Balanced Least Absolute Value Estimator and its applications in navigation problems

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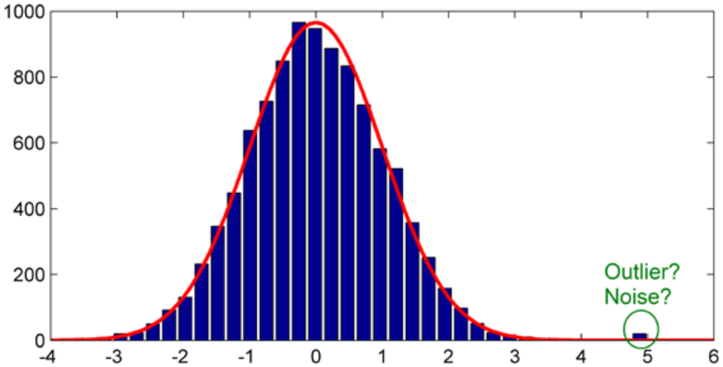
Introduction

Parameter estimation from noisy observations

"An outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs"



The user must specify the threshold for unexpected deviations

Robust estimators

Provide unbiased estimates even in presence of outliers

E.g. L1 estimator for direct observations – median is not influenced by outliers (example: 2 2 3 4 8 90 100)

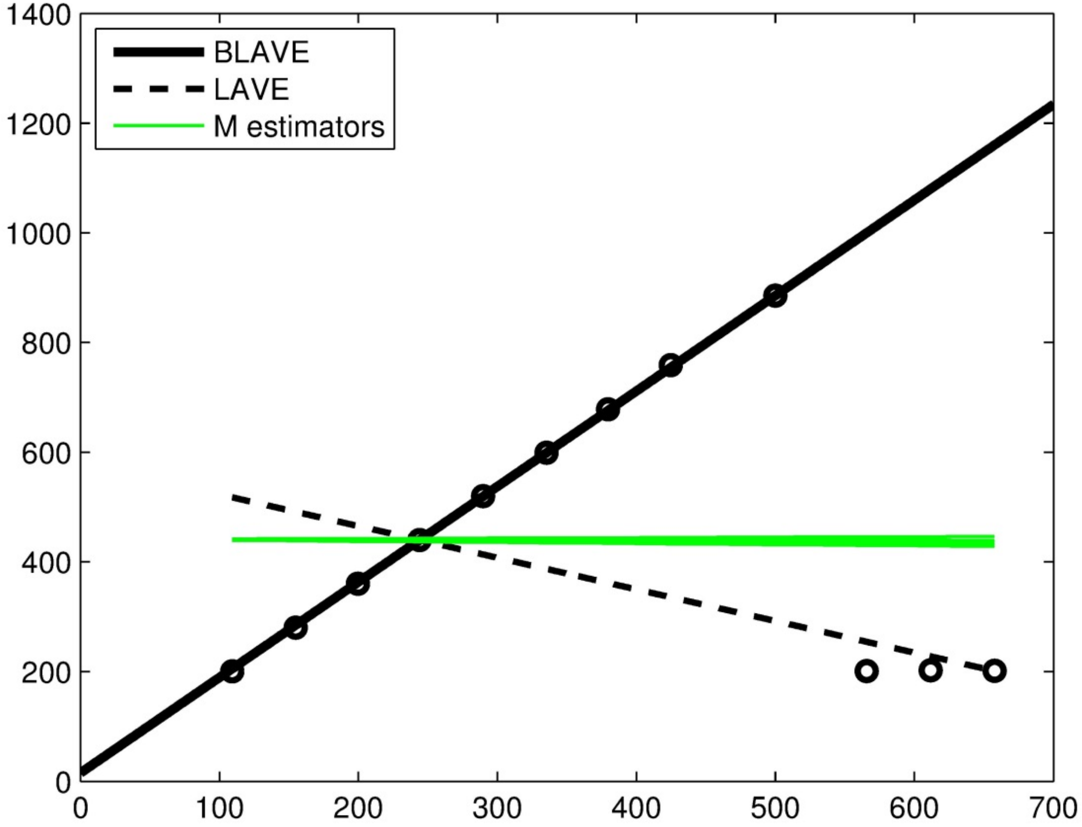
L2 estimator is not robust (mean ≈ 30)

M-estimators minimize a function of residuals (Huber, Cauchy,…) in iterative way

Estimators based on random sampling, e.g. RANSAC

L1 estimator = least absolute value, minimises L1 norm of residuals
RANSAC = RANdom SAmple Consensus

M-estimators vs BLAVE



Disadvantage of M-estimators:

- non-elegant (iterative procedure even for linear problems)

- do not work properly in certain situations

Balancing

Balancing: reweight the observations, so that they have equal influence on the estimates, i.e. $(\mathbf{C}_b)_{ii} = k/n$

$$\underset{n\times 1}{\mathbf{L}} + \underset{n\times 1}{\mathbf{v}} = \underset{n\times k}{\mathbf{A}}\,\underset{k\times 1}{\mathbf{X}}; \quad D(\mathbf{L}) = \sigma_0^2 \underset{n\times n}{\mathbf{I}} \quad \mathbf{L} + \mathbf{v} = \hat{\mathbf{L}} = \mathbf{CL} \qquad \mathbf{C} = \mathbf{A}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$$

Diagonal elements of "hat" matrix **C** - leverage values $0 \le C_{ii} \le 1$

If $C_{ii} = 1$, then the corresponding residual $v_{ii} = 0$

If $C_{ii} = 0$, then the corresponding observation has no influence on estimate of **X**, it is totally controlled by other observations

$$\mathbf{P}_G^{1/2}\mathbf{L} + \mathbf{P}_G^{1/2}\mathbf{v} = \mathbf{P}_G^{1/2}\mathbf{AX} \qquad \mathbf{L}_b + \mathbf{v}_b = \mathbf{A}_b\mathbf{X},\ D(\mathbf{L}_b) = \sigma_0^2 \underset{n\times n}{\mathbf{I}}$$

$$(\mathbf{C}_b)_{ii} = (\mathbf{P}_G^{1/2}\mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{P}_G\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}_G^{1/2})_{ii} = k/n$$

P_G – diagonal matrix of balancing factors

BLAVE

L1-estimate of balanced observations $\mathbf{L}_b + \mathbf{v}_b = \mathbf{A}_b\mathbf{X},\ D\left(\mathbf{L}_b\right) = \sigma_0^2\,\mathbf{I}_{n \times n}$

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \left\| \mathbf{A}_b\mathbf{X} - \mathbf{L}_b \right\|_1$$

has median-like properties: outliers do not influence $\hat{\mathrm{x}}$, the residuals can directly be tested for outliers

Solution by linear programming

The algorithm for computation of the balancing factors is based on QR-decomposition of the design matrix A and it can be found in Jurisch and Kampmann (1998).

The balancing factors exist only if there are no inner restrictions on the observations (straight or latent)

Estimation of camera motion parameters

Estimate rotation and unit translation between two consecutive images using feature matched features

Arrows show motion of features between two images

There is no "perfect" feature matching algorithm: outliers are unavoidable

Images:The KITTI Vision Benchmark Suite, http://www.cvlibs.net/datasets/kitti/raw_data.php

Inliers identified by BLAVE

Difference between ”true” and estimated motion parameters using 400 matched corners

“True” values from GNSS/INS

	RANSAC			BLAVE		
	f	r	d	f	r	d
Translation	0.006 -1.130	0.541 0.629	0.640 -0.879	0.006 (±0.001)	-0.088 (±0.001)	-0.015 (±0.001)
Rotation (deg)	4.608 -7.244	1.645 -1.151	3.849 -5.773	-0.090 (±0.002)	0.012 (±0.001)	0.103 (±0.003)
Time (s)	2.5 s			3.5 s		

Conclusions

RANSAC – standard tool for robust estimation in computer vision applications

Number of iterations (samples) must be chosen based on the expected portion of outliers

Samples can have bad geometry – difficult to identify inliers

BLAVE – elegant solution, processes all observations at once, no iterations (beside due to linearization) are required

Median-like properties, tolerates up to 50% outliers

Applicable for any parameter estimation problem, e.g. positioning with GNSS

Literature

Horemuž M. and Zhao Y. (2014). Motion of moving camera from point matches: comparison of two robust estimation methods. IET Computer Vision, E-first.

Jurisch, R., Kampmann, G.: ‘Vermittelnde Ausgleichungsrechnung mit balancierten Beobachtungen - erste Schritte zu einem neuen Ansatz’, Zeitschrift für Vermessungswesen, 1998, 123, pp.87-92.