Basic Methods for satellite positioning

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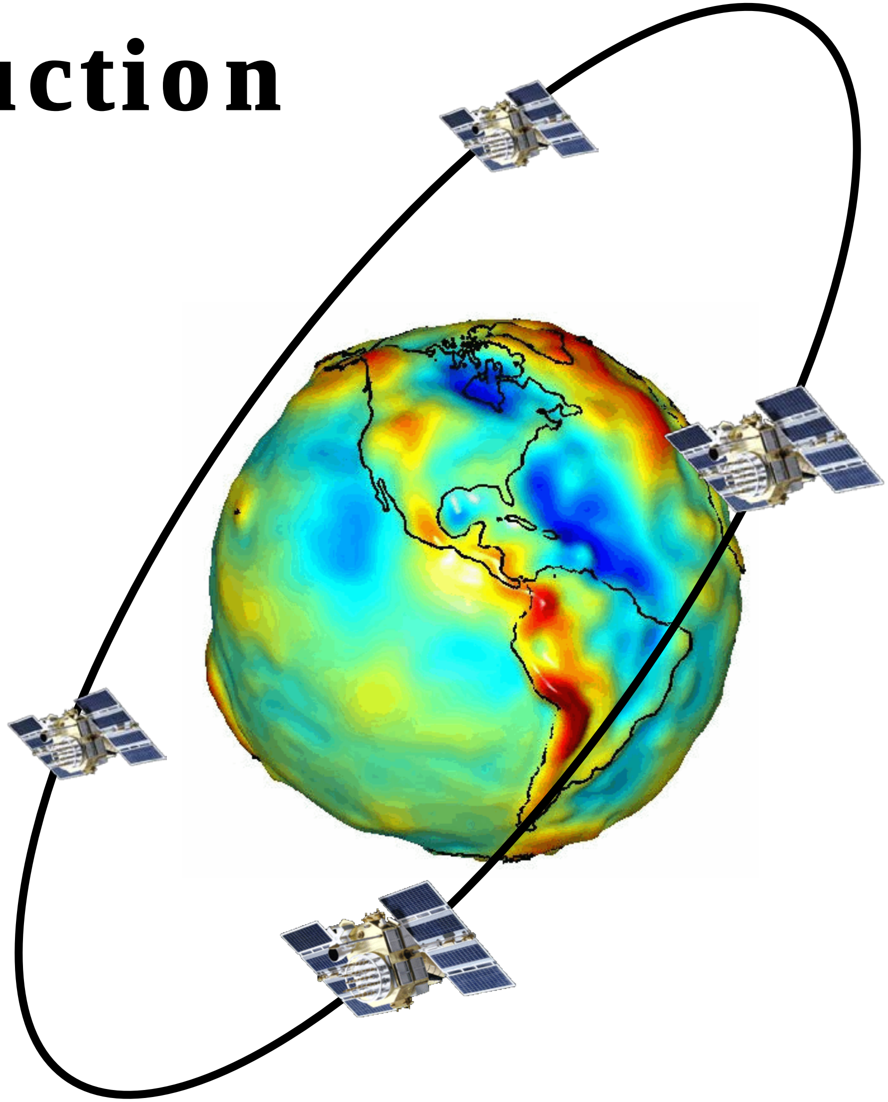
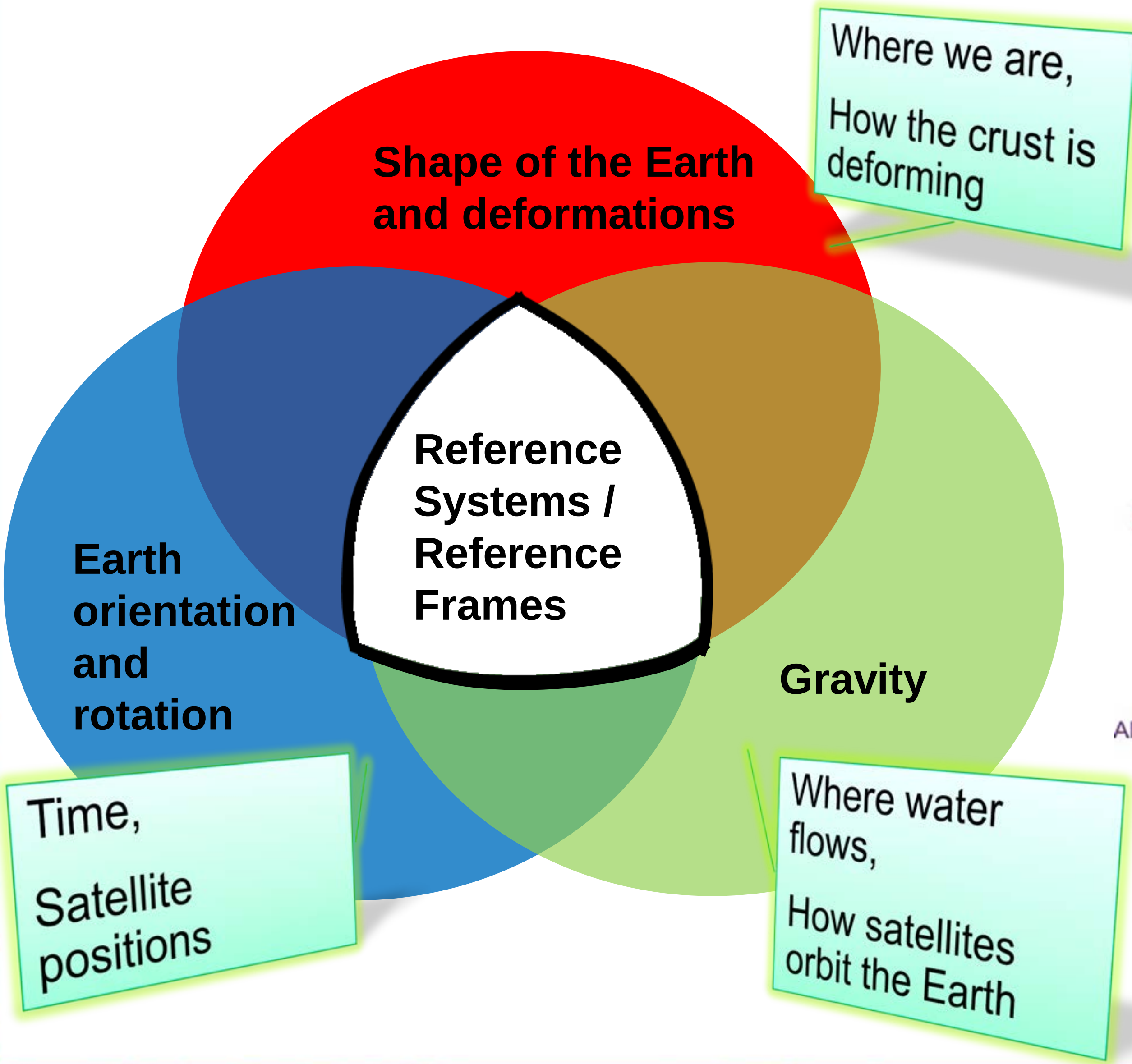
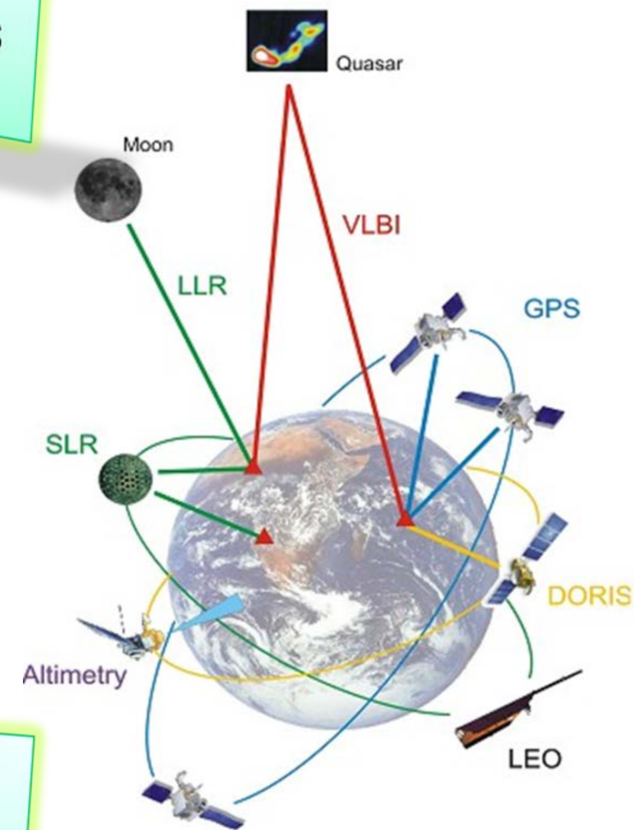
Introduction



Three pillars of geodesy





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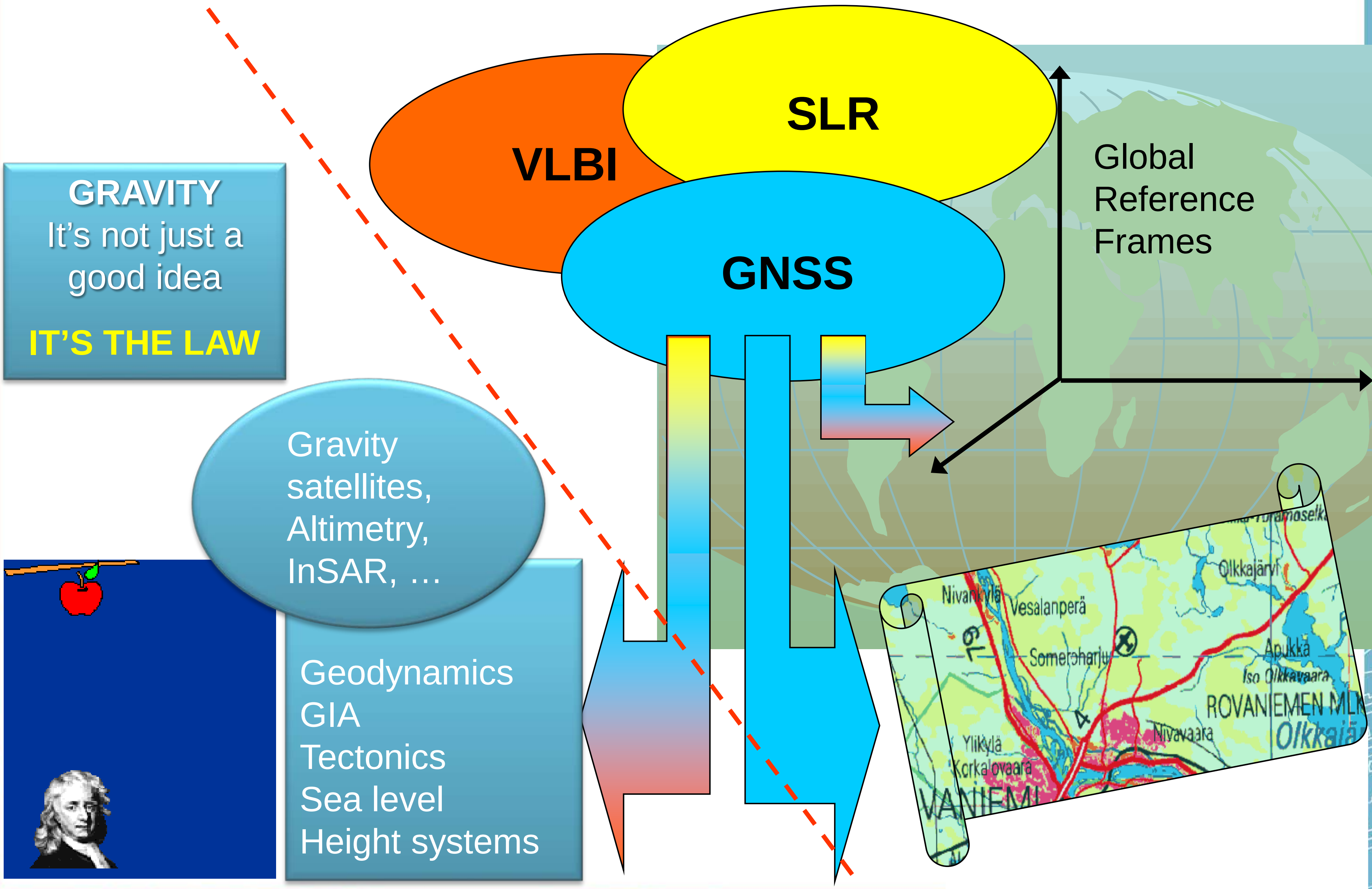
Space geodetic techniques



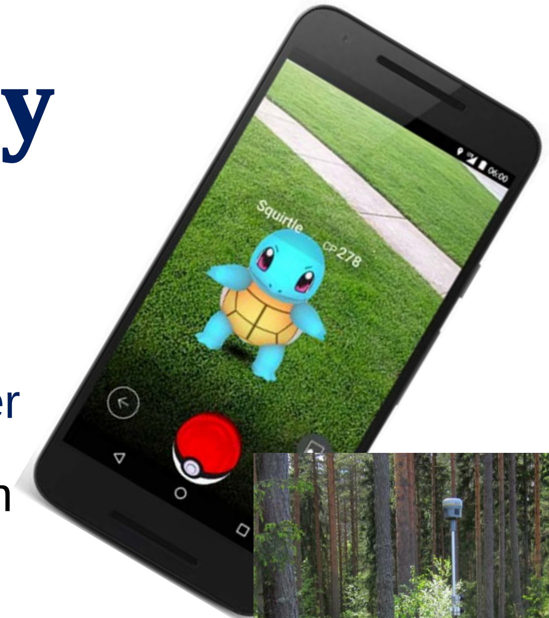
GNSS accuracy

CODE

- **10 m**
- **1 m**
  - ← Navigation / code, single receiver
  - ← DGNSS / code, reference station

PHASE

- **0.1 m**
  - ← RTK / phase + reference station
- **0.01 m**
  - ← Static, phase observations, network, post processing
- **0.001m**
  - ← Permanent stations, time series, changes

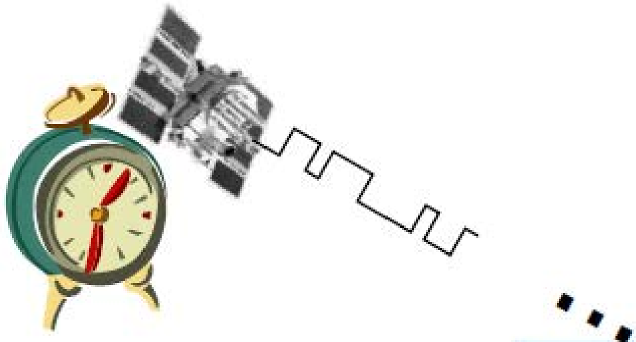


Jaakko Kuokkanen

Basic signals

We need the distances to the satellites

Signal flight time – code pseudorange *R*



$$\Delta t = t_R - t^S = \Delta t_0 + \Delta\delta$$

$$R' = c\,\Delta t = c\Delta t_0 + c\Delta\delta$$

$$R = \rho + c\Delta\delta + \ldots$$

Δt = observed flight time

t_R = receiver time

t^S = satellite time

Δt_0 = actual flight time

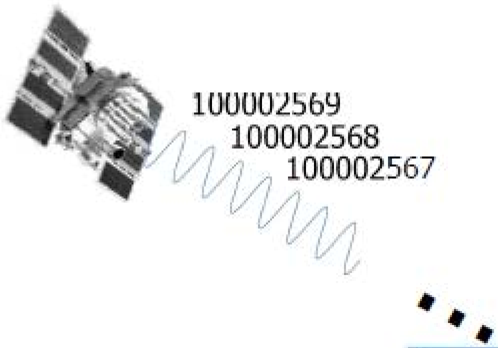
$\Delta\delta$ = sum of clock erros

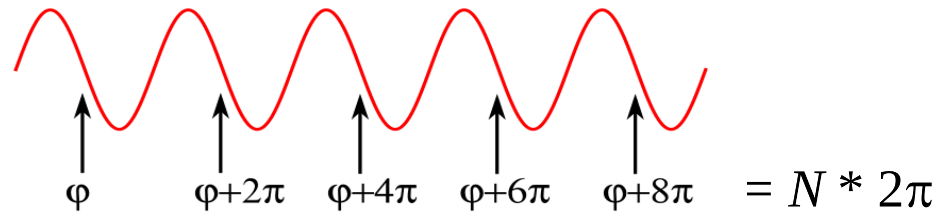
c = speed of light

ρ = actual distance

Carrier phase – carrier pseudorange L

$$L = (N + \Delta\varphi)\lambda + c\delta t + \dots$$

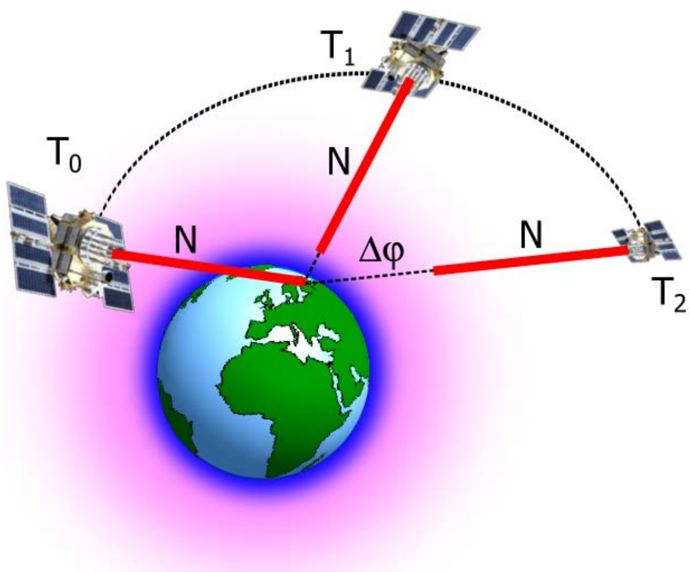




- N = full wavelengths
- $\Delta\varphi$ = partial phase
- λ = carrier wavelength
- δt = clock error
- c = speed of light

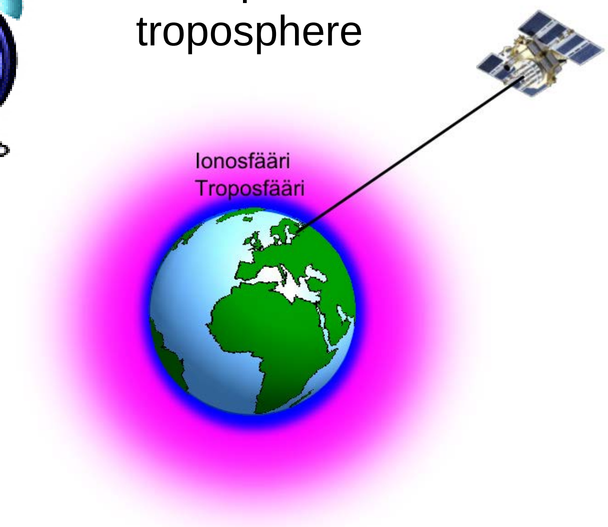
Carrier pseudorange

$$L = \overbrace{(N + \Delta\varphi)\lambda}^{\text{Actual distance to a satellite}} + \overbrace{c\delta t + \Delta^{\text{ion}} + \Delta^{\text{trop}} + \ldots^{\text{et al.}}}^{\text{Distance-like "artefacts"}}$$



Clock
error

Ionosphere and
troposphere



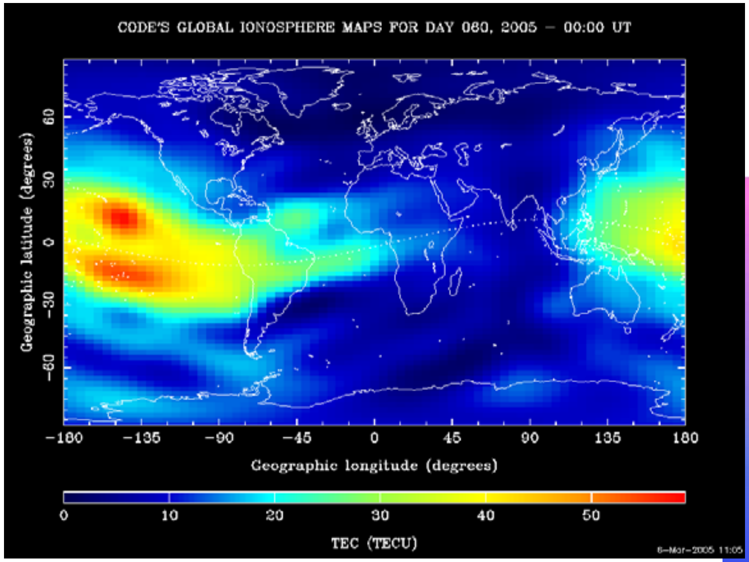
We want to solve for
this part of the equation

We want to get rid of
this part of the equation

$$L = (N + \Delta\varphi)\lambda + c\delta t + \Delta^{\text{ion}} + \Delta^{\text{trop}} + \ldots^{\text{et al.}}$$

Rolf

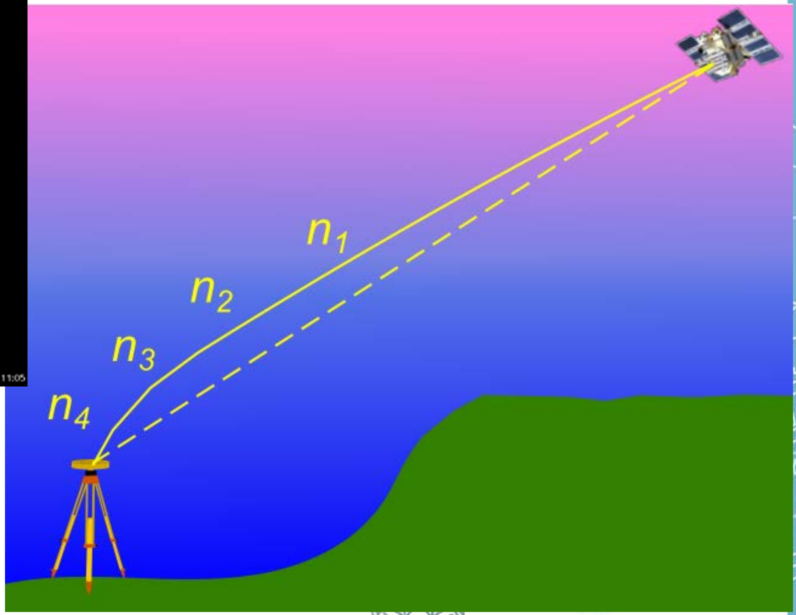
Depends on the signal frequency



Knut

Jan

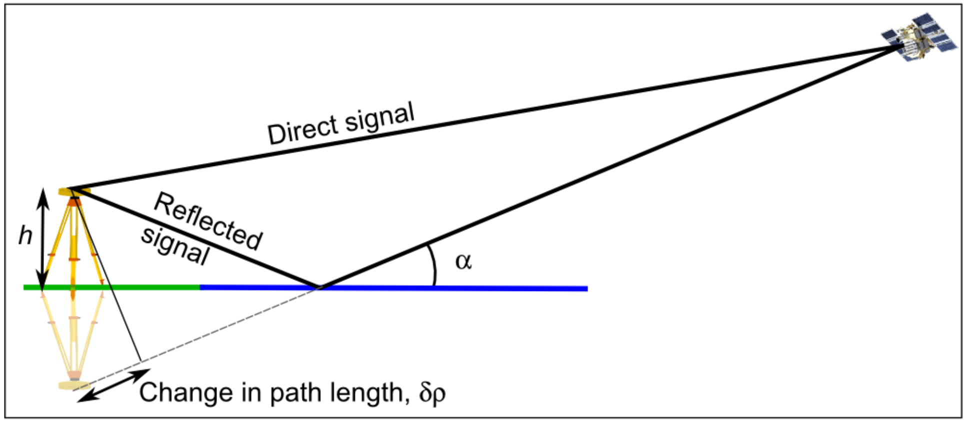
Independent of the signal frequency

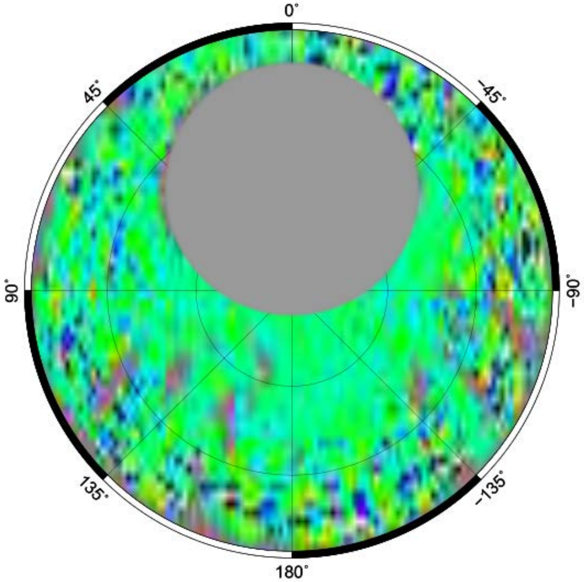


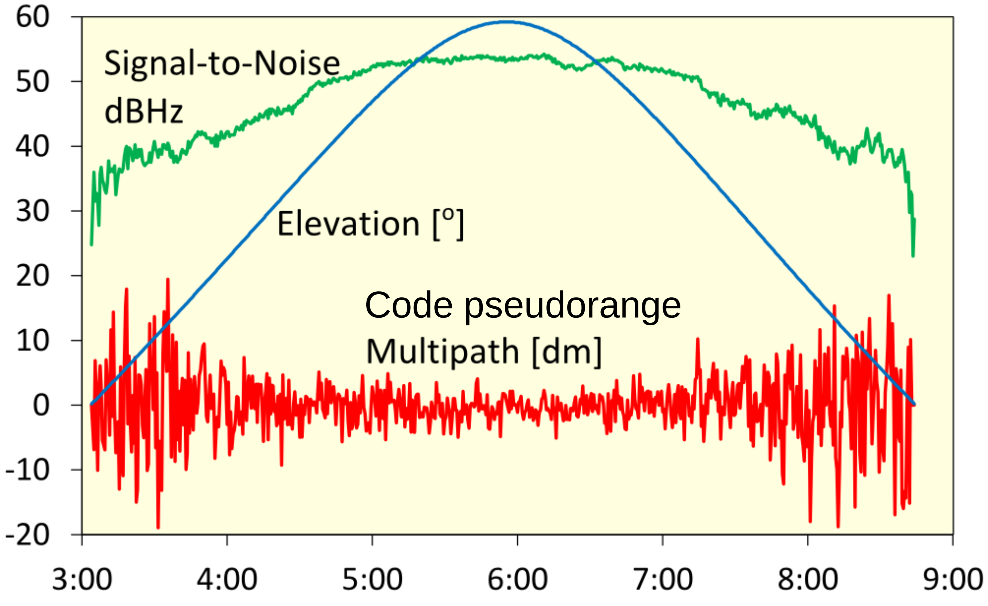
$$L = (N + \Delta\varphi)\lambda + c\delta t + \Delta^{\text{ion}} + \Delta^{\text{trop}} + \ldots$$

et al.

Multipath

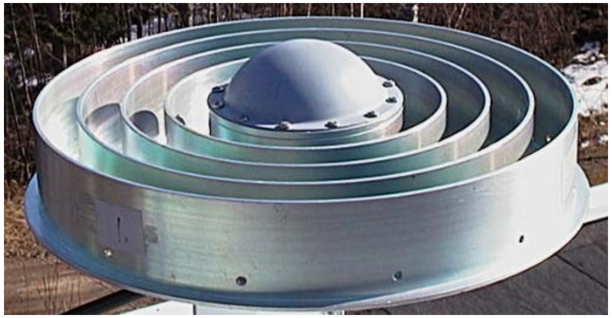




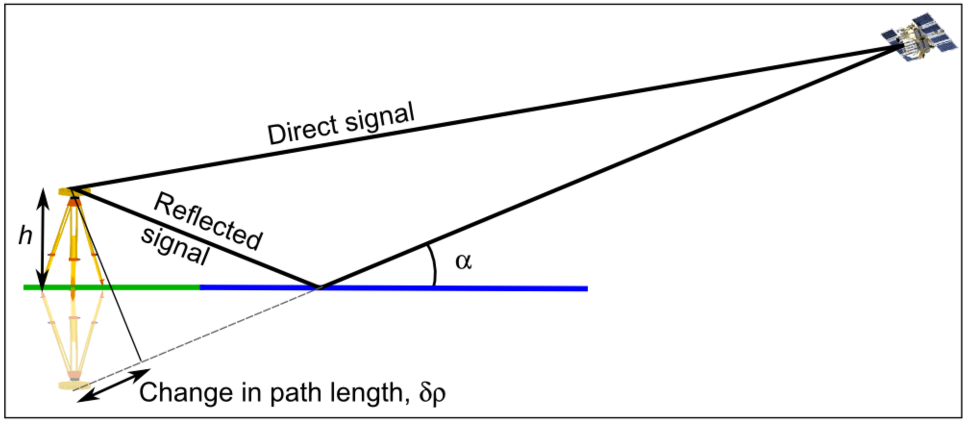


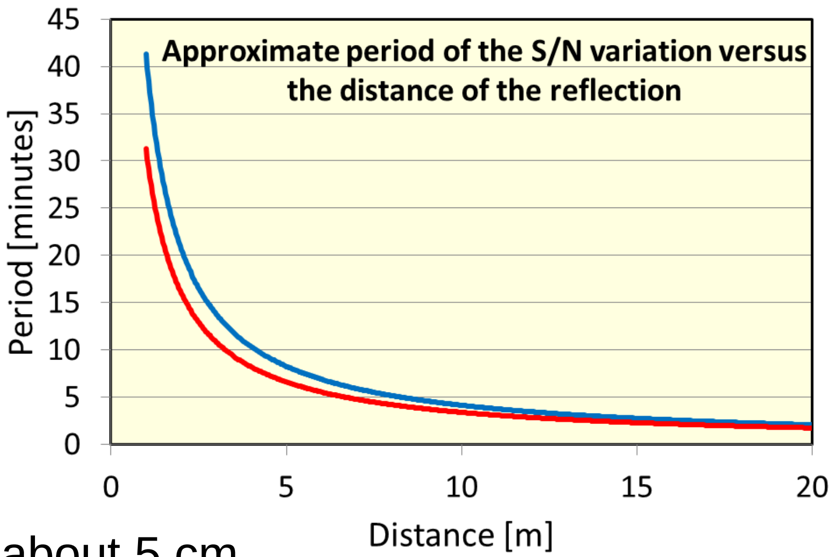
Solution: ??

Mitigation: Choke Ring antennas



Multipath in carrier pseudorange





Maximum phase shift 90º = ¼ wavelength i.e. about 5 cm
In practice it is smaller, normally below 1 cm

Phase shift of the original and reflected signal $$\Delta\Phi = \frac{\delta\rho}{\lambda} = \frac{2h}{\lambda}\sin\alpha$$

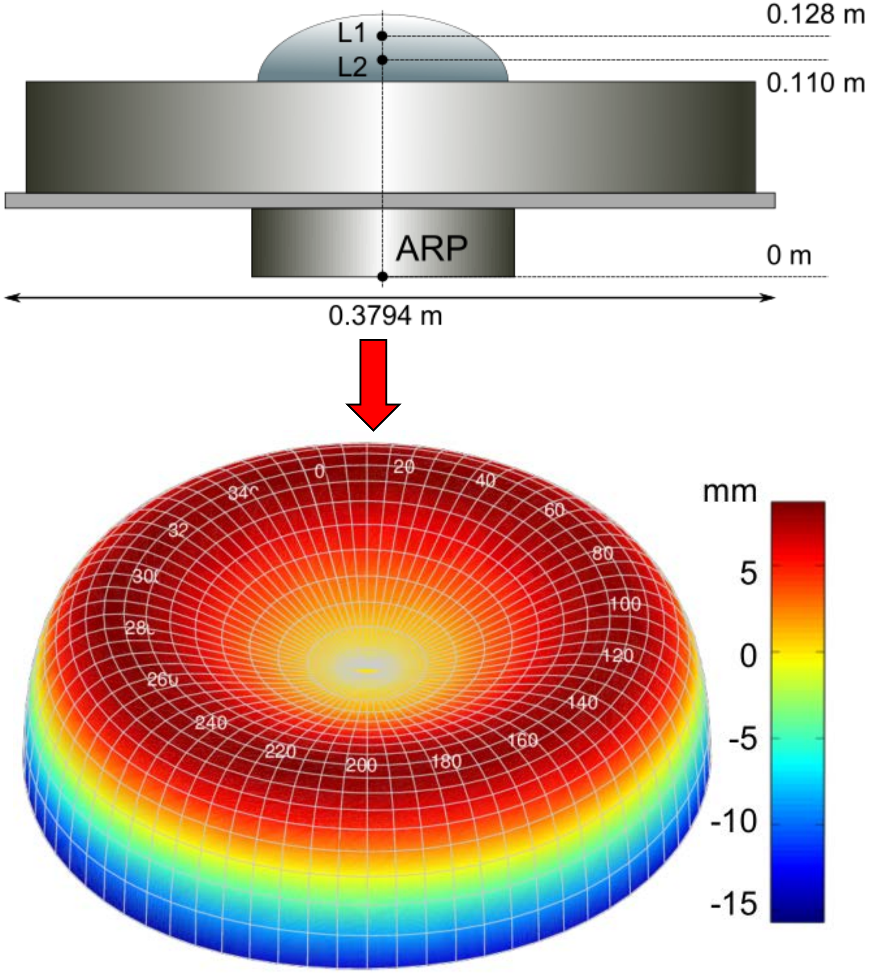
The speed of change depends of the distance of reflection $$\frac{\mathrm{d}\Delta\Phi}{\mathrm{d}t} = \frac{2h}{\lambda}\frac{\mathrm{d}\sin\alpha}{\mathrm{d}t} = \frac{2h}{\lambda}\cos\alpha\frac{\mathrm{d}\alpha}{\mathrm{d}t}$$

This is seen as the pseudo-periodic variation of the signal-to-noise ratio

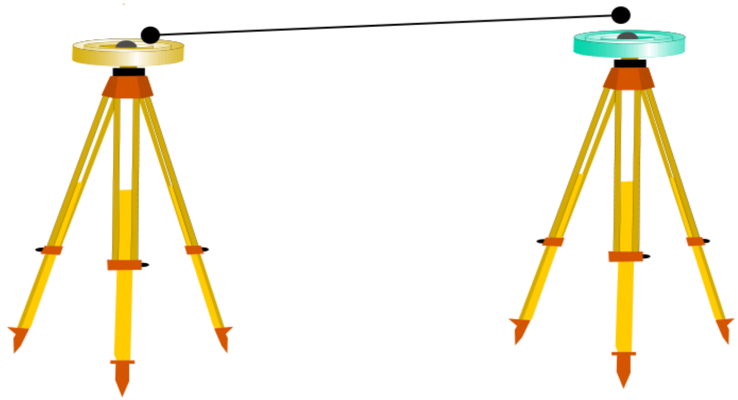
$$L = (N + \Delta\varphi)\lambda + c\delta t + \Delta^{\text{ion}} + \Delta^{\text{trop}} + \ldots$$

et al.

Antenna phase center



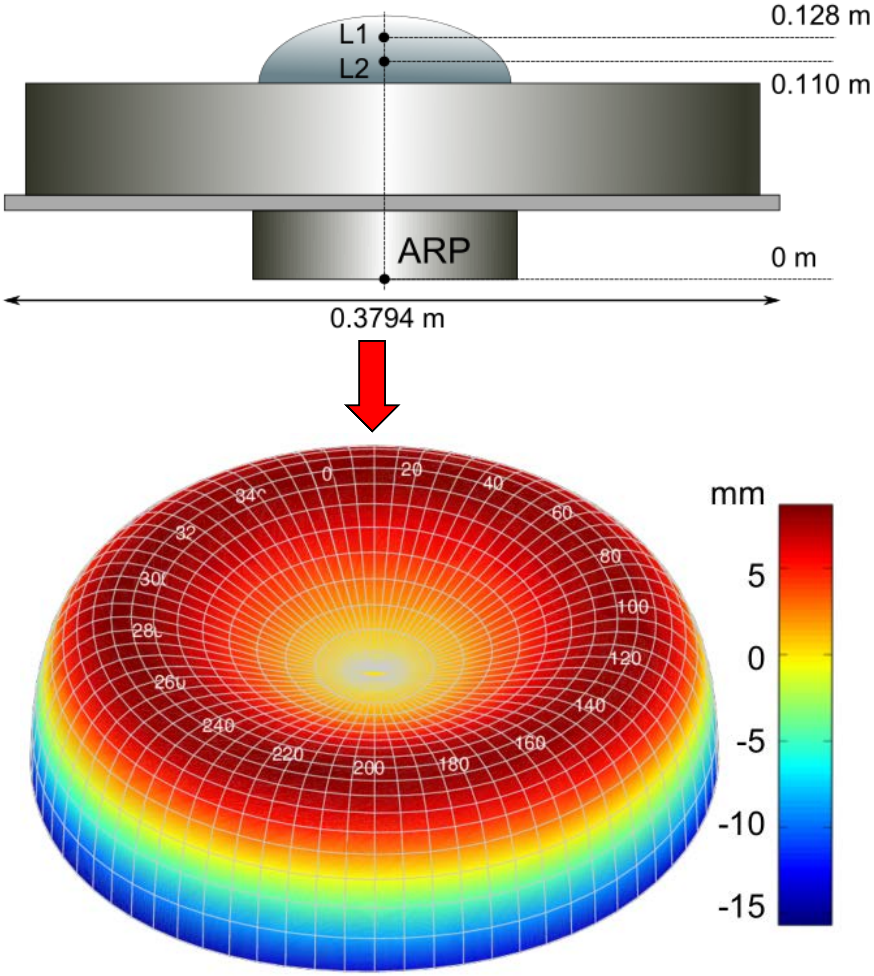
Ulla Kallio



- Identical antennas: Phase center behaves similarly and cancels out in differences
- Different antennas: Does not cancel out

Solution: Antenna calibration
Use calibration tables in processing

Antenna phase center variation



Ulla Kallio

- PCO = Phase Center Offset
- PCV = Phase Center Variation

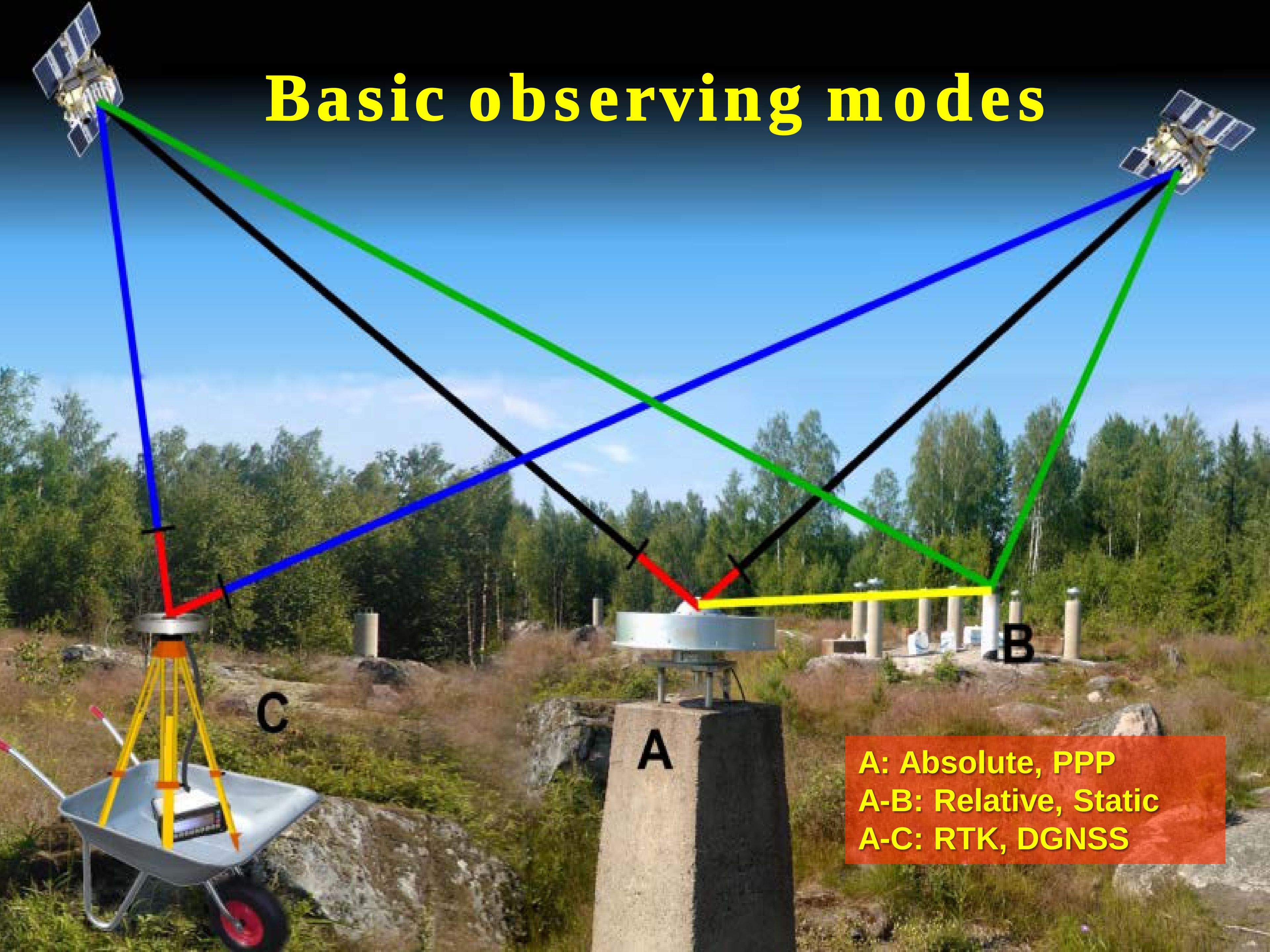
- Individually calibrated antennas
- Type calibrated antennas

Calibration values in ANTEX files

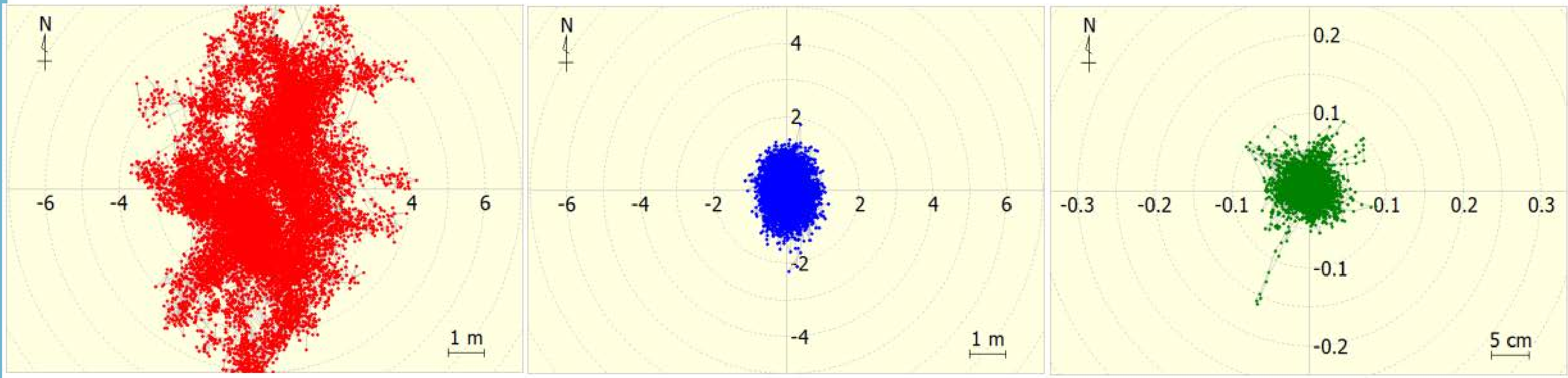
Use official names for antennas (+ possible radomes) in post processing to ensure that correct ANTEX files are used

PCO and PCV corrects the observations to the ARP (Antenna Reference Point)

Basic observing modes



Observing modes, Accuracy



Absolute positioning

- Single receiver
- Navigation
- A few meter accuracy
- Simple, fast
- No external information

Differential positioning

- Navigation
- Below meter accuracy
- Needs a reference station and real time data transfer

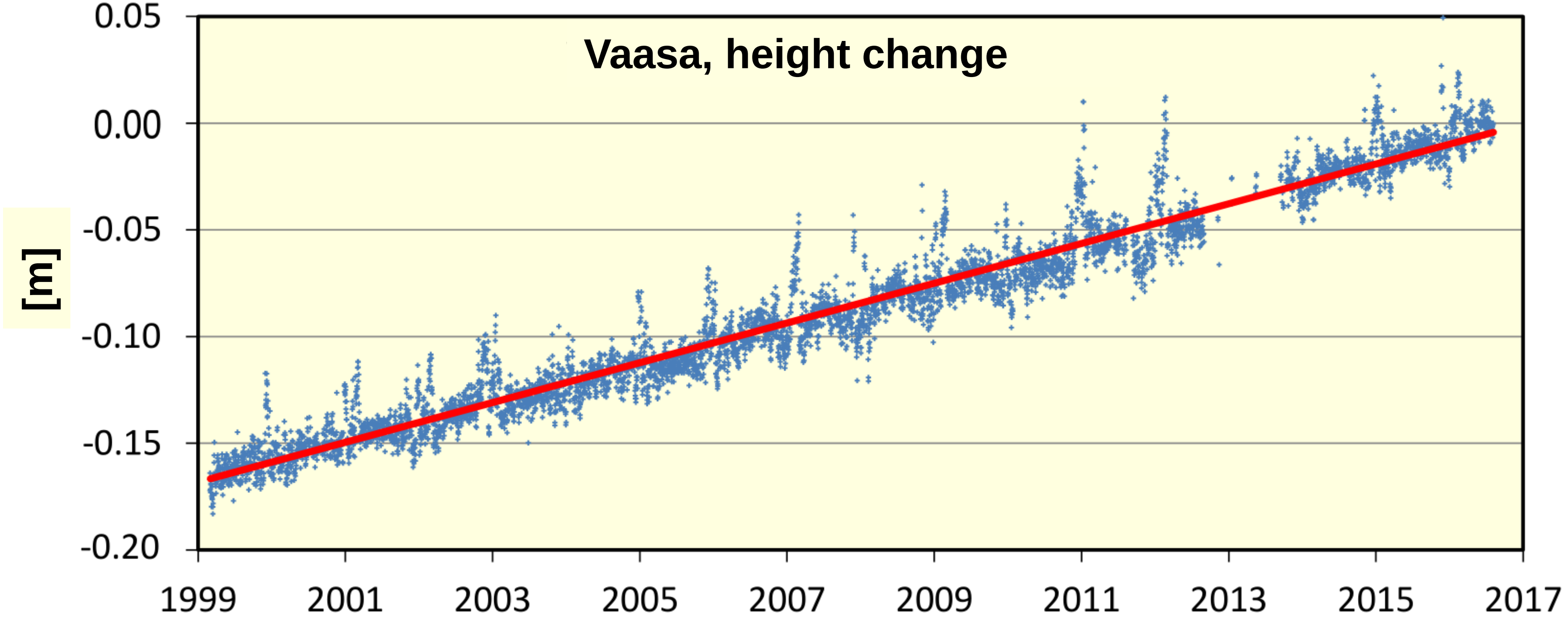
Relative positioning

- Static and kinematic
- Real time or post processing
- Most accurate mode
- Permanent stations

PPP: cm accuracy -> Ola and Axel

RTK -> Anna

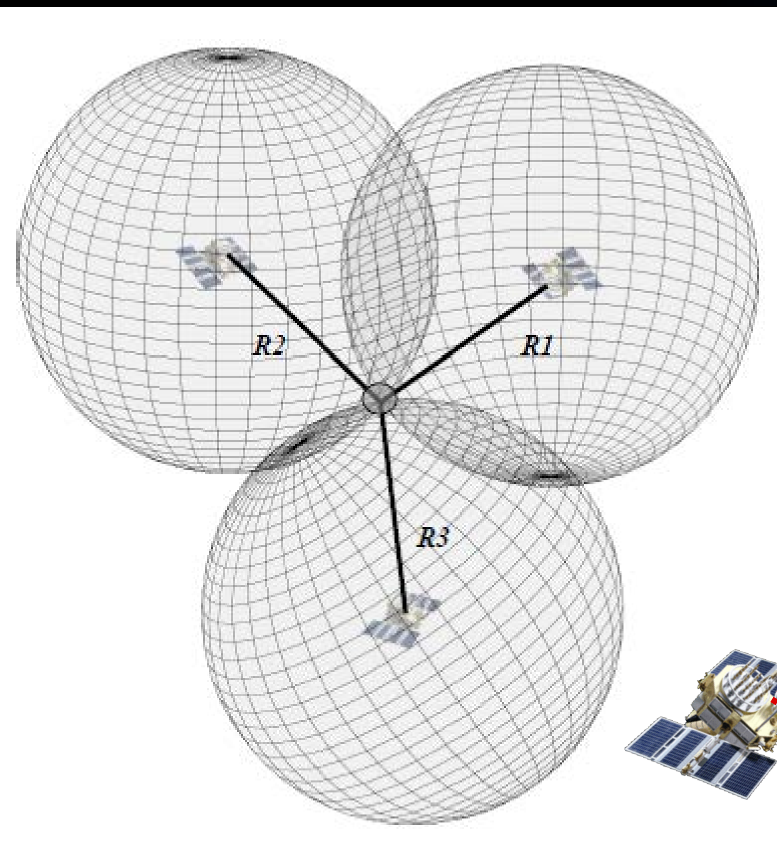
Time series

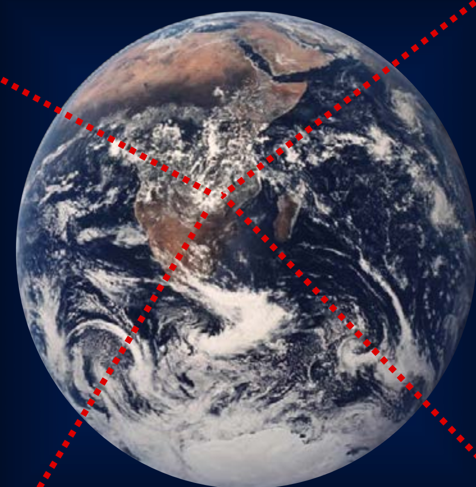


- We need long time series to get sub-cm – sub-mm accuracy in trends. Coordinate values have always some systematic uncertainty components so the coordinate values are not that good. (But how to verify?)

Why four satellites?

(at least)





Unknowns:

X, Y, Z, Δ t

Observable:

Distance to a satellite

Least squares – 1

- **Observables**: Distances to the satellites
- **Unknowns**: **Observer's position (XYZ), receiver clock error**, satellite clock error, integer ambiguities, signal propagation errors, …
- In general: **More observations than unknowns**, solution with the least squares method
- Matrix of observations (= pseudoranges) $\boldsymbol{\ell}$
- Matrix of unknowns, **x** , Design matrix **A**

$$\ell = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \vdots \\ \ell_n \end{pmatrix};\ \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1u} \\ a_{21} & a_{22} & \cdots & a_{2u} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nu} \end{pmatrix};\ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_u \end{pmatrix}$$

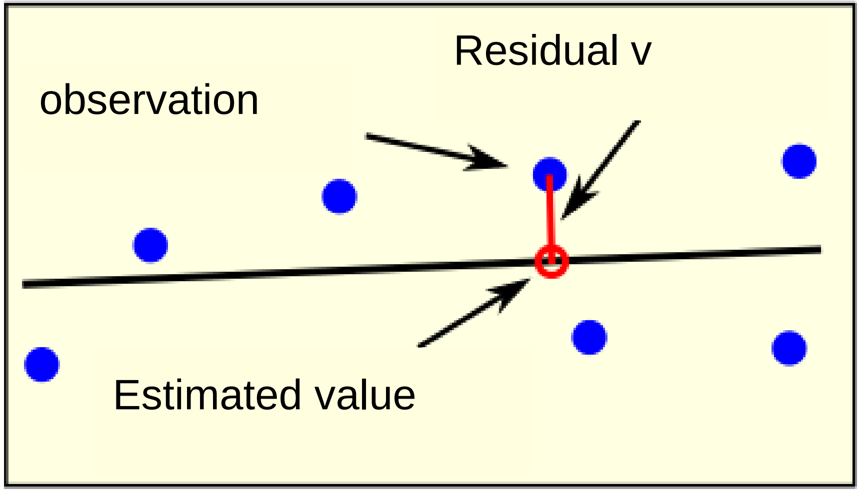
$$\boxed{\ell = \mathbf{A}\mathbf{x}}$$

Least squares – 2

Solution of $\boxed{\ell = \mathbf{A}\mathbf{x} - \mathbf{v}}$

residual vector v is to be minimized

-> solution: $\boxed{\widehat{\mathbf{x}} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\ell}$



The observation equation is nonlinear, must be linearized for the least squares,

- Taylor's series, take only the linear part.
- Estimation of observer's position (ξ, η, ζ)
- Develop in Taylor's series around this point and take the difference to the original position as the new unknown

$$(\Delta\xi_A, \Delta\eta_A, \Delta\zeta_A)'$$

A trash can for unused least squares residuals

Least squares – 3

Receiver *A*, satellite *i*
Satellite position (x,y,z)
Receiver position (ξ, η, ζ)

$$L = \underbrace{\lambda(\varphi + N)} + d\rho + c\Delta\delta + \ldots$$

$$\rho_A^i(t) = \sqrt{(x^i(t) - \xi_A)^2 + (y^i(t) - \eta_A)^2 + (z^i(t) - \zeta_A)^2}$$

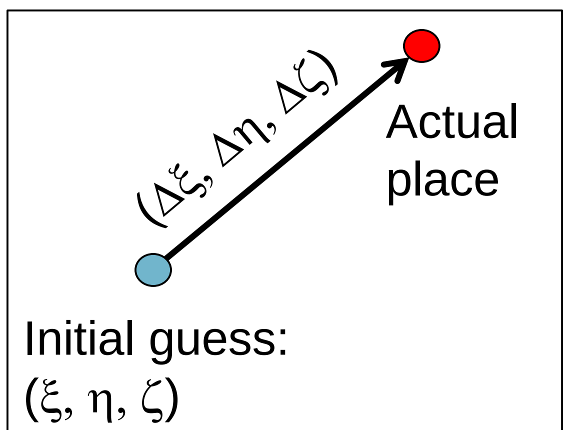
$$= \rho_{A,0}^i(t) - \frac{x^i(t) - \xi_{A,0}}{\rho_{A,0}^i(t)}\Delta\xi_A - \frac{y^i(t) - \eta_{A,0}}{\rho_{A,0}^i(t)}\Delta\eta_A - \frac{z^i(t) - \zeta_{A,0}}{\rho_{A,0}^i(t)}\Delta\zeta_A$$

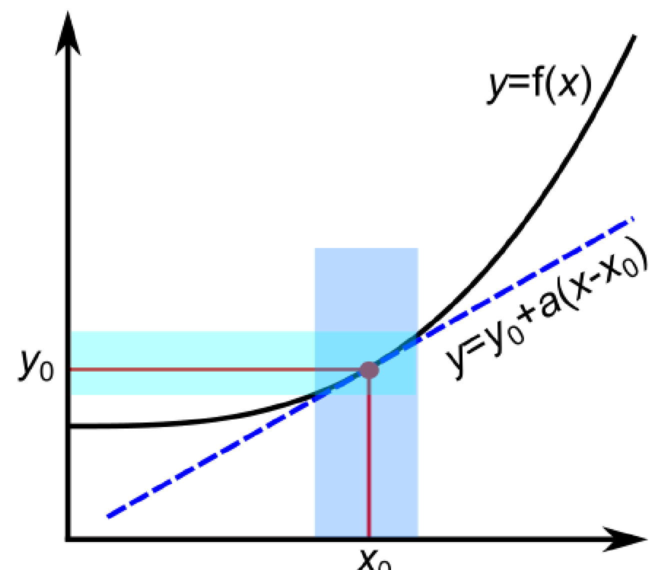
$$= \rho_{A,0}^i(t) - a_{\xi,A}^i \Delta\xi_A - a_{\eta,A}^i \Delta\eta_A - a_{\zeta,A}^i \Delta\zeta_A - c\delta_A(t)$$

$$a_{\xi,A}^i = \frac{x^i(t) - \xi_{A,0}}{\rho_{A,0}^i(t)}$$

$$a_{\eta,A}^i = \frac{y^i(t) - \eta_{A,0}}{\rho_{A,0}^i(t)}$$

$$a_{\zeta,A}^i = \frac{z^i(t) - \zeta_{A,0}}{\rho_{A,0}^i(t)}$$





Positioning using code

- Absolute positioning – Single receiver
- 4 unknowns, 4 satellites, 4 pseudoranges
- **Can be solved from one epoch**

A piece of cake to solve these…

$$\rho_A^i(t) - \rho_{A,0}^i(t) - c\delta^i(t) = \ell_A^i =$$
$$-a_{\xi,A}^i \Delta\xi_A - a_{\eta,A}^i \Delta\eta_A - a_{\zeta,A}^i \Delta\zeta_A - c\delta_A(t)$$

$$\ell = \begin{pmatrix} \ell_A^1 \\ \ell_A^2 \\ \ell_A^3 \\ \ell_A^4 \end{pmatrix}; \quad \mathbf{A} = \begin{pmatrix} a_{\xi,A}^1 & a_{\eta,A}^1 & a_{\zeta,A}^1 & -c \\ a_{\xi,A}^2 & a_{\eta,A}^2 & a_{\zeta,A}^2 & -c \\ a_{\xi,A}^3 & a_{\eta,A}^3 & a_{\zeta,A}^3 & -c \\ a_{\xi,A}^4 & a_{\eta,A}^4 & a_{\zeta,A}^4 & -c \end{pmatrix}; \quad \mathbf{x} = \begin{pmatrix} \Delta\xi_A \\ \Delta\eta_A \\ \Delta\zeta_A \\ \delta_A(t) \end{pmatrix}.$$

Positioning using carrier – 1

- One epoch: 8 unknowns, 4 satellites, 4 pseudoranges
- **Cannot be solved from one epoch!**

$$\lambda \varphi_A^i(t) - \rho_{A,0}^i(t) - c\delta^i(t) = \ell^i_A =$$

$$-a^i_{\xi,A}\Delta\xi_A - a^i_{\eta,A}\Delta\eta_A - a^i_{\zeta,A}\Delta\zeta_A - c\delta_A(t) + \lambda N_A^i.$$

$$\mathbf{A} = \begin{pmatrix} a^1_{\xi,A} & a^1_{\eta,A} & a^1_{\zeta,A} & \lambda & 0 & 0 & 0 & -c \\ a^2_{\xi,A} & a^2_{\eta,A} & a^2_{\zeta,A} & 0 & \lambda & 0 & 0 & -c \\ a^3_{\xi,A} & a^3_{\eta,A} & a^3_{\zeta,A} & 0 & 0 & \lambda & 0 & -c \\ a^4_{\xi,A} & a^4_{\eta,A} & a^4_{\zeta,A} & 0 & 0 & 0 & \lambda & -c \end{pmatrix}; \quad \mathbf{x} = \begin{pmatrix} \Delta\xi_A \\ \Delta\eta_A \\ \Delta\zeta_A \\ N_A^1 \\ N_A^2 \\ N_A^3 \\ N_A^4 \\ \delta_A(t) \end{pmatrix}$$

Sorry, guys….

We have these nasty ambiguity parameters which screw up the whole thing

Positioning using carrier – 2

- **Adding a new satellite won't help**: every new satellite will bring one new unknown (integer ambiguity) and one observable (pseudorange to the new satellite) = ± 0
- **Adding a new epoch will help**: If the receivers' positions are fixed, we will have only one new clock unknown, but four observables = +3
- Three epochs, four satellites = 12 observables, 10 unknowns
- **JESS!**

- …but what happens if the receiver is moving?
- On every epoch we will get **three new coordinate unknowns** = ± 0
- **Solution**: Anna will explain this ;-)

Positioning using carrier – 3

- Three epochs, four satellites, static receivers

$$
\ell = \begin{pmatrix} \ell_A^1(t_1) \\ \ell_A^2(t_1) \\ \ell_A^3(t_1) \\ \ell_A^4(t_1) \\ \ell_A^1(t_2) \\ \ell_A^2(t_2) \\ \ell_A^3(t_2) \\ \ell_A^4(t_2) \\ \ell_A^1(t_3) \\ \ell_A^2(t_3) \\ \ell_A^3(t_3) \\ \ell_A^4(t_3) \end{pmatrix}; \quad
\mathbf{x} = \begin{pmatrix} \Delta\xi_A \\ \Delta\eta_A \\ \Delta\zeta_A \\ N_A^1 \\ N_A^2 \\ N_A^3 \\ N_A^4 \\ \delta_A(t_1) \\ \delta_A(t_2) \\ \delta_A(t_3) \end{pmatrix} \qquad
\mathbf{A} = \begin{pmatrix}
a^1_{\xi,A}(t_1) & a^1_{\eta,A}(t_1) & a^1_{\zeta,A}(t_1) & \lambda & 0 & 0 & 0 & -c & 0 & 0 \\
a^2_{\xi,A}(t_1) & a^2_{\eta,A}(t_1) & a^2_{\zeta,A}(t_1) & 0 & \lambda & 0 & 0 & -c & 0 & 0 \\
a^3_{\xi,A}(t_1) & a^3_{\eta,A}(t_1) & a^3_{\zeta,A}(t_1) & 0 & 0 & \lambda & 0 & -c & 0 & 0 \\
a^4_{\xi,A}(t_1) & a^4_{\eta,A}(t_1) & a^4_{\zeta,A}(t_1) & 0 & 0 & 0 & \lambda & -c & 0 & 0 \\
a^1_{\xi,A}(t_2) & a^1_{\eta,A}(t_2) & a^1_{\zeta,A}(t_2) & \lambda & 0 & 0 & 0 & 0 & -c & 0 \\
a^2_{\xi,A}(t_2) & a^2_{\eta,A}(t_2) & a^2_{\zeta,A}(t_2) & 0 & \lambda & 0 & 0 & 0 & -c & 0 \\
a^3_{\xi,A}(t_2) & a^3_{\eta,A}(t_2) & a^3_{\zeta,A}(t_2) & 0 & 0 & \lambda & 0 & 0 & -c & 0 \\
a^4_{\xi,A}(t_2) & a^4_{\eta,A}(t_2) & a^4_{\zeta,A}(t_2) & 0 & 0 & 0 & \lambda & 0 & -c & 0 \\
a^1_{\xi,A}(t_3) & a^1_{\eta,A}(t_3) & a^1_{\zeta,A}(t_3) & \lambda & 0 & 0 & 0 & 0 & 0 & -c \\
a^2_{\xi,A}(t_3) & a^2_{\eta,A}(t_3) & a^2_{\zeta,A}(t_3) & 0 & \lambda & 0 & 0 & 0 & 0 & -c \\
a^3_{\xi,A}(t_3) & a^3_{\eta,A}(t_3) & a^3_{\zeta,A}(t_3) & 0 & 0 & \lambda & 0 & 0 & 0 & -c \\
a^4_{\xi,A}(t_3) & a^4_{\eta,A}(t_3) & a^4_{\zeta,A}(t_3) & 0 & 0 & 0 & \lambda & 0 & 0 & -c
\end{pmatrix}
$$

Kinematic positioning – 1

- Three epochs, four satellites, moving receivers = 12 observations, 16 unknowns!

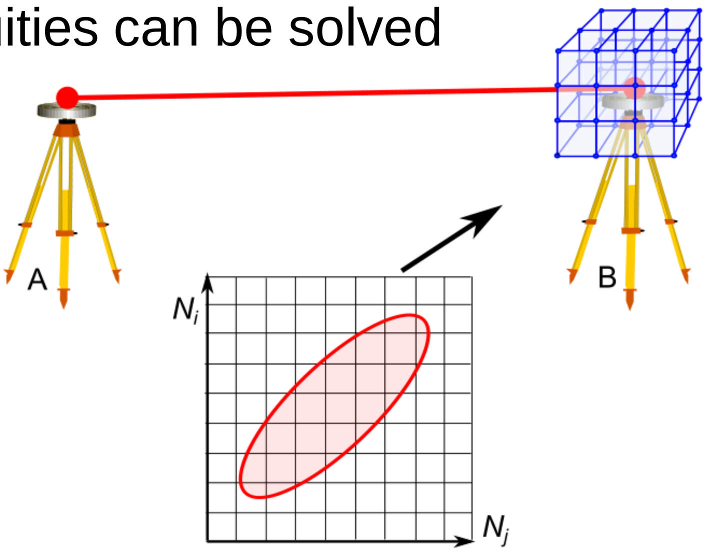
$$\ell = \begin{pmatrix} \ell^1_A(t_1) \\ \ell^2_A(t_1) \\ \ell^3_A(t_1) \\ \ell^4_A(t_1) \\ \ell^1_A(t_2) \\ \ell^2_A(t_2) \\ \ell^3_A(t_2) \\ \ell^4_A(t_2) \\ \ell^1_A(t_3) \\ \ell^2_A(t_3) \\ \ell^3_A(t_3) \\ \ell^4_A(t_3) \end{pmatrix}; \quad \mathbf{x} = \begin{pmatrix} \Delta\xi_A(t_1) \\ \Delta\eta_A(t_1) \\ \Delta\zeta_A(t_1) \\ \Delta\xi_A(t_2) \\ \Delta\eta_A(t_2) \\ \Delta\zeta_A(t_2) \\ \Delta\xi_A(t_3) \\ \Delta\eta_A(t_3) \\ \Delta\zeta_A(t_3) \\ N^1_A \\ N^2_A \\ N^3_A \\ N^4_A \\ \delta_A(t_1) \\ \delta_A(t_2) \\ \delta_A(t_3) \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix}
a^1_{\xi,A}(t_1) & a^1_{\eta,A}(t_1) & a^1_{\zeta,A}(t_1) & \lambda & 0 & 0 & 0 & -c & 0 & 0 \\
a^2_{\xi,A}(t_1) & a^2_{\eta,A}(t_1) & a^2_{\zeta,A}(t_1) & 0 & \lambda & 0 & 0 & -c & 0 & 0 \\
a^3_{\xi,A}(t_1) & a^3_{\eta,A}(t_1) & a^3_{\zeta,A}(t_1) & 0 & 0 & \lambda & 0 & -c & 0 & 0 \\
a^4_{\xi,A}(t_1) & a^4_{\eta,A}(t_1) & a^4_{\zeta,A}(t_1) & 0 & 0 & 0 & \lambda & -c & 0 & 0 \\
a^1_{\xi,A}(t_2) & a^1_{\eta,A}(t_2) & a^1_{\zeta,A}(t_2) & \lambda & 0 & 0 & 0 & 0 & -c & 0 \\
a^2_{\xi,A}(t_2) & a^2_{\eta,A}(t_2) & a^2_{\zeta,A}(t_2) & 0 & \lambda & 0 & 0 & 0 & -c & 0 \\
a^3_{\xi,A}(t_2) & a^3_{\eta,A}(t_2) & a^3_{\zeta,A}(t_2) & 0 & 0 & \lambda & 0 & 0 & -c & 0 \\
a^4_{\xi,A}(t_2) & a^4_{\eta,A}(t_2) & a^4_{\zeta,A}(t_2) & 0 & 0 & 0 & \lambda & 0 & -c & 0 \\
a^1_{\xi,A}(t_3) & a^1_{\eta,A}(t_3) & a^1_{\zeta,A}(t_3) & \lambda & 0 & 0 & 0 & 0 & 0 & -c \\
a^2_{\xi,A}(t_3) & a^2_{\eta,A}(t_3) & a^2_{\zeta,A}(t_3) & 0 & \lambda & 0 & 0 & 0 & 0 & -c \\
a^3_{\xi,A}(t_3) & a^3_{\eta,A}(t_3) & a^3_{\zeta,A}(t_3) & 0 & 0 & \lambda & 0 & 0 & 0 & -c \\
a^4_{\xi,A}(t_3) & a^4_{\eta,A}(t_3) & a^4_{\zeta,A}(t_3) & 0 & 0 & 0 & \lambda & 0 & 0 & -c
\end{pmatrix}$$

Kinematic positioning – 2

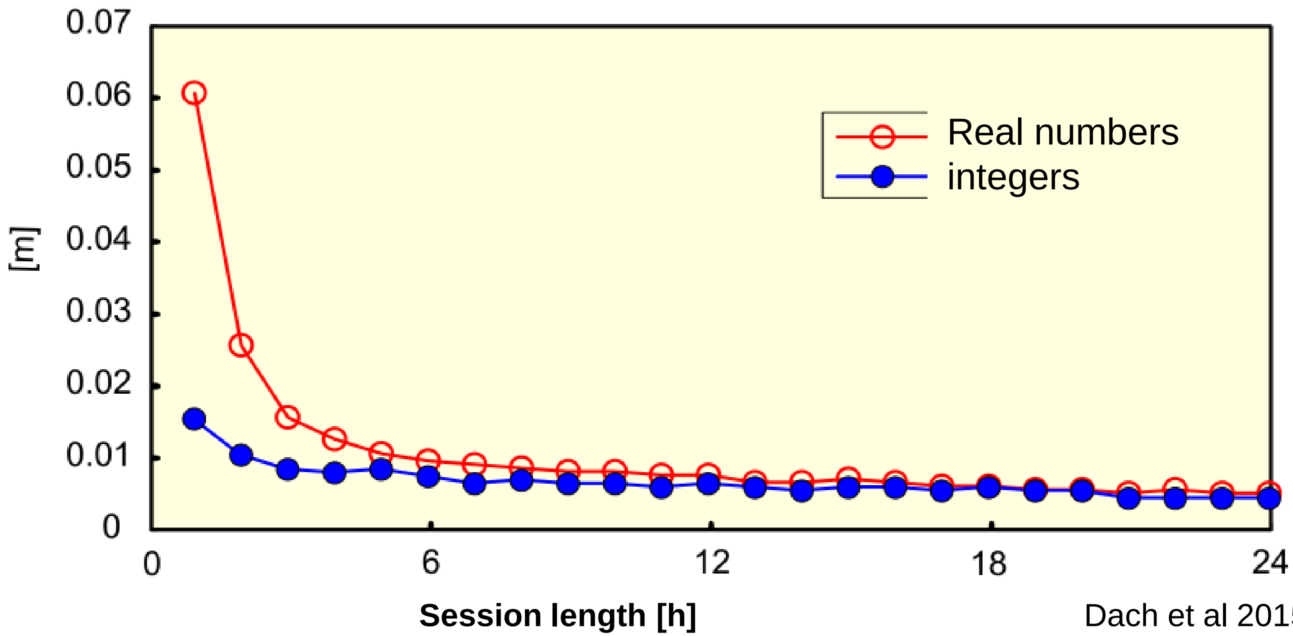
- We have to get rid of integer ambiguities -> approaching then the situation we have in code solution
- Static observations before and after kinematic measurement?
  - YES, but… Ambiguities can be solved but what if you have a cycle slip in the middle of kinematic?
- Better to solve for on-the-fly (OTF)
- Smoothing code with carrier until ambiguities can be solved
- How to restrict the search space

Example: Code uncertainty 2 m, seek length 4 m = 20 wavelengths. 8 satellites = 7 double differences = 20^7 possible integer combinations = 1 280 000 000



Ambiguity resolution

- Least squares solution gives ambiguities as real numbers
- The shorter the session, the more important it is to seek the integers
- In kinematic measurement a must



Dach et al 2015

DOP = Dilution of Precision

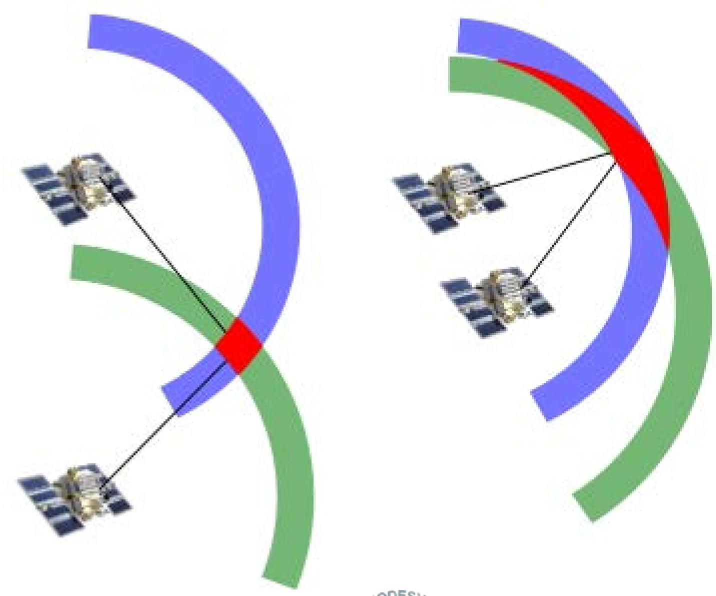
$$(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1} = \begin{pmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{pmatrix}$$

$$\begin{cases} \mathrm{GDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}} \\ \mathrm{PDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz}} \\ \mathrm{TDOP} = \sqrt{q_{tt}} \end{cases}$$

Measure of the satellite geometry; affects solution uncertainty

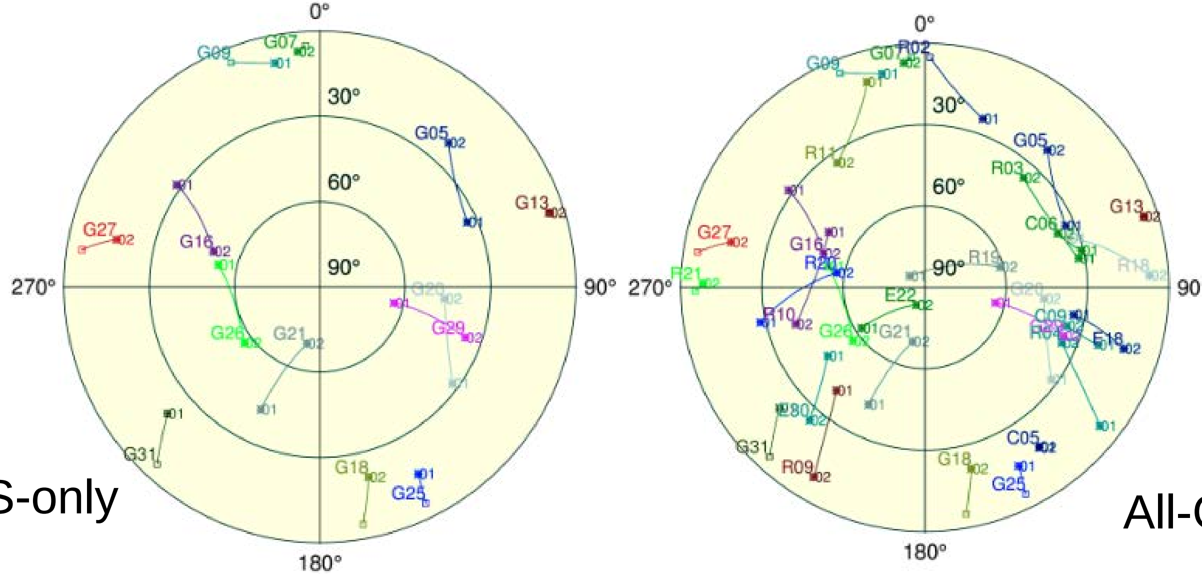
”Rule-of-thumb:”

<2	Excellent
2 – 5	Good
5 – 10	Fair / Still usable
10 – 20	Poor, avoid unless you’re desperate
> 20	Better to forget the whole thing



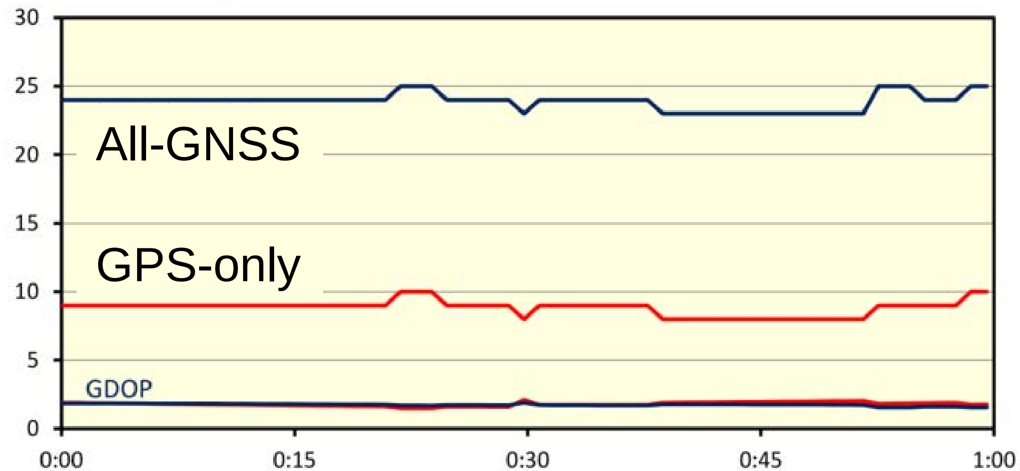
We do not need observations for DOP

DOP = Dilution of Precision

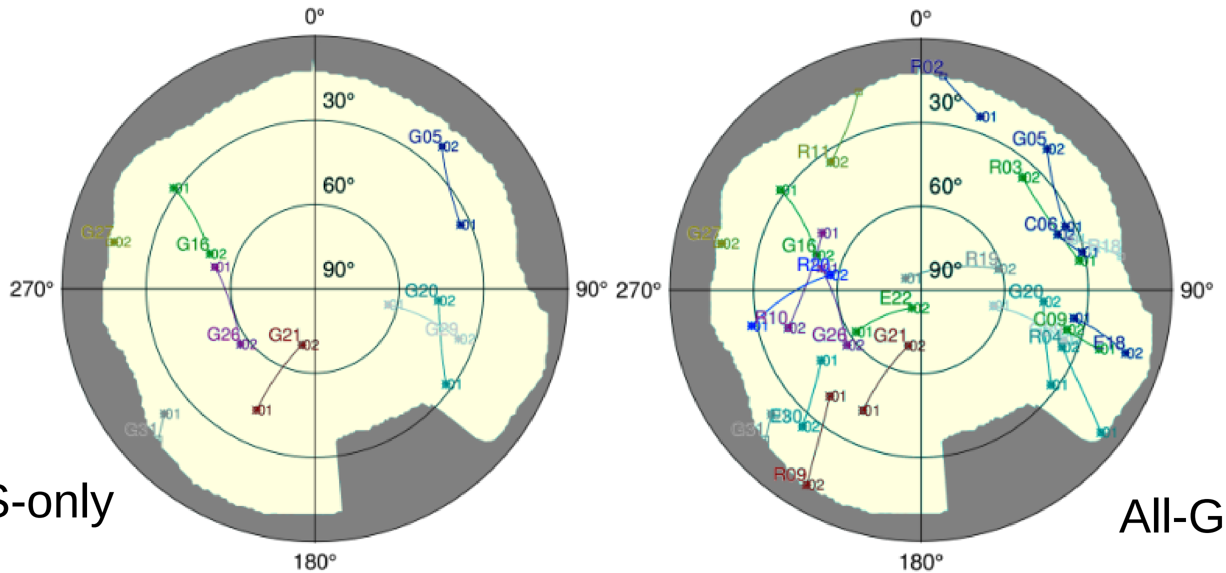


GPS-only

All-GNSS

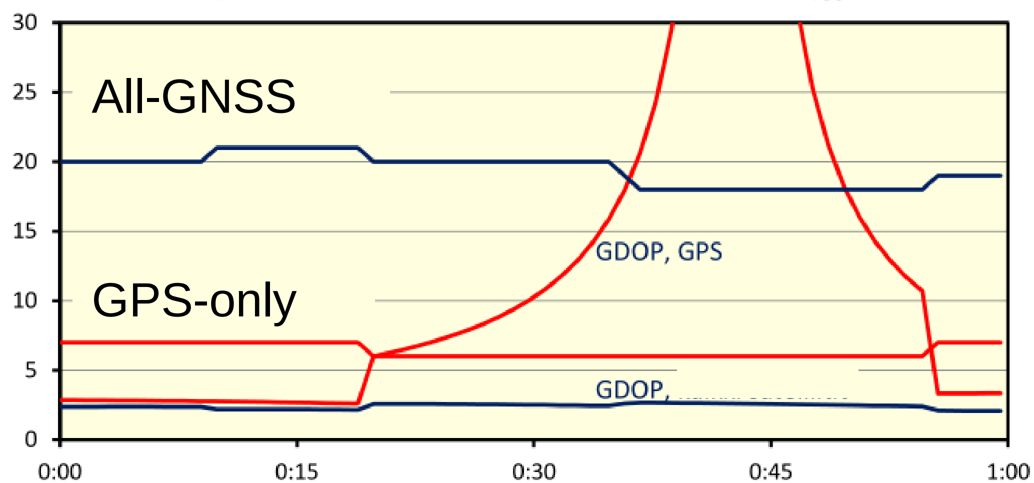


DOP = Dilution of Precision



GPS-only

All-GNSS



DOP = Dilution of Precision

	X	Y	Z
Mets	2892.570	1311.843	5512.634
PRN 2	-21785.163	13532.811	-5589.180
PRN 5	-19971.284	5358.610	16710.088
PRN 21	10449.520	12258.497	21863.323
PRN 26	25036.026	3300.852	8305.403
PRN 29	4867.645	25920.216	3355.990
PRN 31	21506.405	-2994.918	-15432.550

$$\mathbf{A} = \begin{pmatrix} -0.8311 & 0.4116 & -0.3739 & -1.0000 \\ -0.8869 & 0.1570 & 0.4344 & -1.0000 \\ 0.3585 & 0.5193 & 0.7757 & -1.0000 \\ 0.9882 & 0.0888 & 0.1246 & -1.0000 \\ 0.0797 & 0.9930 & -0.0870 & -1.0000 \\ 0.6566 & -0.1519 & -0.7388 & -1.0000 \end{pmatrix}$$

$$\mathbf{Q} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}$$

$$\mathbf{Q} = \begin{pmatrix} 0.3517 & 0.1500 & 0.0098 & 0.0721 \\ 0.1500 & 1.4180 & -0.2751 & 0.4798 \\ 0.0098 & -0.2751 & 0.7263 & -0.0756 \\ 0.0721 & 0.4798 & -0.0756 & 0.3307 \end{pmatrix}$$

$$\mathrm{GDOP} = \sqrt{\mathrm{Tr}\ \mathbf{Q}} = 1.7$$

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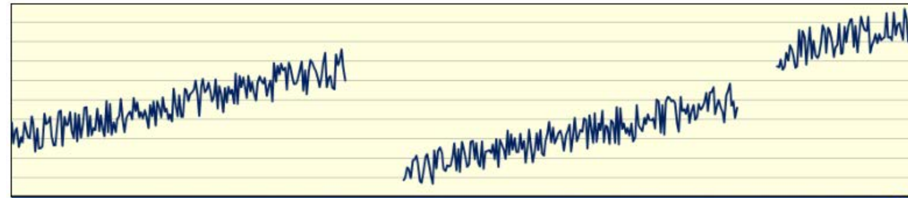
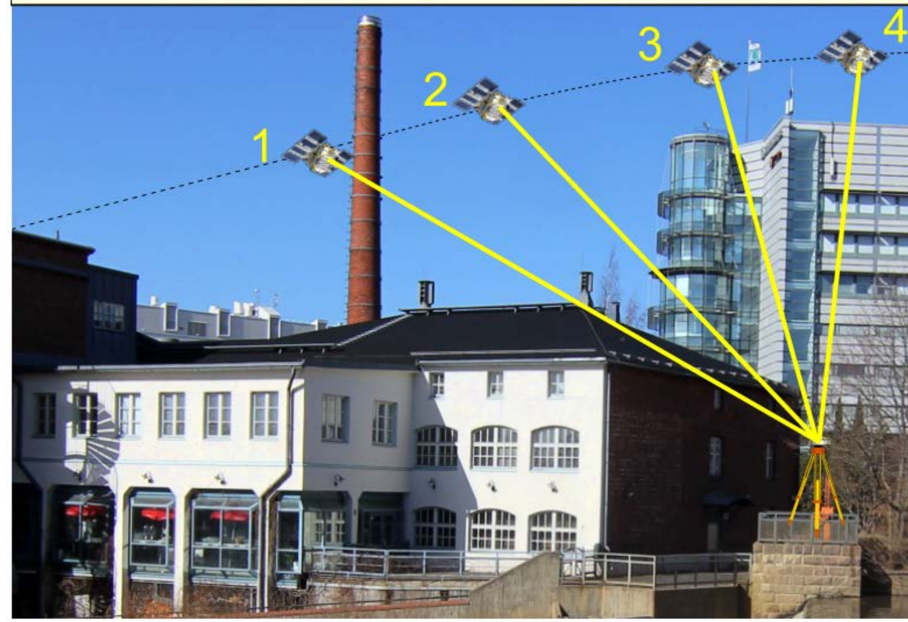
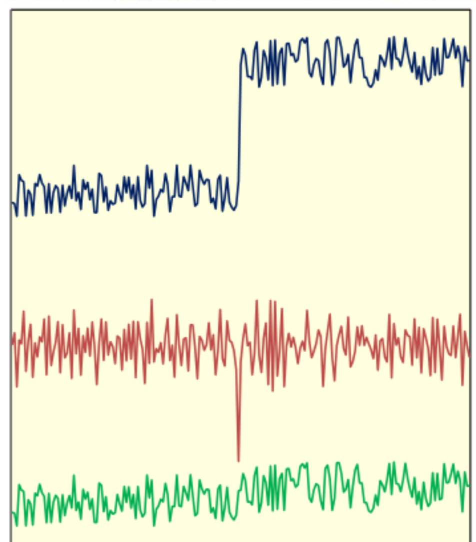
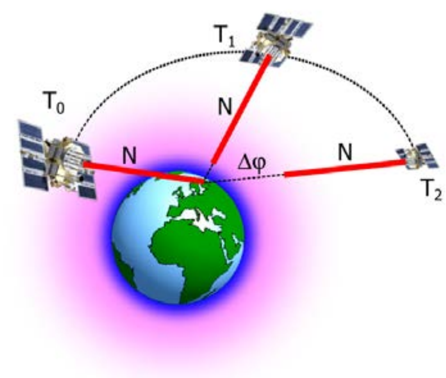
Cycle slips

- Receiver lost the lock to the satellite
- After recovering the connection, a new integer ambiguity is created
- On phase one sees a jump = **cycle slip**
- New unknown in the observation equation
- Must be removed before computation
- Epoch-wise differences to detect and remove a slip









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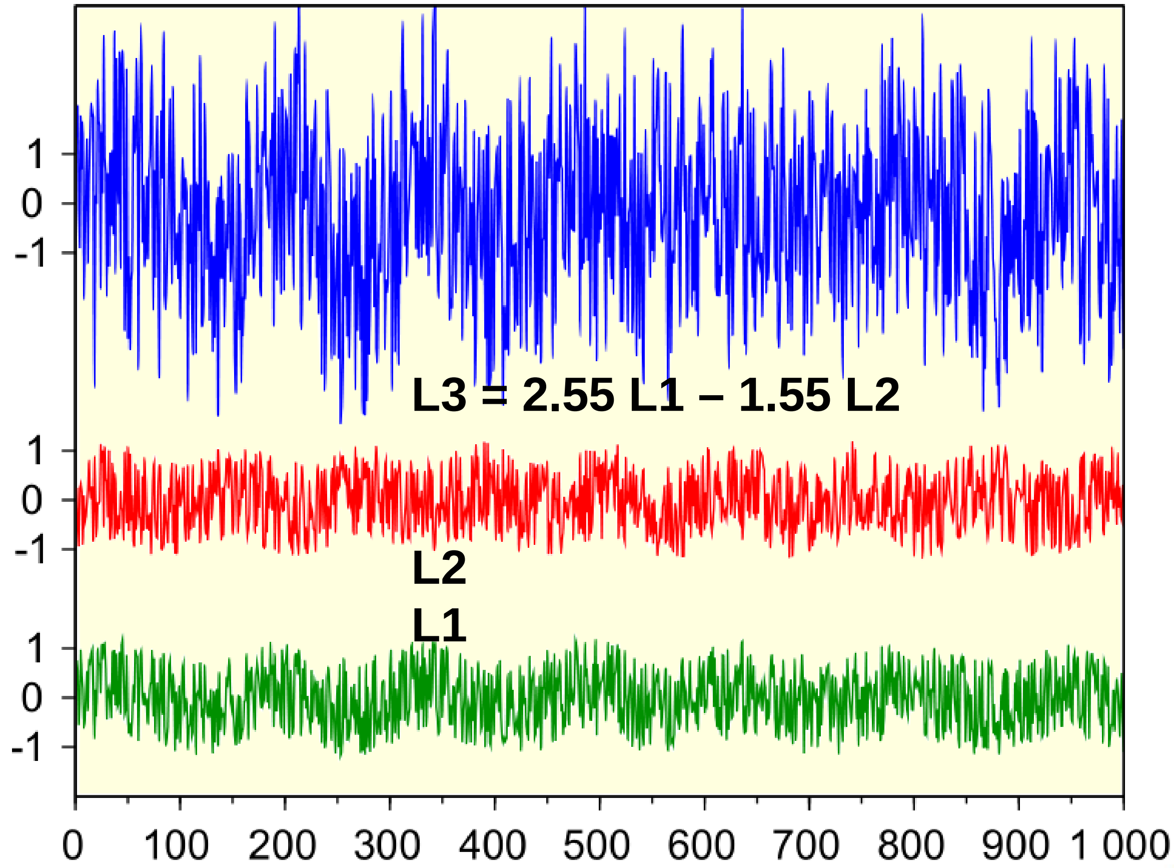
Combinations and differences

Signal and noise

- Signals contain random noise
- Signal-to-Noise ratio S/N affects the results
- Every operation on a signal tends to increase noise thus making the S/N ratio worse
- Avoid unnecessary operations and consider pros and cons
- You have been warned



Benefit of combining signals

$$L = (N + \Delta\varphi)\lambda \boxed{+ c\delta t + \Delta^{\text{ion}} + \Delta^{\text{trop}} + \ldots}$$

- The pseudorange equation contains that part we want to get rid of
- Some sources of error can be modelled, some not
- Combining frequencies, receivers, satellites, epochs,… we can get rid of or mitigate the effect of the error source
- We have to pay the price to the Mother Nature: S/N is getting worse
- Each time must be considered whether the operation is useful or not and will it improve our solution or not

Ionosphere-free combination L3

$$\begin{cases} L_1 = (N_1 + \Delta\varphi_1)\lambda_1 + c\delta t + \Delta_1^{\text{ion}} + \Delta^{\text{trop}} + \ldots \\ L_2 = (N_2 + \Delta\varphi_2)\lambda_2 + c\delta t + \Delta_2^{\text{ion}} + \Delta^{\text{trop}} + \ldots \end{cases}$$

$$L = x_1 L_1 + x_2 L_2$$

$$\Delta^{ion} \propto \frac{TEC}{f^2}$$

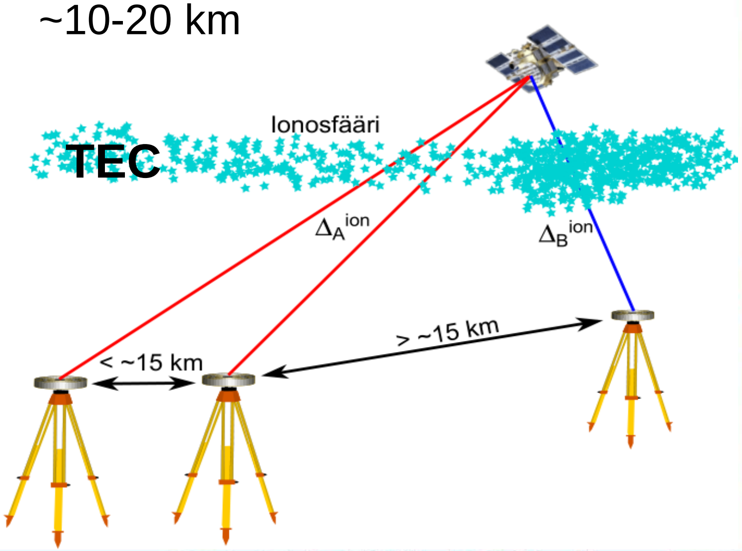
TEC = Total Electron Content
Describes ionospheric activity

$$x_1 \Delta_1^{ion} + x_2 \Delta_2^{ion} = 0 \quad \Rightarrow$$

$$x_1 = \frac{f_1^2}{f_1^2 - f_2^2}; \quad x_2 = -\frac{f_2^2}{f_1^2 - f_2^2}$$

$$\boxed{L3 = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2}$$

- Ionosphere refraction depends on the signal frequency
- Multiplying two frequencies by suitable numbers and adding signals together, we can eliminate the ionosphere
- Noise is 3 times bigger than that of the original signal
- Use when receiver distances > ~10-20 km

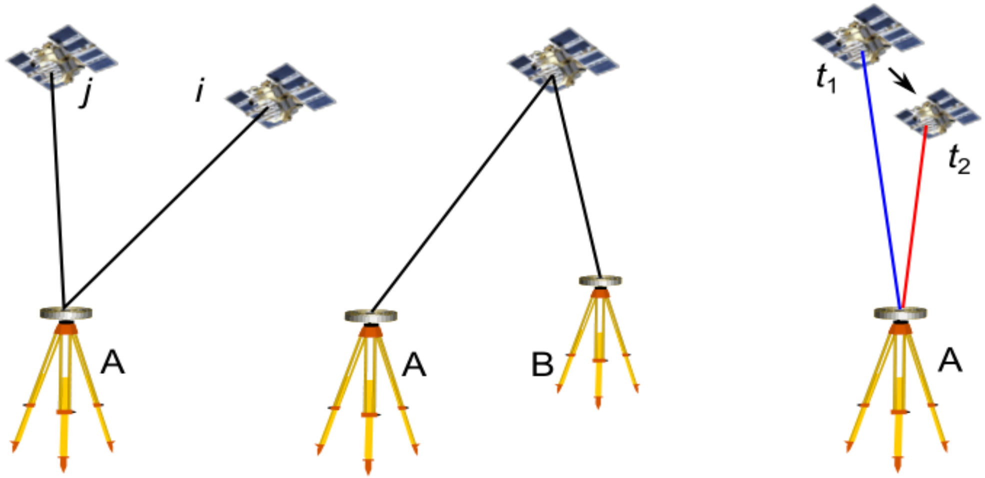


Single Difference

Satellite – satellite difference

Receiver – receiver difference

Epoch – epoch difference



$$\Delta L^i_{AB} = \lambda(\varphi^i_B + N^i_B) + d\rho^i_B + c\Delta\delta^i_B + \Delta^{\mathrm{ion},i}_B + \Delta^{\mathrm{trop},i}_B + \Delta_{etal} - [\lambda(\varphi^i_A + N^i_A) + d\rho^i_A + c\Delta\delta^i_A + \Delta^{\mathrm{ion},i}_A + \Delta^{\mathrm{trop},i}_A + \Delta_{etal}]$$

$$= \lambda(\varphi^i_{AB} + N^i_{AB}) + d\rho^i_{AB} + c\Delta\delta_{AB} + \Delta^{\mathrm{ion},i}_{AB} + \Delta^{\mathrm{trop},i}_{AB} + \Delta_{etal,AB}$$

- In relative measurements, the receiver – receiver difference is used
- Eliminates or reduces: satellite orbit and clock errors, ionosphere and troposphere refraction
- Does not affect on receiver clock errors, near-receiver errors like multipath or antenna phase center variation

Double Difference

Difference of two single differences

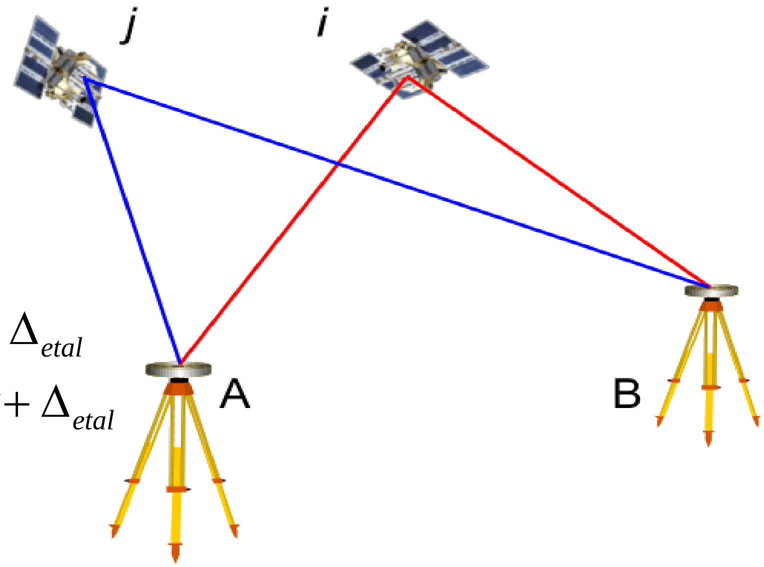
$$\nabla\Delta L_{AB}^{ij} = \Delta L_{AB}^{j} - \Delta L_{AB}^{i}$$

$$= \lambda(\varphi_{AB}^{j} + N_{AB}^{j}) + d\rho_{AB}^{j} + c\Delta\delta_{AB} + \Delta_{AB}^{\text{ion,j}} + \Delta_{AB}^{\text{trop,j}} + \Delta_{etal}$$

$$-\lambda(\varphi_{AB}^{i} + N_{AB}^{i}) + d\rho_{AB}^{i} + c\Delta\delta_{AB} + \Delta_{AB}^{\text{ion,i}} + \Delta_{AB}^{\text{trop,i}} + \Delta_{etal}$$

$$= \lambda(\varphi_{AB}^{ij} + N_{AB}^{ij}) + d\rho_{AB}^{ij} + \Delta_{AB}^{\text{ion,ij}} + \Delta_{AB}^{\text{trop,ij}} + \Delta_{etal}$$

$$\approx \lambda(\varphi_{AB}^{ij} + N_{AB}^{ij})$$



- Eliminates receiver clock errors
- Ionosphere, troposphere, … errors getting smaller
- Integer ambiguities remain
- Double difference is widely used for GNSS computation

Triple difference

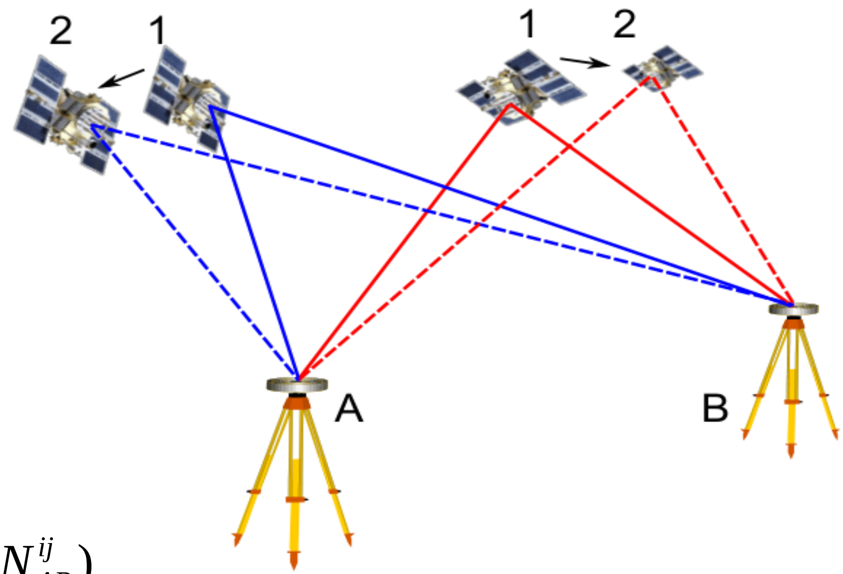
Difference of two double differences between two epochs

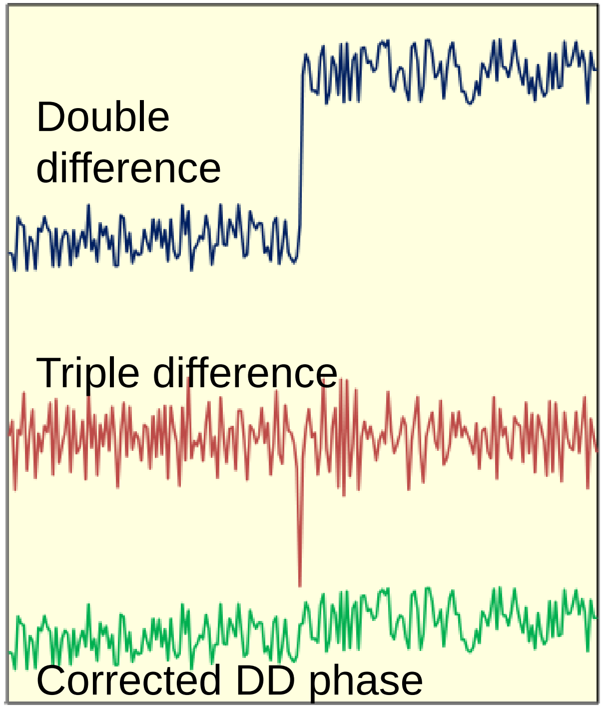
$$\mathbf{D3}L_{AB}^{ij}(t_{12}) = \nabla\Delta L_{AB}^{ij}(t_2) - \nabla\Delta L_{AB}^{ij}(t_1)$$

$$= \lambda(\varphi_{AB}^{ij}(t_2) + N_{AB}^{ij}) - \lambda(\varphi_{AB}^{ij}(t_1) + N_{AB}^{ij})$$

$$= \lambda\varphi_{AB}^{ij}(t_{12})$$

- Eliminates the integer ambiguity!
- Used for seeking and estimating the cycle slips in phase observations
- Noise is increased, therefore not used for final computations





Undifferenced phase observations

- Advantage of differences: error sources eliminated or minimized
- Disadvantage of differences: correlation between stations, propagation of common errors (or is this good??), amount of computation in large networks

- Undifferenced phases (used. e.g. in PPP)
  - Error sources affect on their full power
  - Need to use: precise satellite orbits, precise clock corrections, ionosphere elimination, troposphere elimination/modelling,…
  - No reference stations, coordinates in the reference frame of satellites at the epoch of observations
  - Advantage especially in large networks

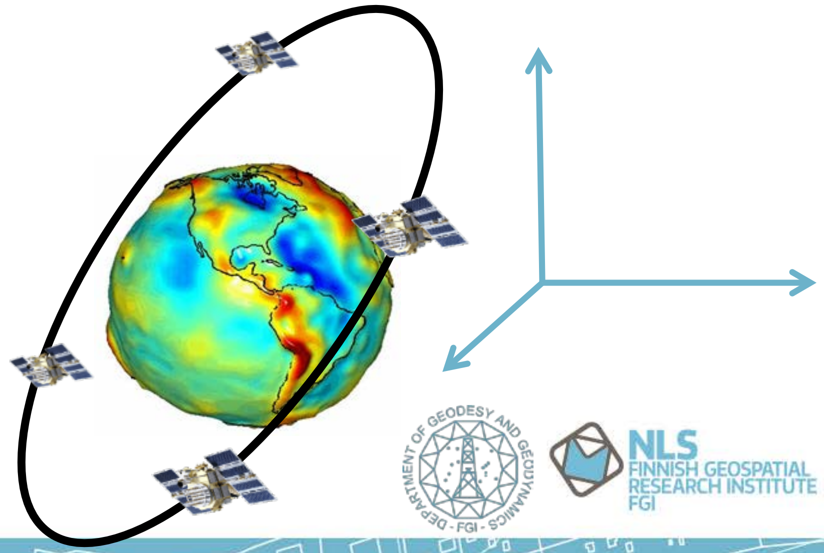
Reference Frames

Celestial and Terrestrial Reference Frames

- **Celestial Reference Frame** (CRF), maintained by VLBI observations. E.g. GNSS satellite orbits are computed in this frame
- **Terrestrial Reference Frame** (TRF), major contribution from GNSS, but also VLBI, SLR, DORIS. Positions on the Earth are given in an Earth-bound Frame.
- Linked together with the **Earth Orientation Parameters** (EOP) Precession/nutation + Polar motion + UT1

Celestial frame will not rotate with Earth
Terrestrial frame is co-rotating with Earth

- We need the EOP (in real time) to get connection between CRF and TRF
- This is transparent to users who work in TRF; even GNSS precise orbits (IGS) are given in ITRF



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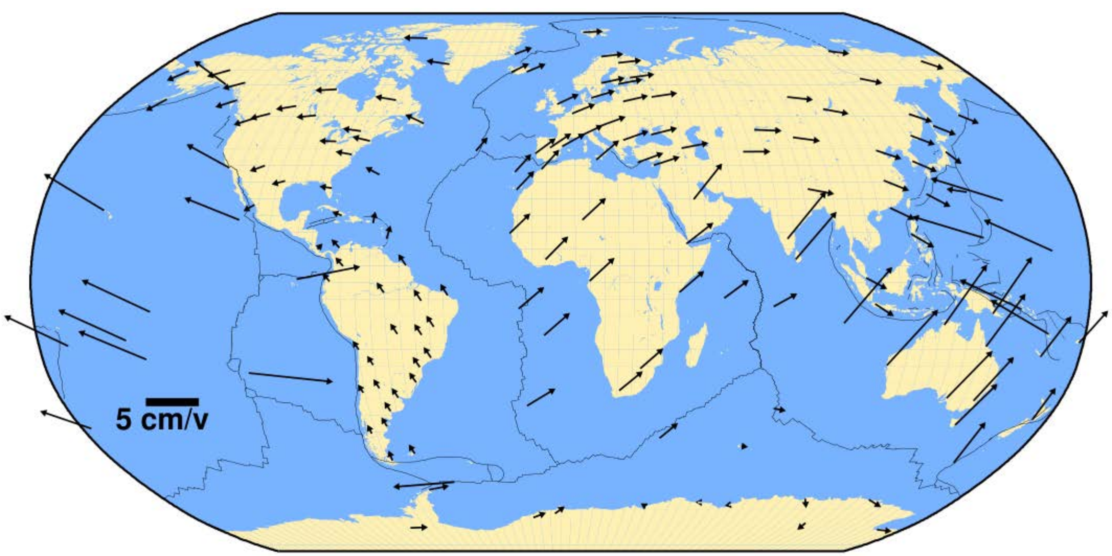
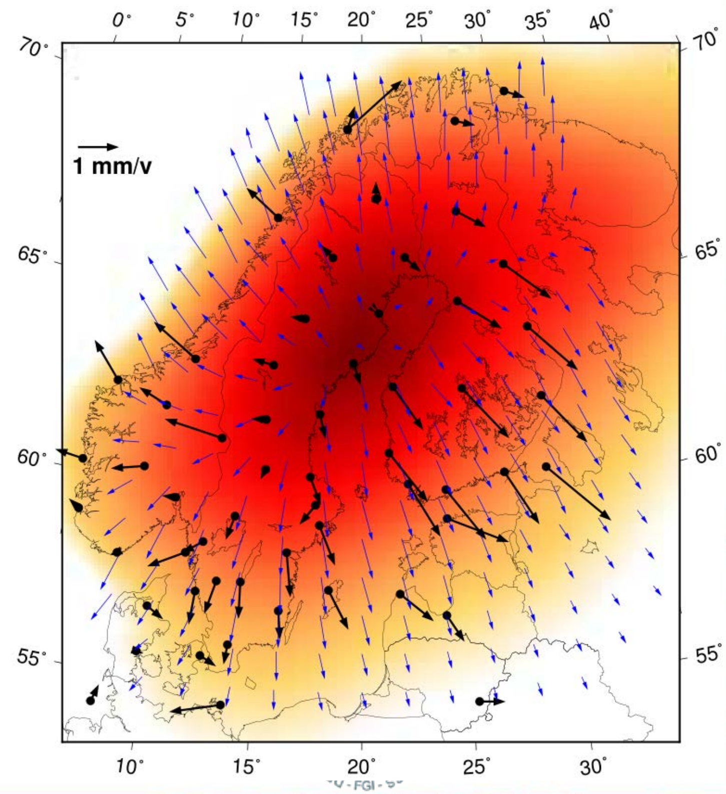
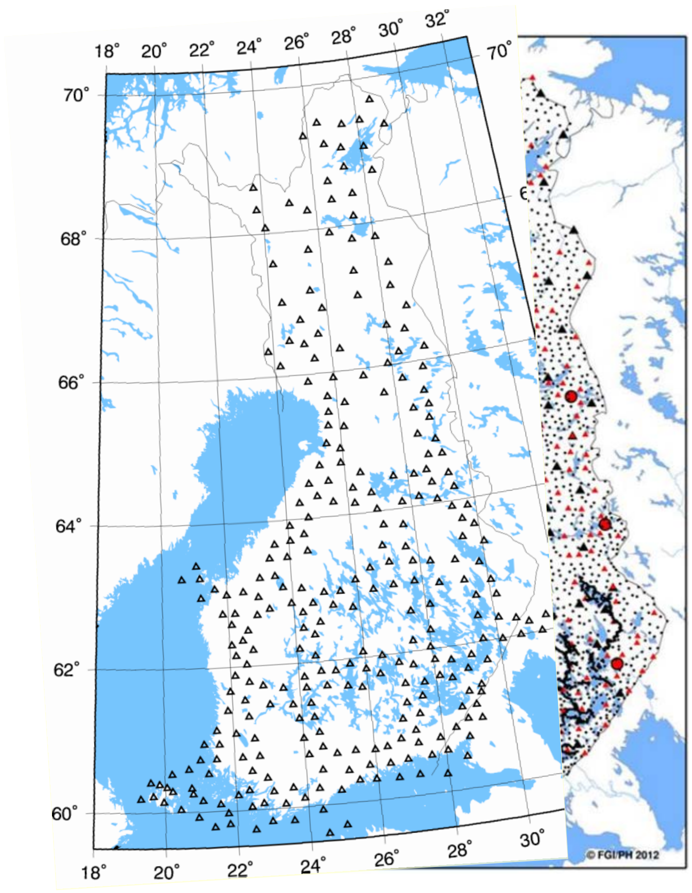
From global to local reference frame

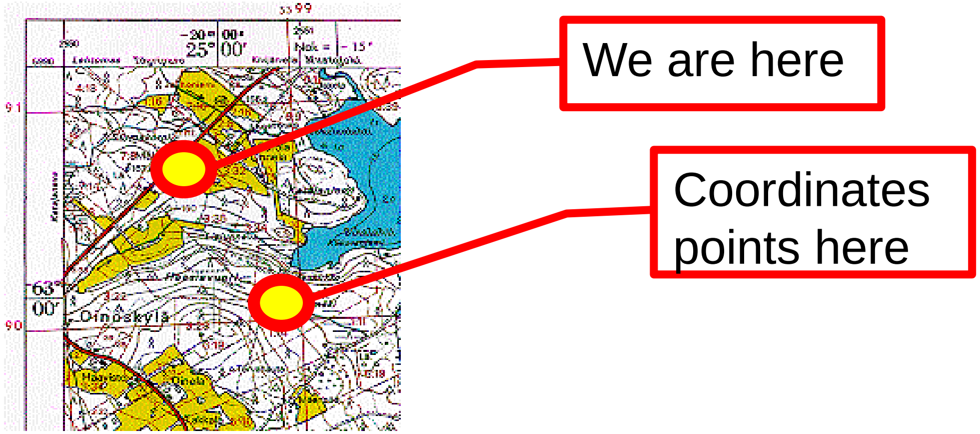
- The very basic issue in GNSS positioning:
  - Measurements are made in a global reference frame (ITRFxx)
  - Results are needed in a local reference frame, e.g. Jokkmokk municipality reference frame
- How we can do this?
- Depends on …





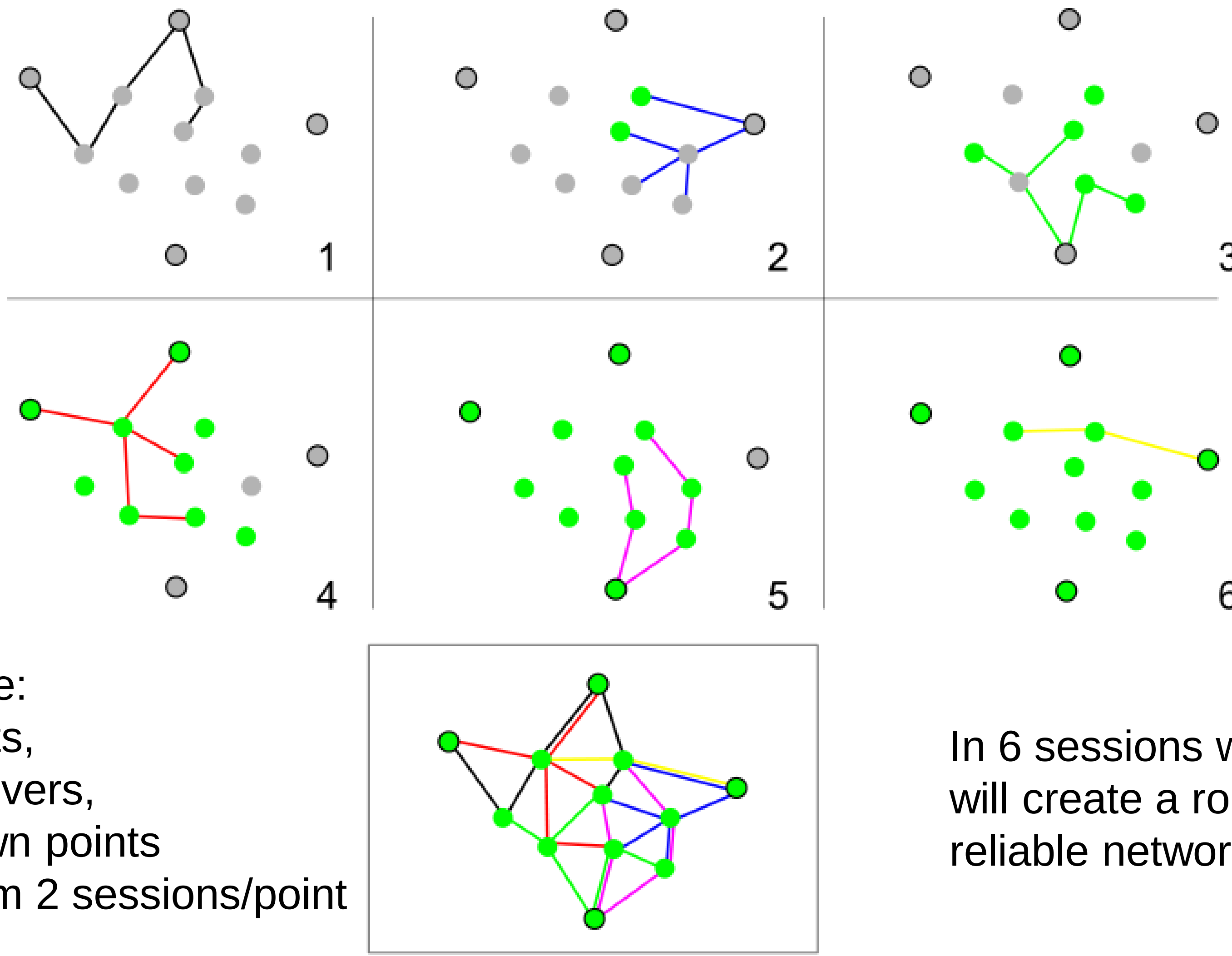
From local frame to local frame





- Adjusting the old coordinates to match with the new reference frame
- Art of coordinate transformation and uncertainties of the transformation
- Local crustal movements

Case 1: Network solution, step 1

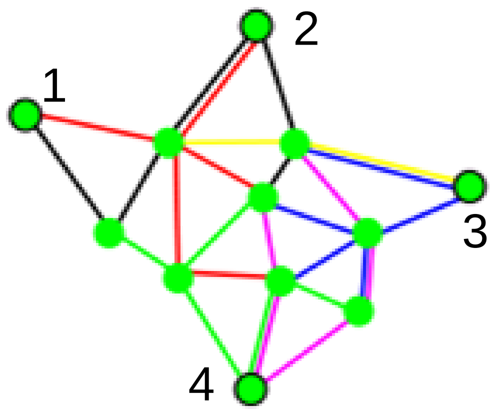


Example:
12 points,
6 receivers,
4 known points
Minimum 2 sessions/point

In 6 sessions we will create a robust reliable network

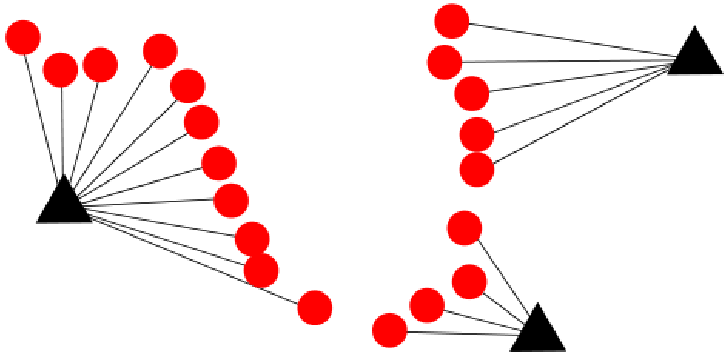
Case 1: Network solution, step 2

- Coordinates of 1, 2, 3 and 4 are known both in old local frame and in the frame used for the GNSS measurement -> transformation parameters can be determined
- Minimum of three common points are needed
- Residuals of the fit will give an estimation of the adjustment uncertainty
- New points can be adjusted in the existing reference frame with a known uncertainty
- No need to apply crustal deformation, global tectonic movements or epoch-related changes
- Residuals will depend on the coordinate transformation applied; best in small networks



Case 2: RTK-type configuration

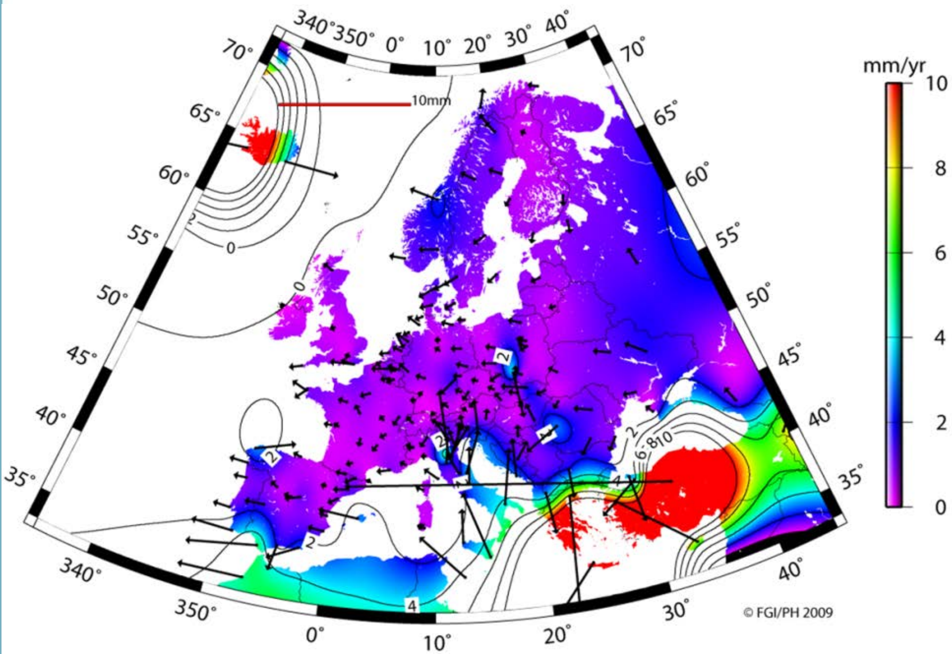
- Reference stations have coordinates in local frame and in the frame used for the GNSS measurement
- No network, new points cannot be adjusted in the local frame
- External information is needed: coordinate transformation to the local reference frame and information on the crustal deformation to make the transformation from one epoch into another
- Reference stations can be used for controlling the transformation

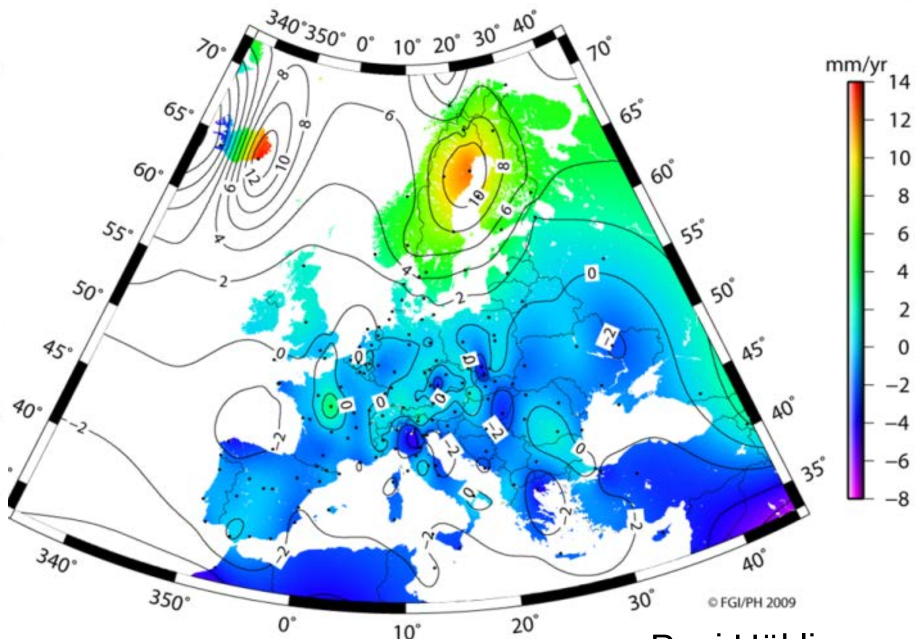


Case 3: PPP-type configuration

- Coordinates are measured without reference stations
- No connection to the local reference frame, no information on regional crustal deformation -> models
- If the uncertainty of the deformation is, say, 0.5 mm/y, extrapolation over 10 years will introduce a 5 mm uncertainty in the coordinate values
- Additionally, uncertainty on the transformation from the global reference frame of the satellites (ITRFxx, epoch of the date) in the epoch of the local frame plus transformation from global to local

Crustal deformation within a continent





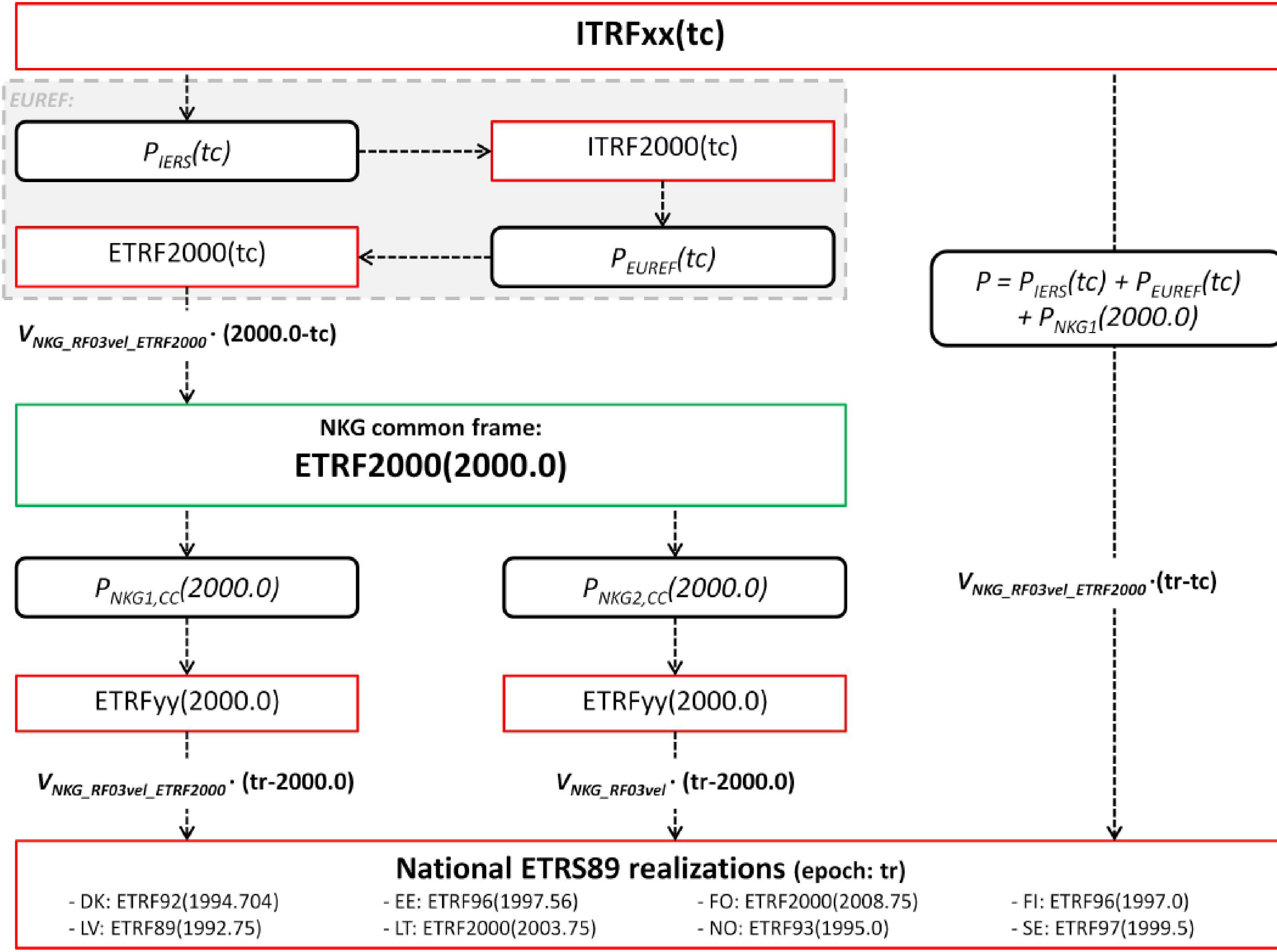
Pasi Häkli

Horizontal and vertical deformation of Eurasian plate. There are large differences within the continent. No single model can describe the motion.

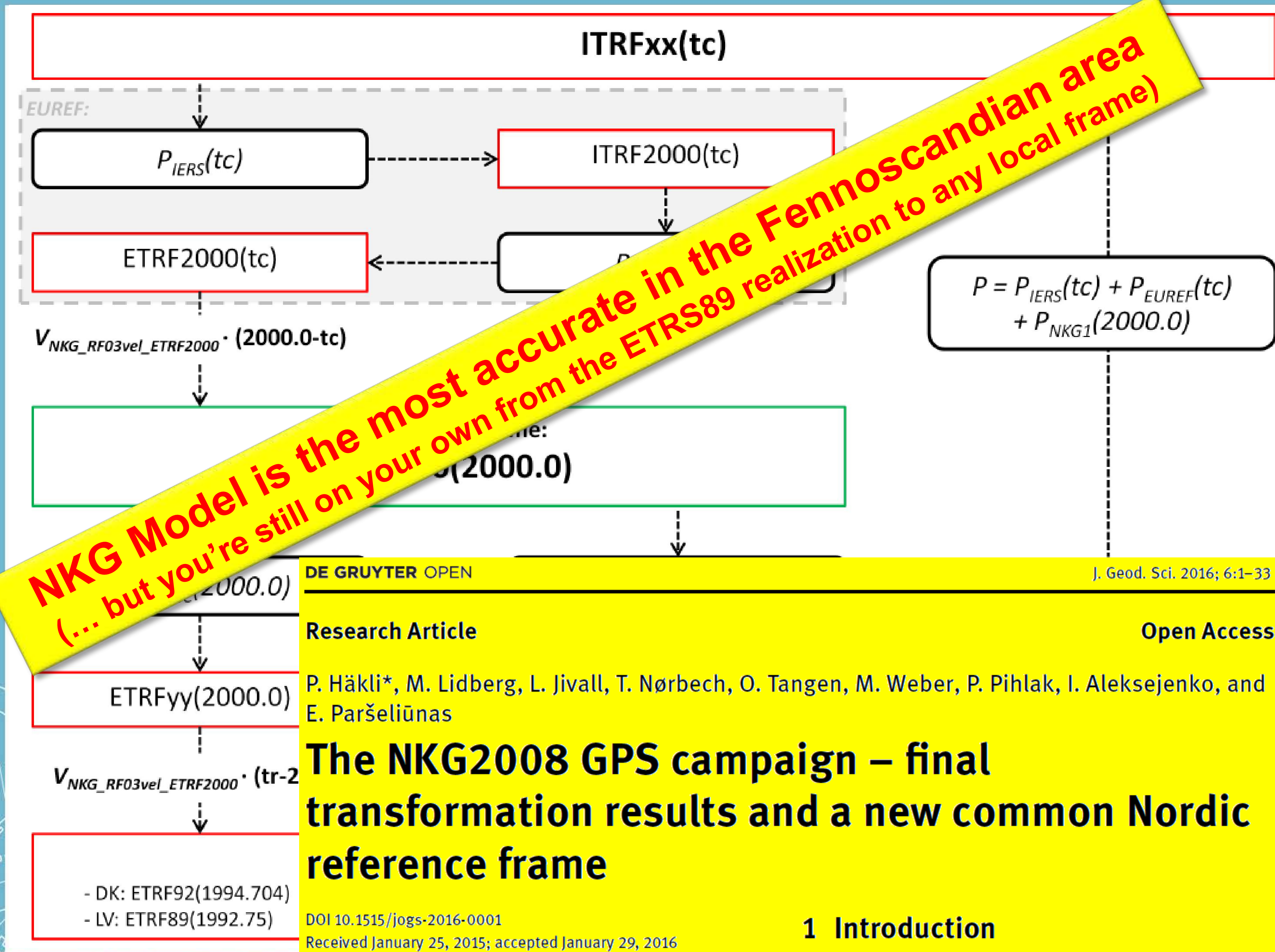
Continuous monitoring the motion, improving models, updating reference systems,…

Transformations

- **Computation should be done in the same reference frame as the satellite orbits are given**
  - IGS precise orbits: ITRF2008 / IGS08 (soon ITRF2014)
  - Broadcast ephemeris: WGS84 (but it is within a few cm the same as ITRF2008)
  - National realisations of ETRS89
    (These are close enough of global, so that in smaller networks and navigation, RTK &c one can directly measure in ETRFyy)
- Transformation to the reference frame of the user
  - Epoch shift (crustal motions)
  - Reference frame difference (geocentric)
  - Reference frame difference 2 (old non-geocentric systems)



ITRFxx(tc)
EUREF:
$P_{IERS}(tc)$
ITRF2000(tc)
ETRF2000(tc)
$P_{EUREF}(tc)$
$V_{NKG\_RF03vel\_ETRF2000} \cdot$ (2000.0-tc)
$P = P_{IERS}(tc) + P_{EUREF}(tc) + P_{NKG1}(2000.0)$
NKG common frame:
ETRF2000(2000.0)
$P_{NKG1,CC}(2000.0)$
$P_{NKG2,CC}(2000.0)$
$V_{NKG\_RF03vel\_ETRF2000} \cdot$(tr-tc)
ETRFyy(2000.0)
ETRFyy(2000.0)
$V_{NKG\_RF03vel\_ETRF2000} \cdot$ (tr-2000.0)
$V_{NKG\_RF03vel} \cdot$ (tr-2000.0)
National ETRS89 realizations (epoch: tr)
- DK: ETRF92(1994.704)
- EE: ETRF96(1997.56)
- FO: ETRF2000(2008.75)
- FI: ETRF96(1997.0)
- LV: ETRF89(1992.75)
- LT: ETRF2000(2003.75)
- NO: ETRF93(1995.0)
- SE: ETRF97(1999.5)



ITRFxx(tc)
EUREF:
P_IERS(tc)
ITRF2000(tc)
ETRF2000(tc)
P = P_IERS(tc) + P_EUREF(tc) + P_NKG1(2000.0)
V_NKG_RF03vel_ETRF2000 · (2000.0-tc)
(2000.0)
ETRFyy(2000.0)
V_NKG_RF03vel_ETRF2000 · (tr-2
- DK: ETRF92(1994.704)
- LV: ETRF89(1992.75)
NKG Model is the most accurate in the Fennoscandian area
(… but you're still on your own from the ETRS89 realization to any local frame)
DE GRUYTER OPEN
J. Geod. Sci. 2016; 6:1–33
Research Article
Open Access
P. Häkli*, M. Lidberg, L. Jivall, T. Nørbech, O. Tangen, M. Weber, P. Pihlak, I. Aleksejenko, and E. Paršeliūnas
The NKG2008 GPS campaign – final transformation results and a new common Nordic reference frame
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Received January 25, 2015; accepted January 29, 2016
1 Introduction

ETRS89 realizations

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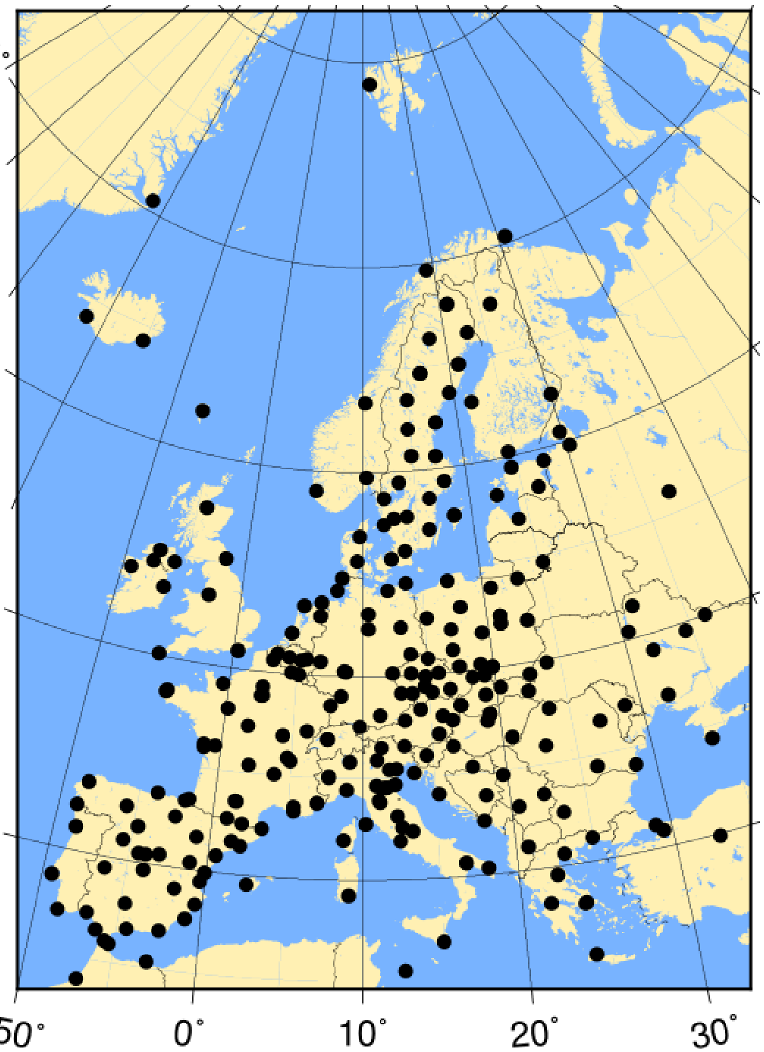
Table 1: Nordic and Baltic ETRS89 realizations.

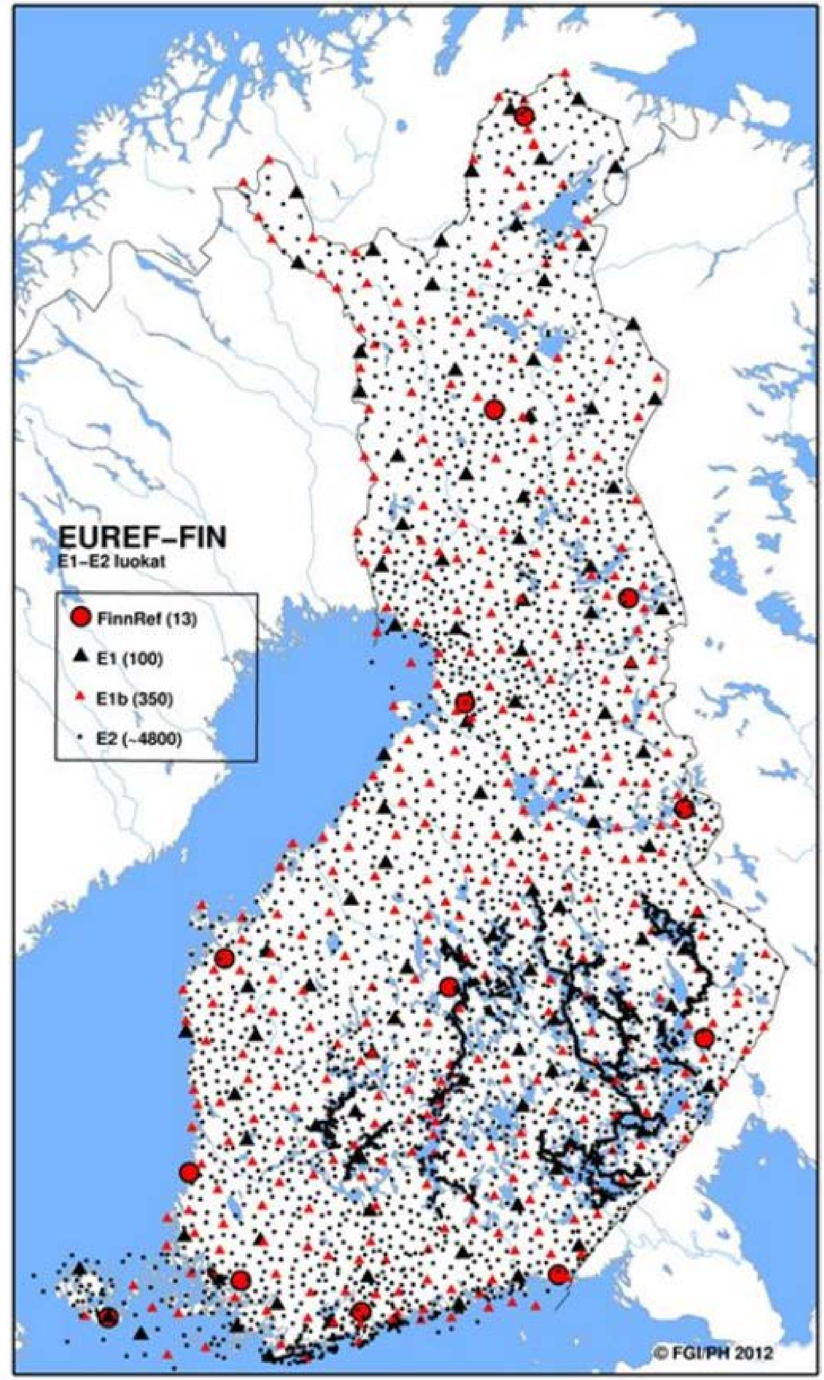
Country	Country ID	Name of realization	ETRF version	Realization epoch
Denmark	DK	EUREF-DK94	ETRF92	1994.704
Estonia	EE	EUREF-EST97	ETRF96	1997.56
Faroe Islands	FO		ETRF2000	2008.75
Finland	FI	EUREF-FIN	ETRF96	1997.0
Latvia	LV	LKS-92	ETRF89	1992.75
Lithuania	LT	EUREF-NKG-2003	ETRF2000	2003.75
Norway	NO	EUREF89	ETRF93	1995.0
Sweden	SE	SWEREF99	ETRF97	1999.5

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRF2000} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRFxx} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + D \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRFxx} + \begin{bmatrix} 0 & -R_3 & R_2 \\ R_3 & 0 & -R_1 \\ -R_2 & R_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRFxx}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ETRF2000} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRF2000} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{R}_3 & \dot{R}_2 \\ \dot{R}_3 & 0 & -\dot{R}_1 \\ -\dot{R}_2 & \dot{R}_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{tc}^{ITRF2000} \cdot (t_c - 1989.0) \quad (2)$$

Active vs. passive reference points





Active vs. passive reference points

- **Passive points**: traditional surveying markers attached on stable surface (bedrock, boulders, buildings, large constructions…)
- Keeping their coordinates fixed and measuring new points relative to them will give you the new points also in the same reference frame
- Network is deformed with time; periodical renewal necessary

- **Active points**: Permanent GNSS stations
- Coordinates up-to-date in global reference frame
- Models are needed to get coordinates in the local frame (e.g. ETRS89 national realization)

"*No single Nation can do this alone*" UN GA Resolution on Global geodetic reference frames
NKG has a 60+ years of experience and co-operation on this

Cadastre ≈ coordinate reference system + precise positioning + current registry information + legal issues

	Datum type	Precise positioning	Reference system in registers	Changes to current register	Legal issues
Now	Static	ETRS89 (regional)	ETRS89, EVRS	None	None
Near future	Semi-dynamic	ITRS (global)	ETRS89, EVRS	None	None(?)
Future?	Dynamic	ITRS (global)	ITRS, EVRS(?)	Positions: N,E,H →X,Y,Z,VX,VY,VZ,t	Changing coordinates

Compiled by Pasi, Hannu and Markku 08/2016

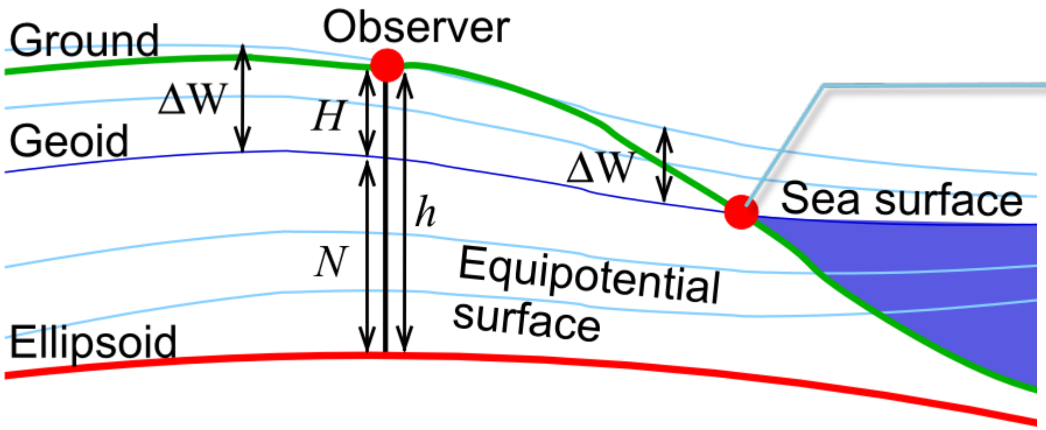
Height

Water is always flowing downhill

…but GNSS does not know it

M.C. Escher

Geoid and GNSS



W_0 = agreed value for the geopotential of the geoid

- Heights *H* are based on gravity-related equipotential surfaces
- One surface has a special meaning, the **geoid**
- Heights *H* refer to the geoid which coincides the sea surface (not quite true… but that is another story)
- GNSS-related heights *h* refer to the ellipsoid
- To connect these two, one needs a **geoid model** giving the height of the geoid *N* from the ellipsoid

$$H = h - N$$

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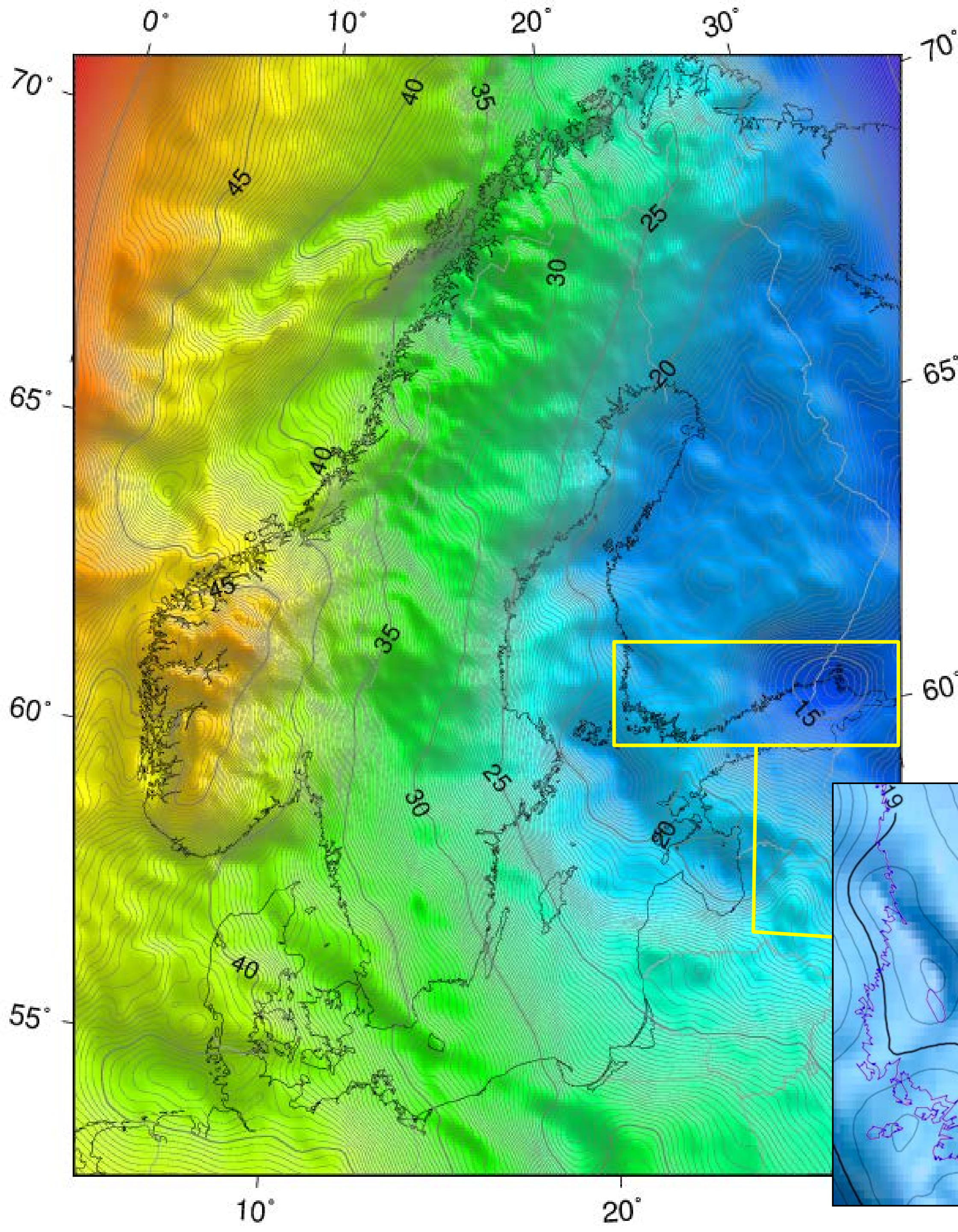
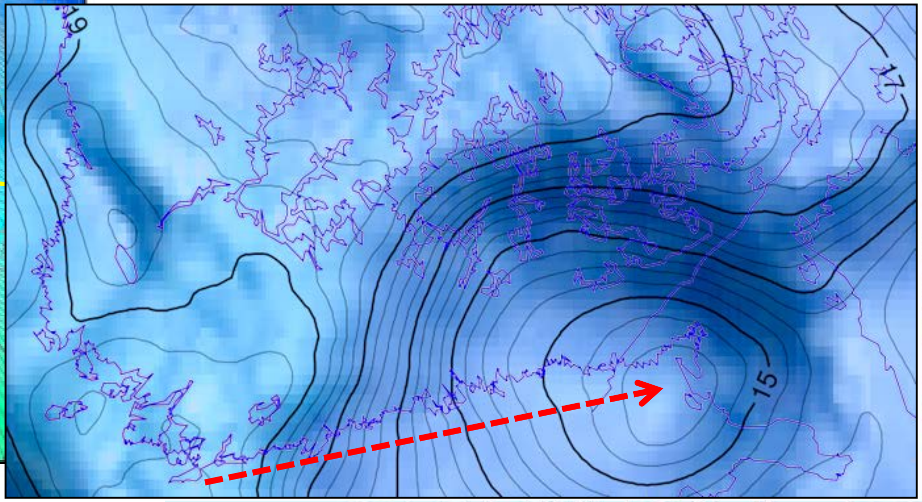


Geoid and GNSS

- An example at the Gulf of Finland
- Going from Hangö to Vyborg GNSS claims the sea surface being more than 4 m lower at Vyborg
- Levellers and the physical reality disagree



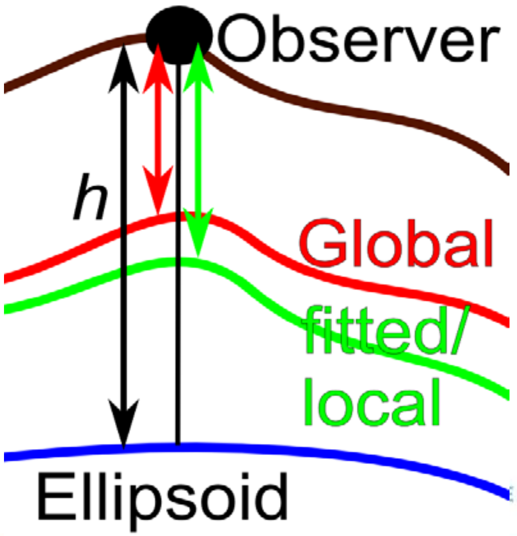
Geoid and GNSS

- Several geoid models available – regional and global ones
- Geoid models are improving all the time
- Differences between models even several decimeters
- Best and most accurate models in Fennoscandian area are the **NKG geoid models**
- They can be fitted to the National height system to get the heights directly in the national system:

$$\left.\begin{aligned} H_{\mathrm{gl}} &= h - N_{\mathrm{global}} \\ H_{\mathrm{NKG}} &= h - N_{\mathrm{NKG}} \\ H_{\mathrm{Local}} &= h - N_{\mathrm{fitted}} \end{aligned}\right\}$$

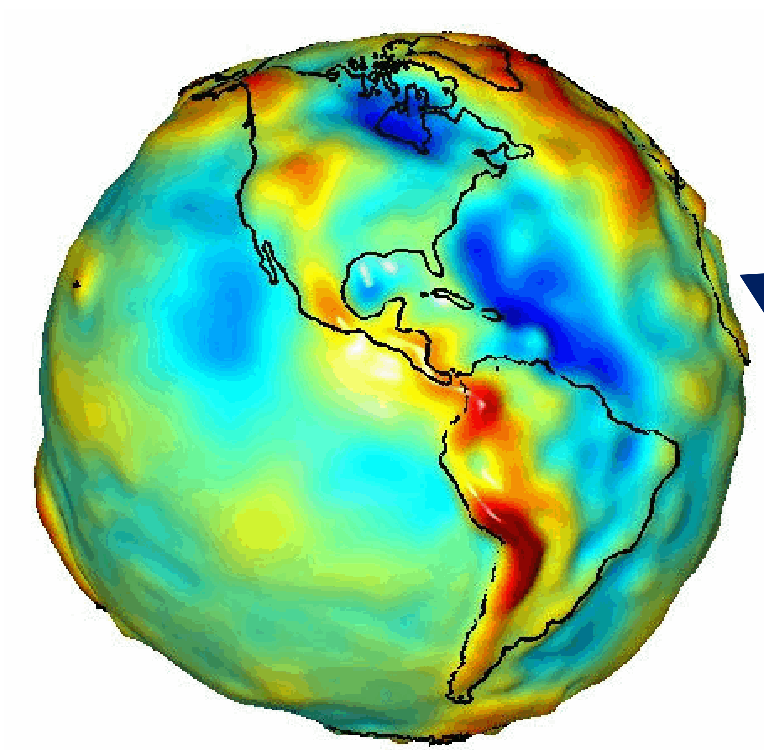
Your heights can be off by decimeters if you do not use a geoid model fitted in the local height system!

e.g. in Finland FIN2005N00 is fitted to the N2000 height system to give directly N2000 heights



Thank you for your attention!

”We have very precise coordinates but where on the Earth they point to?”



Epoch 2016.7

2892570.788 ± 0.001
1311843.445 ± 0.001
5512634.137 ± 0.001