEC \sqrt{1 - \left( \frac{R_E \cos(z)}{R_E + H} \right)^2}$$



# Parameterization

Two basic ways of specifying a scalar field:

- Coefficients for a set of functions
  - Values at a set of Grid points

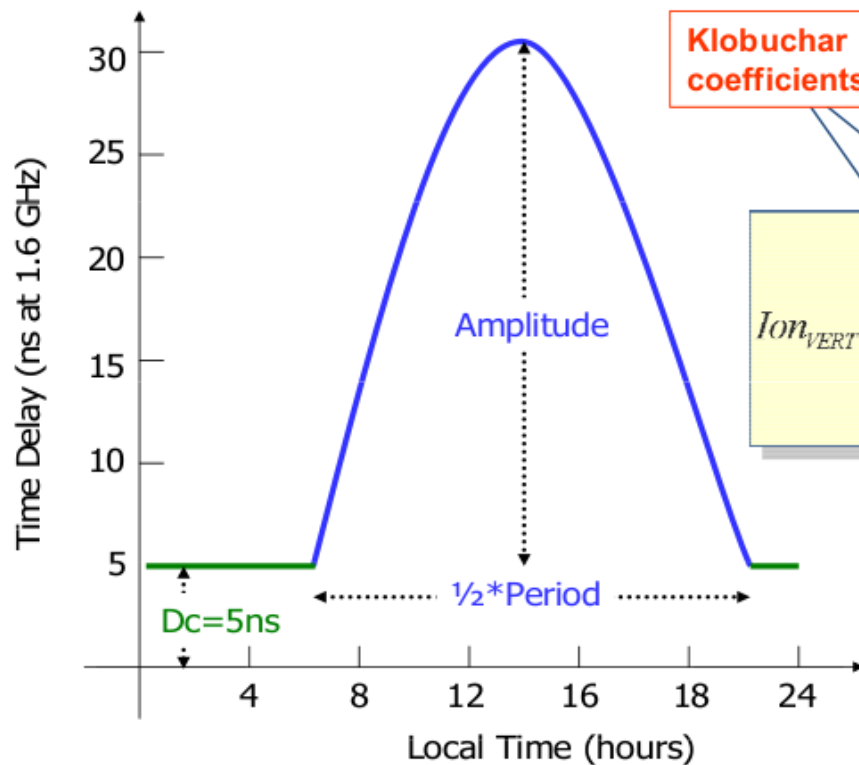


# Parameterization

Two basic ways of specifying a scalar field:

- Coefficients for a set of functions
- Values at a set of Grid points

## Klobuchar model



Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A \cos \left[ \frac{2\pi(t - \Phi)}{P} \right] & (\text{day}) \\ DC & ; \text{ if } \left[ \frac{2\pi(t - \Phi)}{P} \right] > \frac{\pi}{2} \quad (\text{night}) \end{cases}$$

Being:

$$A = \sum_{n=0}^3 \alpha_n \phi^n ; \quad P = \sum_{n=0}^3 \beta_n \phi^n$$

$\phi$  = Geomagnetic Latitude

Where:

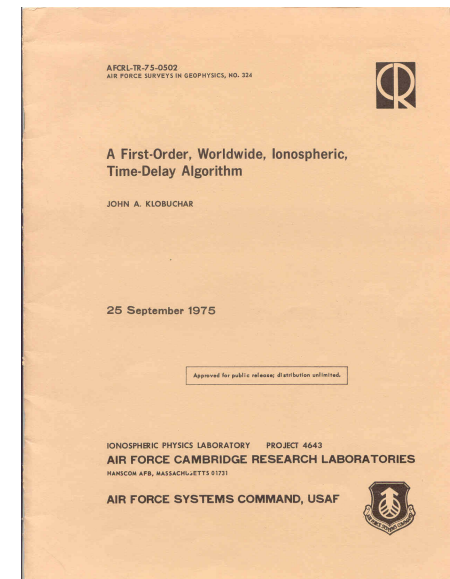
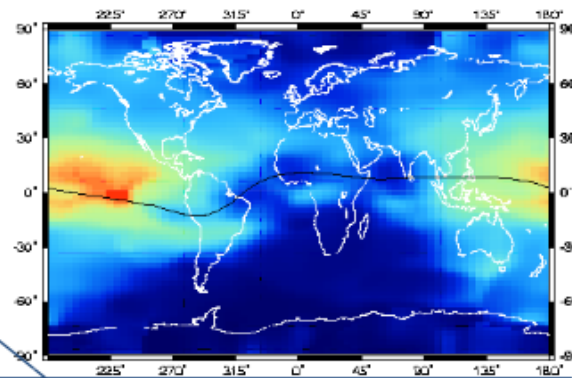
DC= 5ns

$\Phi$ = 14 (ctt. phase offset)

**t = Local Time**

$$Ion_{SLANT} = Ion_{VERT} m(elev)$$

$$m(elev) = 1 + 16(0.53 - elev/\pi)^3$$



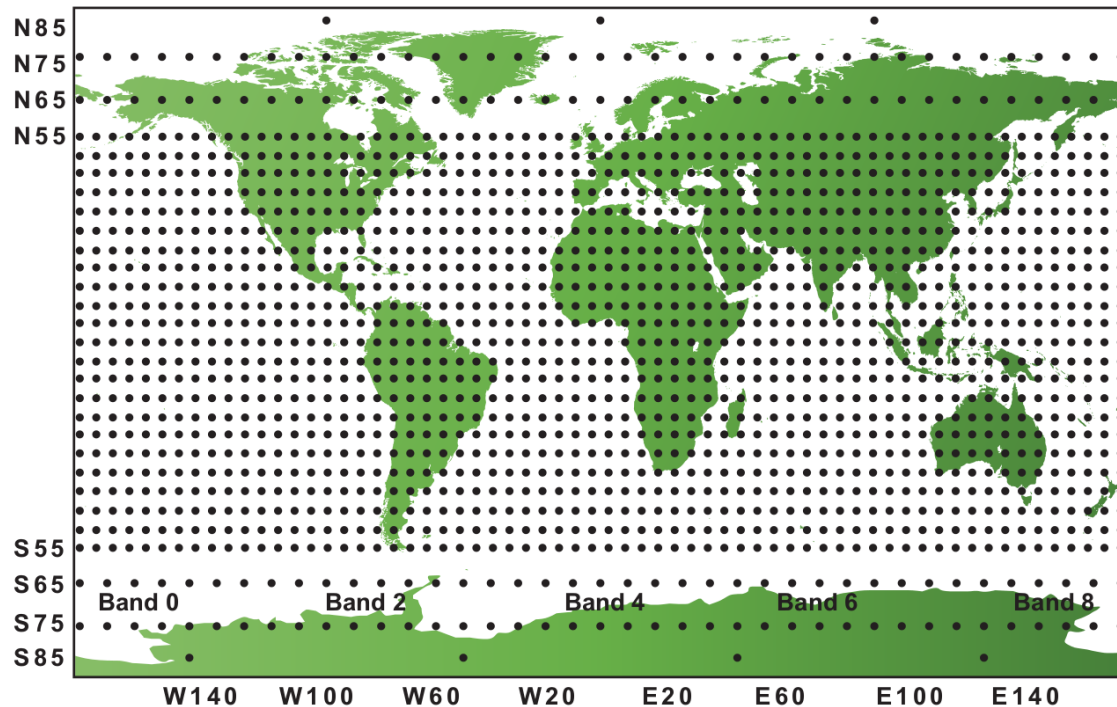
Klobuchar, J., 1987. *Ionospheric Time-Delay Algorithms for Single-Frequency GPS Users*. IEEE Transactions on Aerospace and Electronic Systems (3), pp. 325-331.



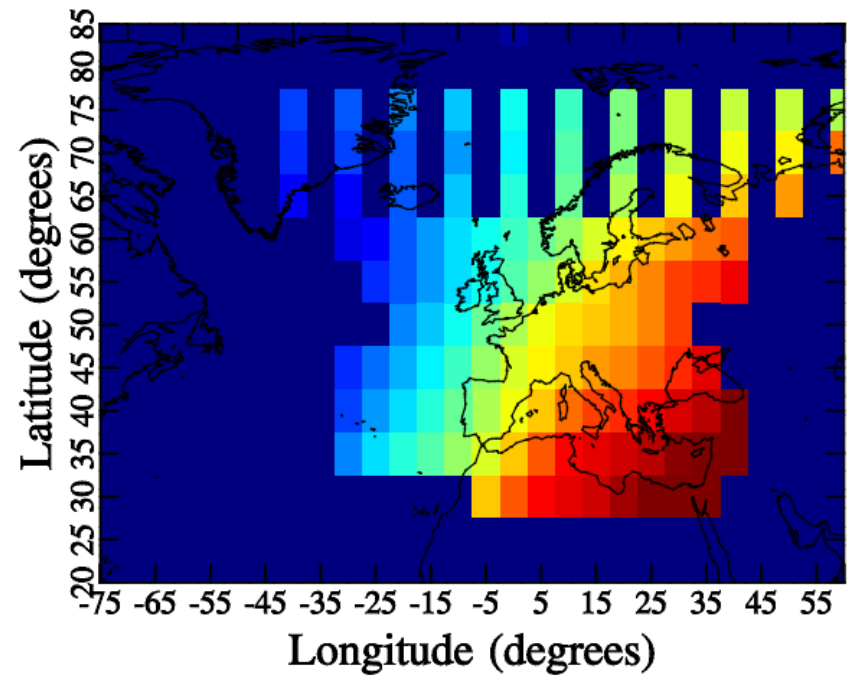
# Parameterization

Two basic ways of specifying a scalar field:

- Coefficients for a set of functions
- Values at a set of Grid points

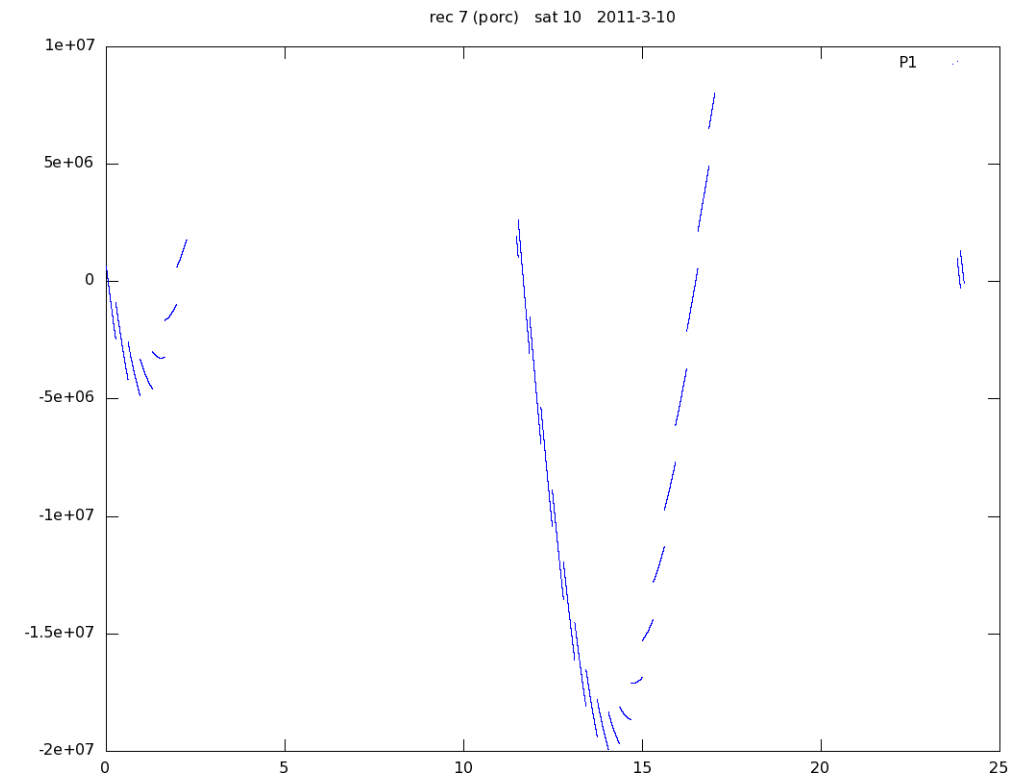
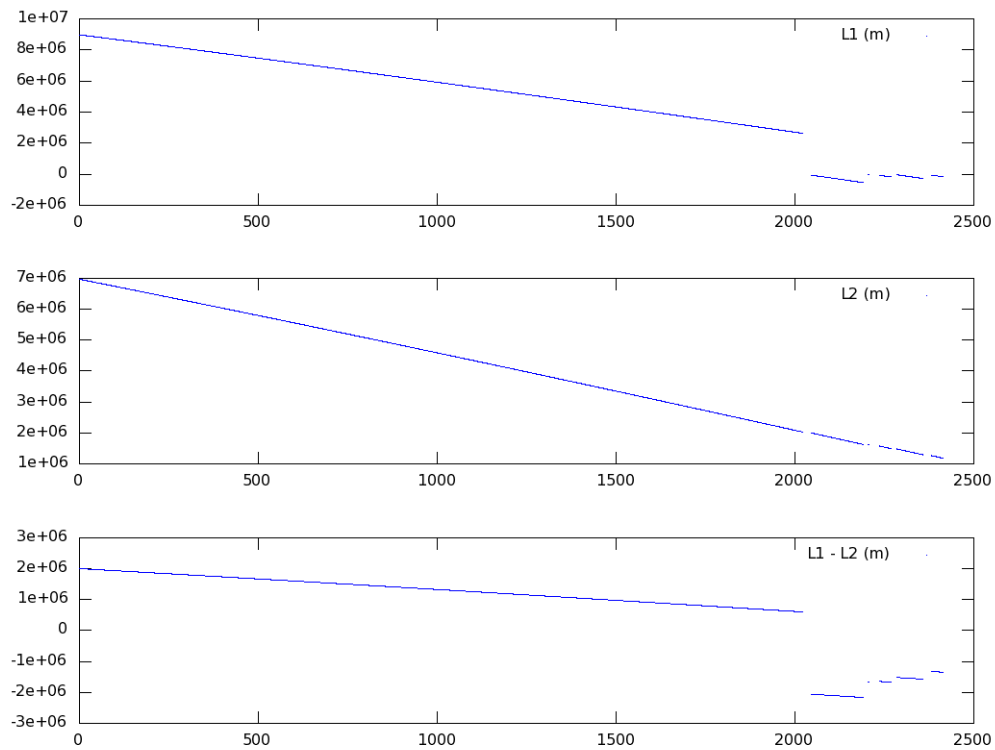


Total Electron Content [Meter]  
2014-02-25 08:45 UTC



# GNSS-specific pre-processing

## Cycle slips & Receiver clock jumps



# Extracting the ionosphere from the measurements

Not frequency dependent

$$P_{r,i}^s = \rho_r^s + \delta \rho_r^s + c(\delta t_r - \delta t^s) + T + \frac{1}{f_i^2} I + b_{P,i}^s + b_{r,P,i} + \epsilon_{P_i}$$

$$L_{r,i}^s = \rho_r^s + \delta \rho_r^s + c(\delta t_r - \delta t^s) + T - \frac{1}{f_i^2} I + b_{L,i}^s + b_{r,L,i} - \lambda_i N_{r,i}^s + \epsilon_{L_i}$$

Geometry-free Linear Combination:

$$P_{GF} = P_{r,1}^s - P_{r,2}^s = \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) I + DCB^s + DCB_r + \epsilon_{P_{GF}}$$

$$L_{GF} = L_{r,1}^s - L_{r,2}^s = - \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) I + b_{L,1}^s + b_{r,L,1} - b_{L,2}^s - b_{r,L,2} - \lambda_1 N_{r,1}^s + \lambda_2 N_{r,2}^s + \epsilon_{L_{GF}}$$



# Extracting the ionosphere from the measurements

Satellite hardware effect.

Can get data from external analysis center or estimate ourself.

Varies very slowly.

Receiver hardware effect.

Often need to estimate ourself.

Varies slowly.

Large noise.

$$P_{GF} = \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) I + DCB^s + DCB_r + \epsilon_{P_{GF}}$$

$$L_{GF} = - \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) I + \underbrace{b_{L,1}^s + b_{r,L,1} - b_{L,2}^s - b_{r,L,2}}_{\text{(Same comments as the code biases, but they generally have smaller values)}} - \lambda_1 N_{r,1}^s + \lambda_2 N_{r,2}^s + \epsilon_{L_{GF}}$$

This is the part that we want.

(Same comments as the code biases, but they generally have smaller values)

Ambiguities. Tricky to get right. Need to be estimated independently for each satellite link, and re-estimated if there is a cycle slip.

Small noise.

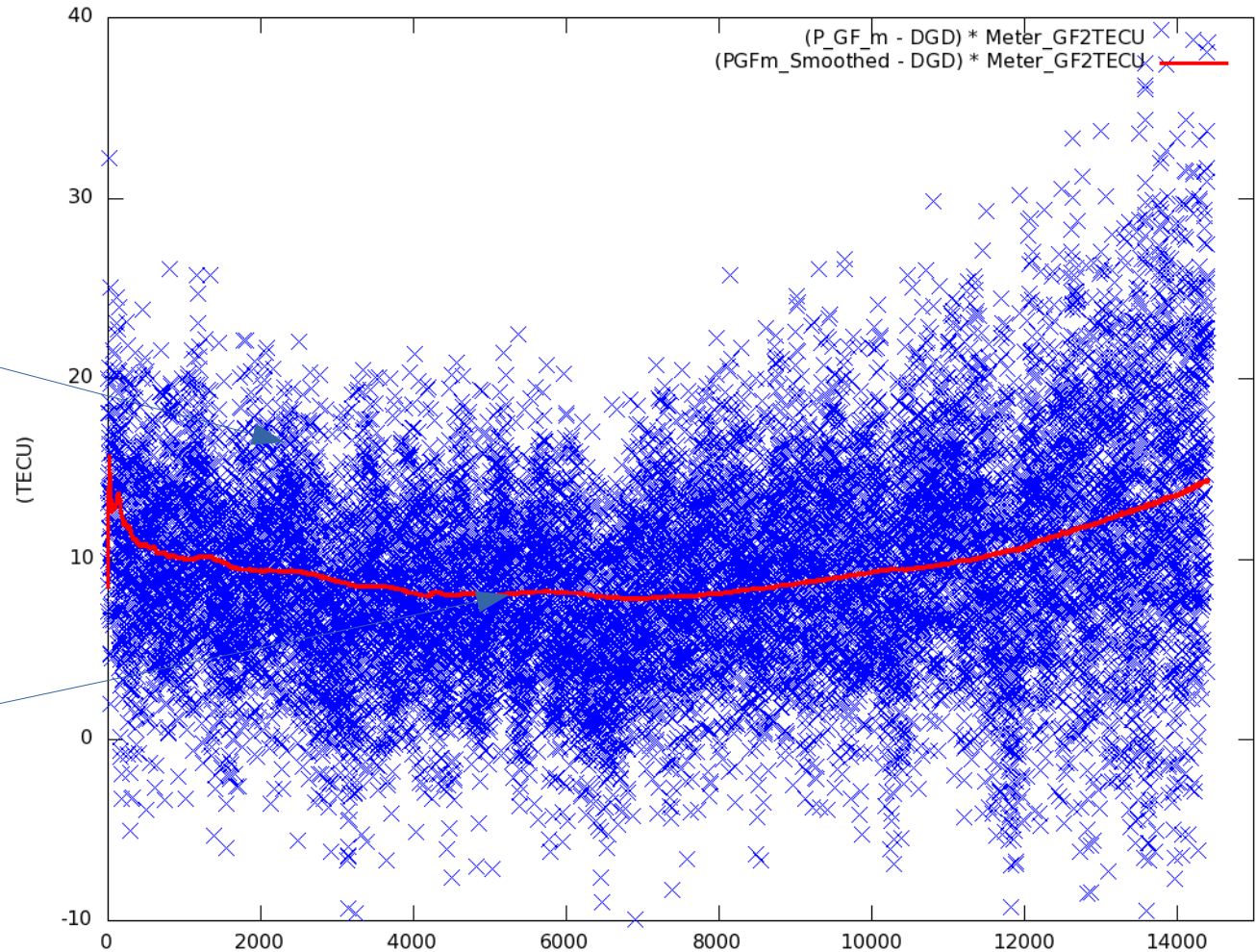
# Phase levelling

A simple technique for combining code and phase measurements

Code measurements give an absolute value for the ionosphere, but are very noisy. (blue crosses)

Phase measurements give a relative value for the ionosphere and have very little noise.


By assimilating data, we can combine these measurements to get a good ionosphere value with little noise. (red line)



# Phase levelling

A simple technique for combining code and phase measurements

The effect of phase levelling is to replace the phase biases and ambiguities with the code biases, while keeping the low noise.


$$L_{GF, Lvl} = -\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) I + \overbrace{DCB^s + DCB_r} + \epsilon_{L_{GF}}$$

# Differential Code Bias Estimation

Anatomy of the equation:

$$L_{GF, Lvl} = -\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) I + DCB^s + DCB_r + \epsilon_{L_{GF}}$$

Scalar field in geographical coordinates.

Correlated in space and time. (points that are close in space and/or time are more likely to be similar)

One number per satellite.  
Not correlated with each other.  
Varies very slowly.

One number per receiver.  
Not correlated with each other.  
Varies slowly.

Measurements from several satellites and receivers are required to solve the equation.

The equations are linked through the ionospheric parameter.

However, one degree of freedom remains. This is normally handled by introducing the constraint that the average of the satellite biases should be equal to zero.

(Other constraints may also be used. Alternatively, if hardware calibration was available to set the value of one or more of the DCB, the degree of freedom would not exist.)

# Apply biases and mapping

$$L_{GF,Lvl} = -\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) I + DCB^s + DCB_r + \epsilon_{L_{GF}}$$

$$STEC = \frac{-L_{GF,Lvl} + DCB^s + DCB_r}{1e16 * 40.3 * \left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)}$$

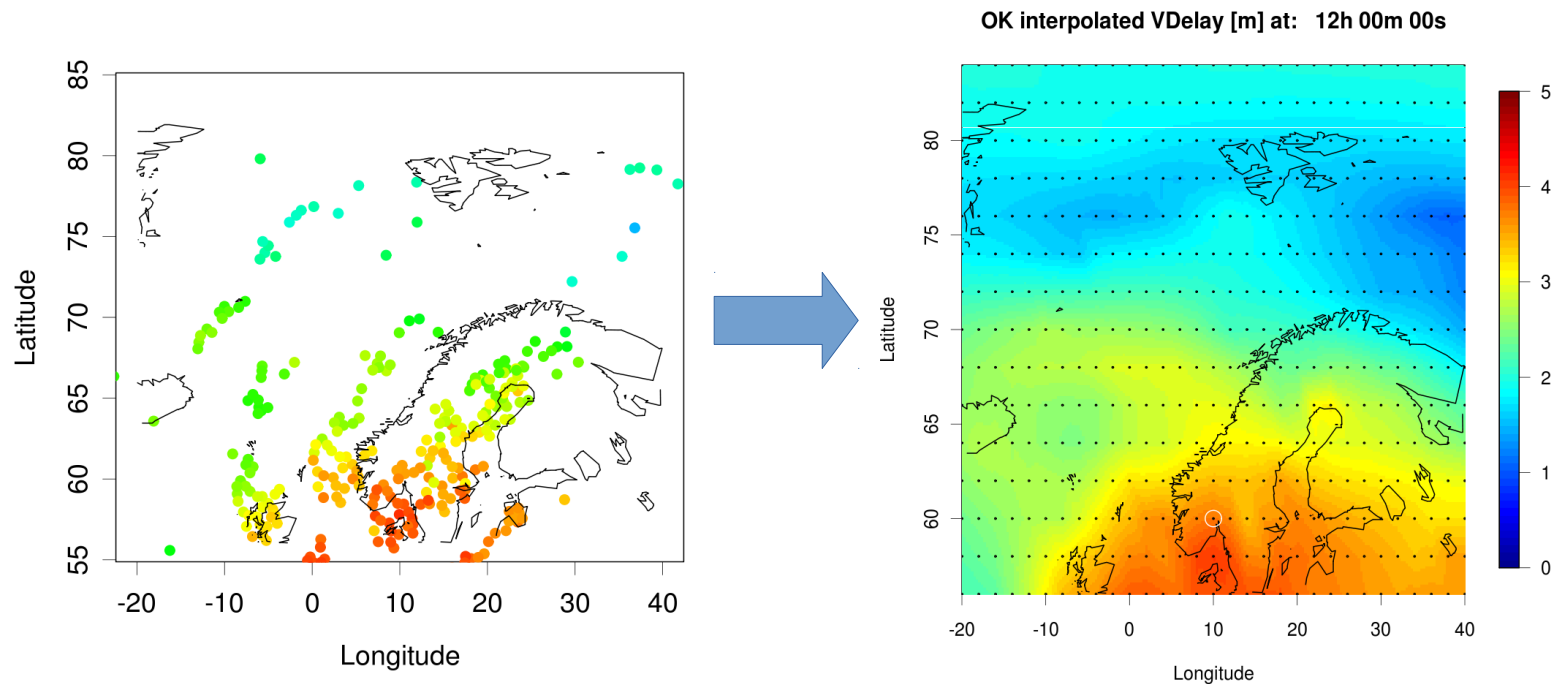
(STEC is given in the unit TECU, which is defined as  $10^{16}$  electrons per  $m^2$ .)

$$VTEC = STEC \sqrt{1 - \left(\frac{R_E \cos(z)}{R_E + H}\right)^2}$$

# Interpolation to grid points

Kriging interpolation is used to find the value at an arbitrary coordinate based on the value at the measurement points.

It takes into account the spatial variability of the ionosphere through the use of a covariance function. (Which describes how strongly two points are related as a function of the distance between them)

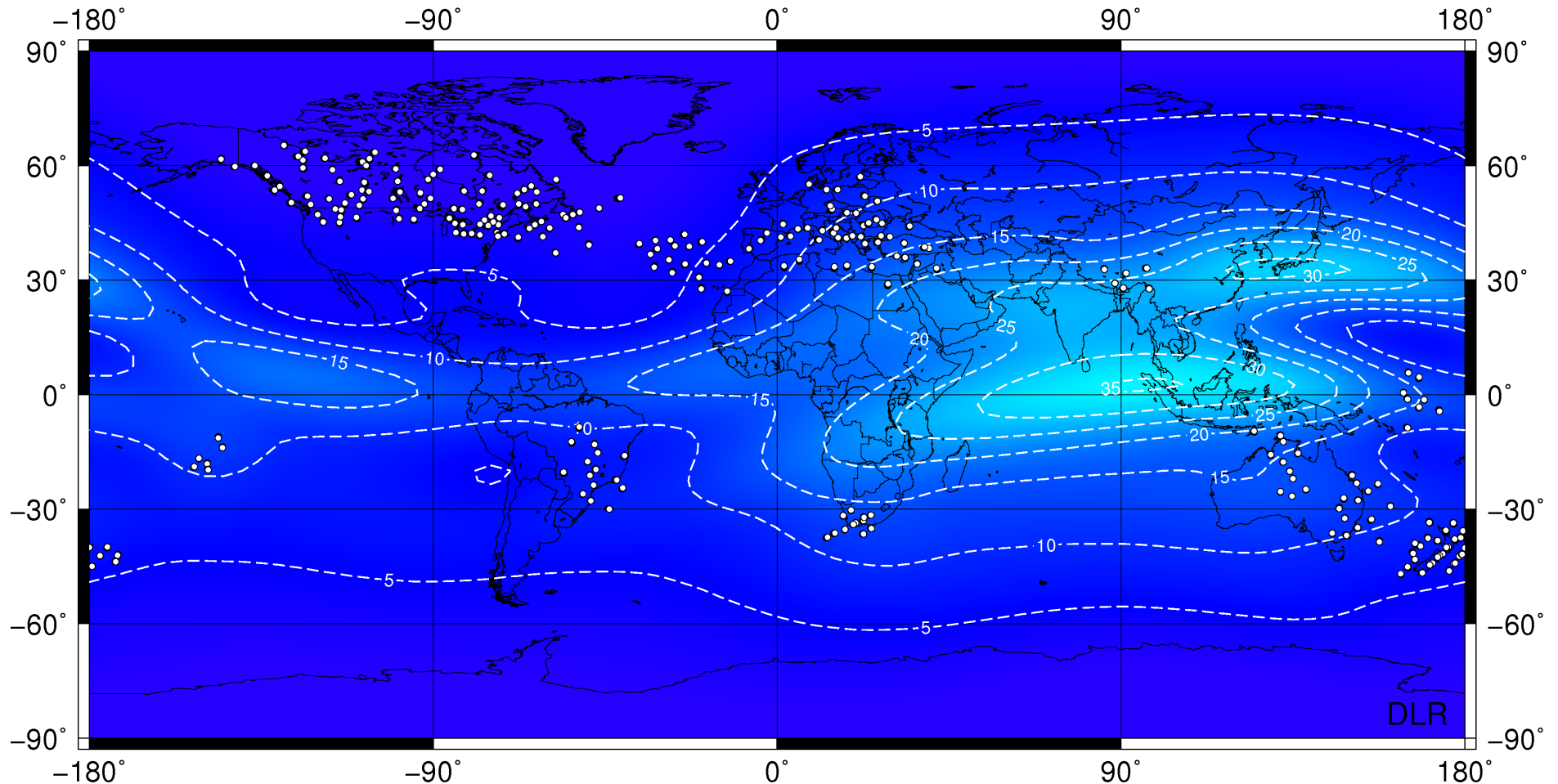




# Filling the gaps

Total Electron Content (TEC)

2016-08-26 08:00:00 UT



Ionospheric Range Error (L1) / m

0.00 1.62 3.24 4.86 6.48 8.10 9.72 11.34 12.96 14.58 16.20 17.82 19.44 21.06



0 10 20 30 40 50 60 70 80 90 100 110 120 130

TEC / TECU

# Question time!

