The relation between the rate of change of gravity and vertical displacement \( \frac{\dot{g}}{u} \) in former glaciated areas

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\(^1\text{Olsson (2013), Olsson et al (in press)}\)
What are we talking about?

$\dot{g}$ $\ddot{u}$

Diagram:
- A chair
- A standing figure
- Arrows indicating movement or forces
- Curved lines suggesting a path or trajectory

Legend:
- LANTMÄTERIET
Why study the relation between $\dot{g}$ and $\dot{u}$?

- A long history of observations of both in Fennoscandia
- Different observables of the same phenomenon (GIA)
- Their ratio contain information on the underlying physics
- A trustworthy relation allows to combine $\dot{u}$ and $\dot{g}$ and strengthen the overall accuracy

- Published ratios are rough estimates for areas with present day ice mass variations (elastic + viscous contribution)\(^1\)

- Are these ratios valid also for Fennoscandia?
- Accurate enough for our purposes?
- Is the ratio constant?
- If not, how does it vary, how much and why?

We have...

... predicted $\dot{g}$ and $\dot{u}$ with a GIA-model

... studied their relation with respect to e.g.
  ... different earth model parameters
  ... different ice sheet geometry
  ... evolution in time
  ... local effects (elastic signal and direct attraction)

... made some conclusions
The GIA-model

- Normal mode approach, 1 dimensional earth rheology
- Sea Level Equation with time dependent coast line geometry
- Ice model: ICE-5G
- Earth model: PREM

<table>
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<tr>
<th>Model name</th>
<th>Comp.</th>
<th>Incomp.</th>
<th>Lithospheric thickness [km]</th>
<th>Upper mantle viscosity $[10^{21} \text{ Pa s}]$</th>
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</table>

Predictions of $\dot{g}$
Linear regression

\[ \dot{g} = C \dot{u} + g_0 \]

<table>
<thead>
<tr>
<th>Region</th>
<th>( C ) [( \mu \text{Gal/mm} )]</th>
<th>( g_0 ) [( \mu \text{Gal/yr} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fennoscandia</td>
<td>-0.163</td>
<td>( \sim 0.03 )</td>
</tr>
<tr>
<td>North America</td>
<td>-0.152</td>
<td>( \sim 0.12 )</td>
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</table>
Residuals

\[ \epsilon = \dot{g} - (C \dot{u} + g_0) \]
Local effects?

- Direct attraction from relative sea level change?
- High degree elastic deformation terms?
High degree elastic deformation terms
Direct attraction from relative sea level changes

If distance to sea $> 10 \times$ height above sea then
Summary and conclusions

• From our GIA-model (Normal Mode, 1D, Maxwell) follows that:
  – Within a specific region (Fennoscandia or North America) the relation between \( \dot{g} \) and \( \dot{u} \) is almost linear with a ratio of -0.163 and -0.152 \( \mu \text{Gal/mm} \) respectively.
  – These values are not sensitive to the choice of earth model parameters.
  – Estimating \( \dot{g} \) from \( \dot{u} \) using this linear relation in the uplift area deviates less than ±0.02 \( \mu \text{Gal/yr} \) in Fennoscandia and less than ±0.1 \( \mu \text{Gal/yr} \) in North America, compared to full modelling of \( \dot{g} \).
• The observational accuracy is expected to be ±0.1 \( \mu \text{Gal/yr} \) after 15-25 years of annual or semiannual AG observations.
• Local effects, such as direct attraction from sea level variations and high degree elastic deformation, do not affect the results other than in extreme cases (distance to the sea < 10H).
• 3D earth?