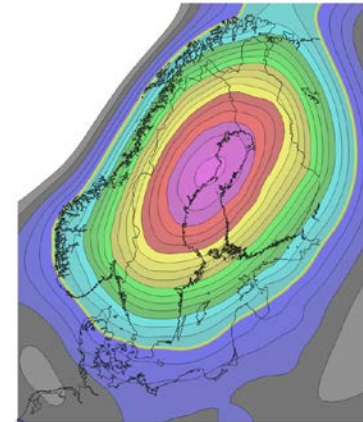
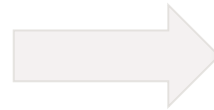
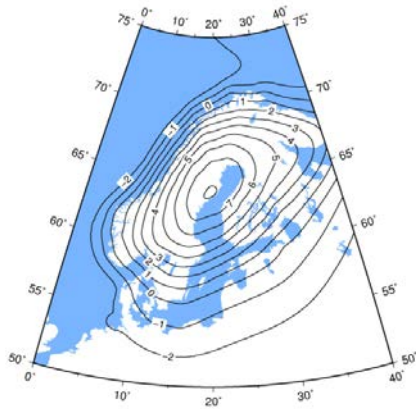


A new updated empirical land uplift model



Olav Vestøl, Jonas Ågren, Tarmo Kall, Ivars
Aleksejenko, Eimuntas Paršeliūnas, Andres Rüdja

GENERAL ASSEMBLY 2014

Outline

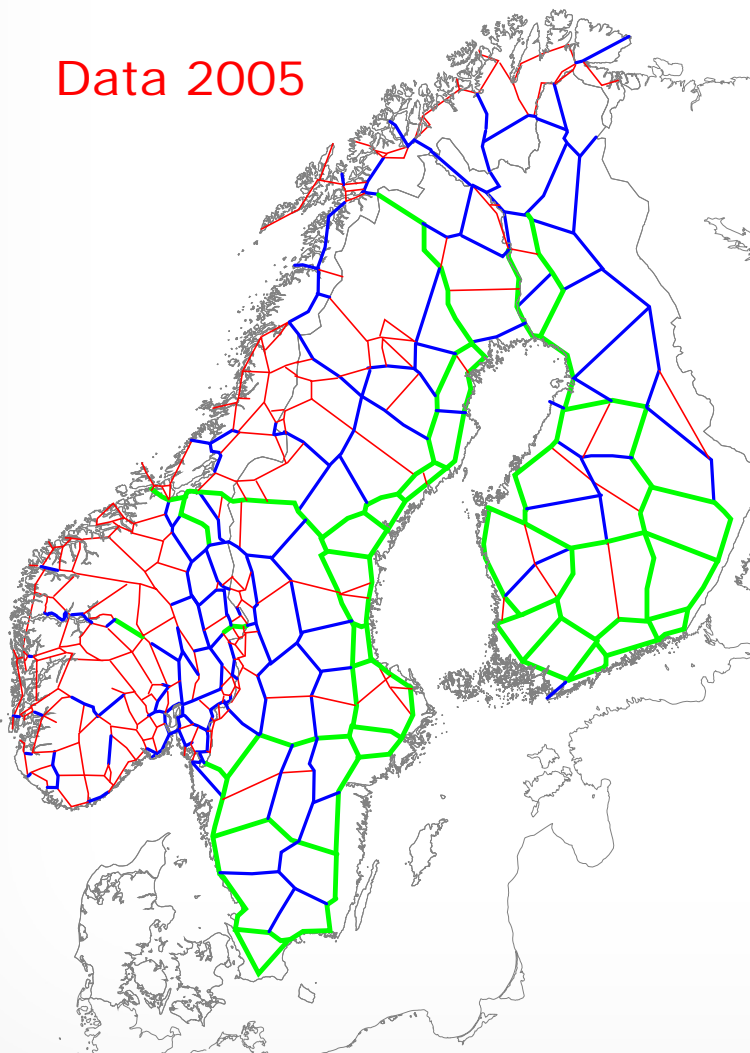
- The observations
- Short about the method
- The result
- Some comparisons

Background

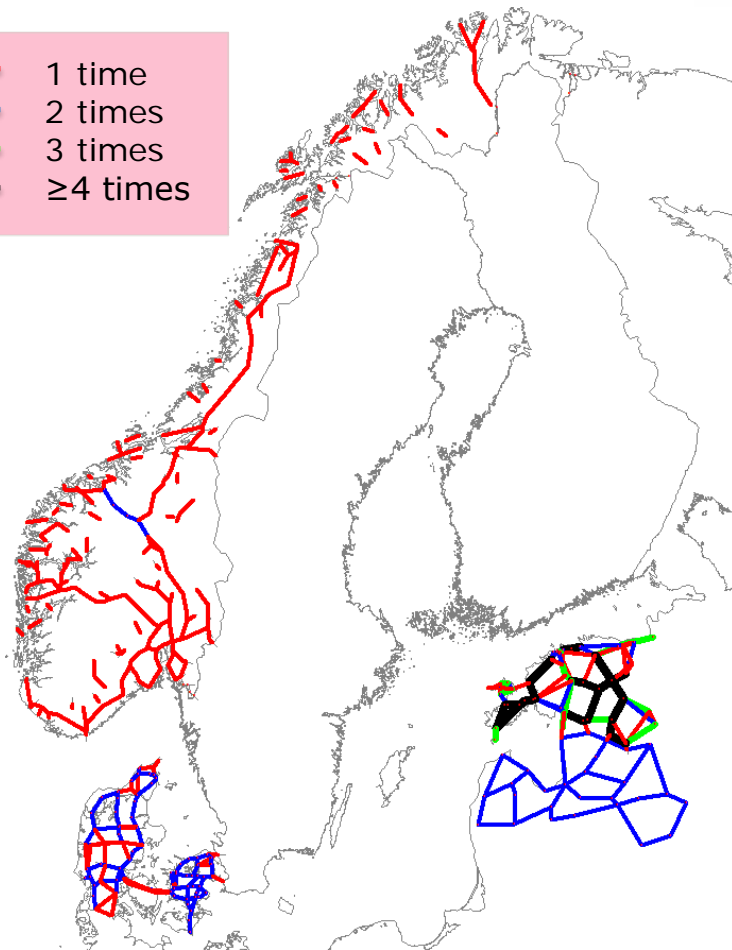
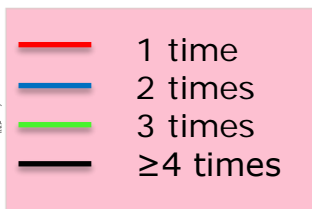
- NKG2005LU was released in 2005.
- Now, 9 years later, more data are available.
- New GIA-models are available

The leveling network

Data 2005

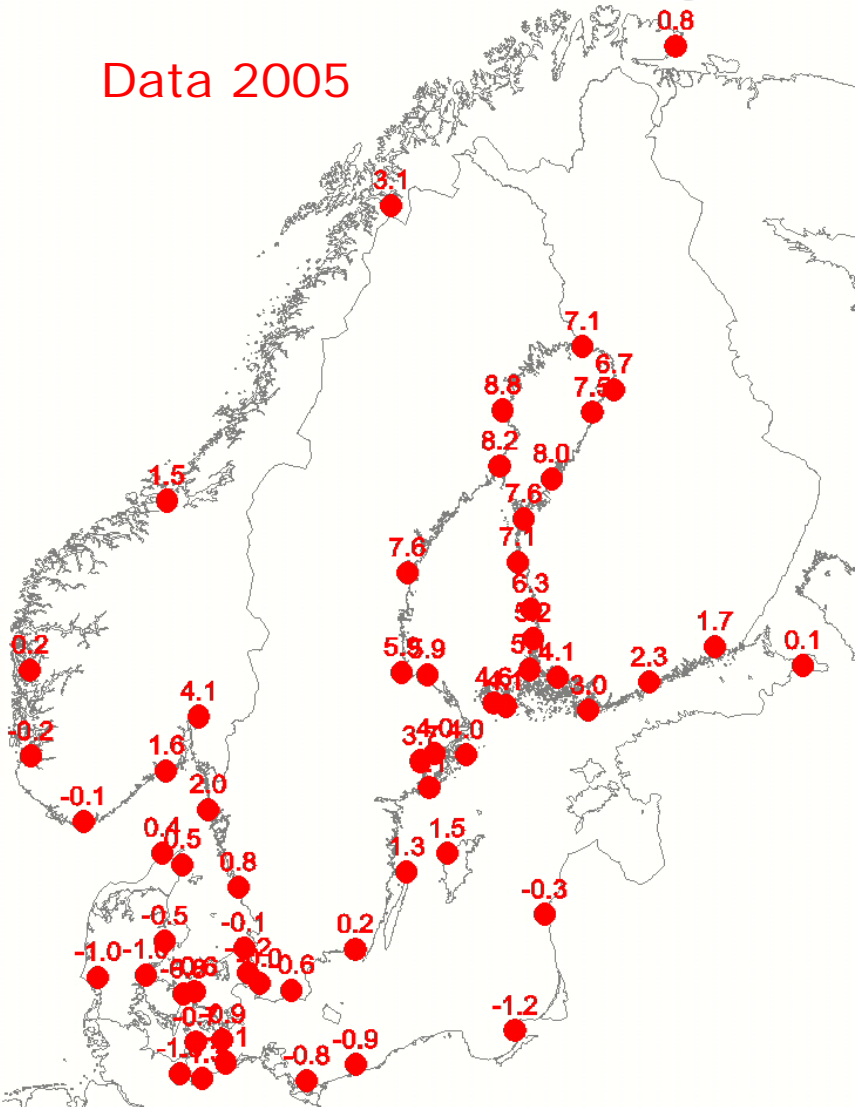


New data included

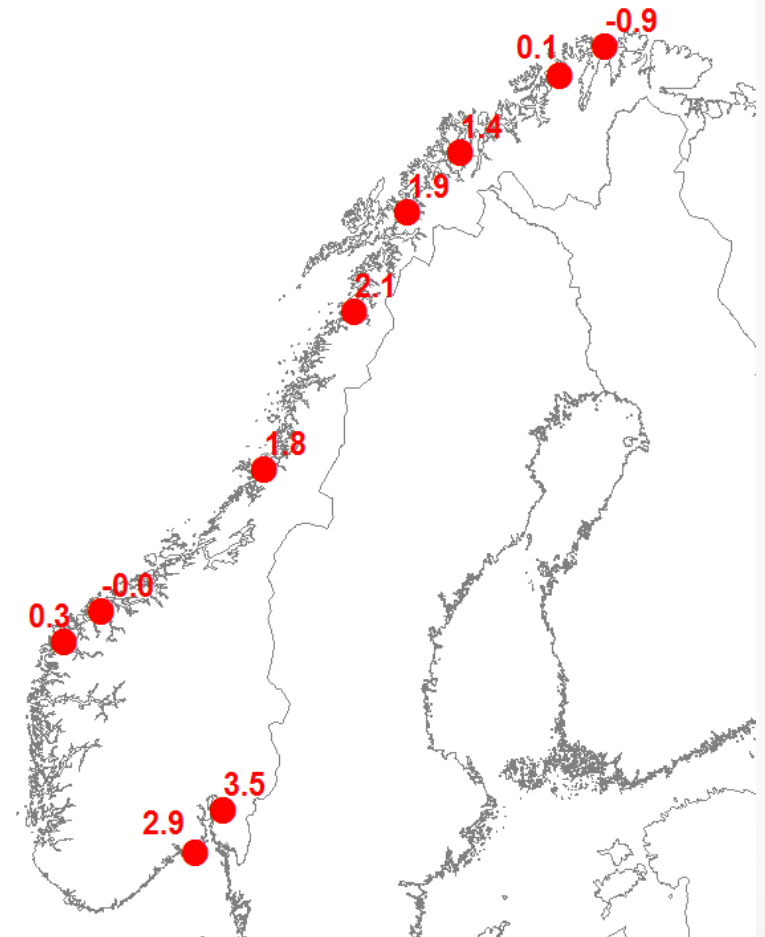


The tide gauges

Data 2005

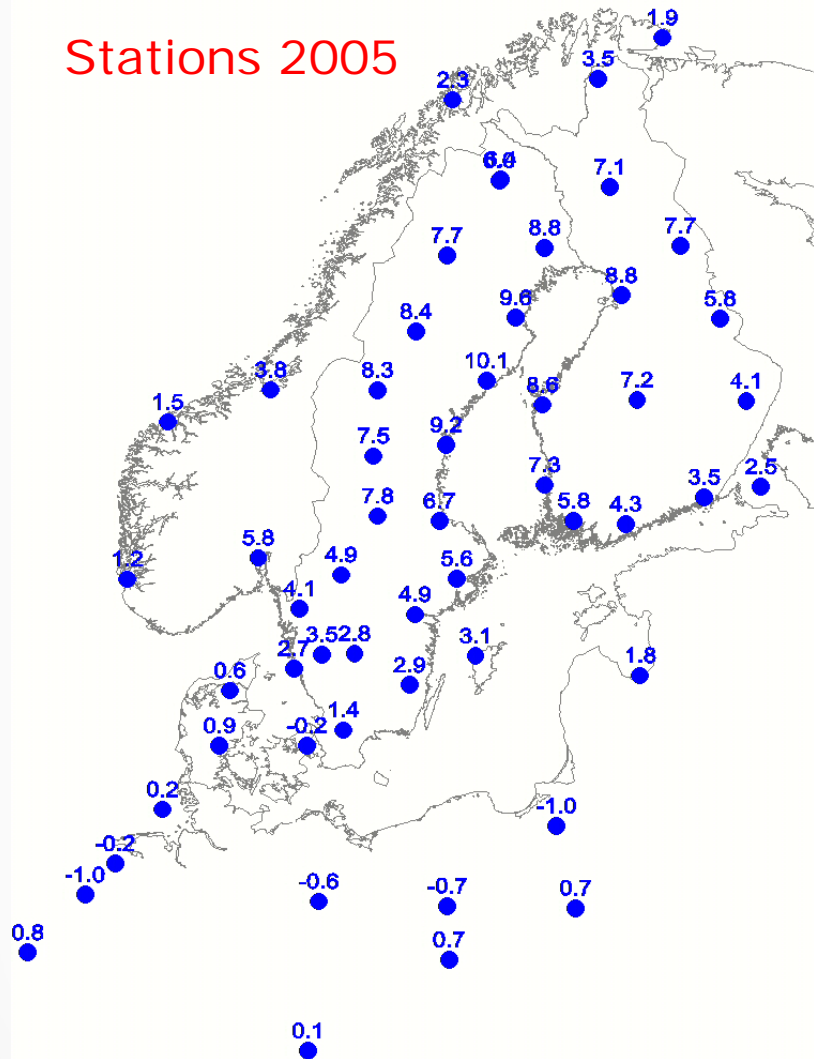


New data included



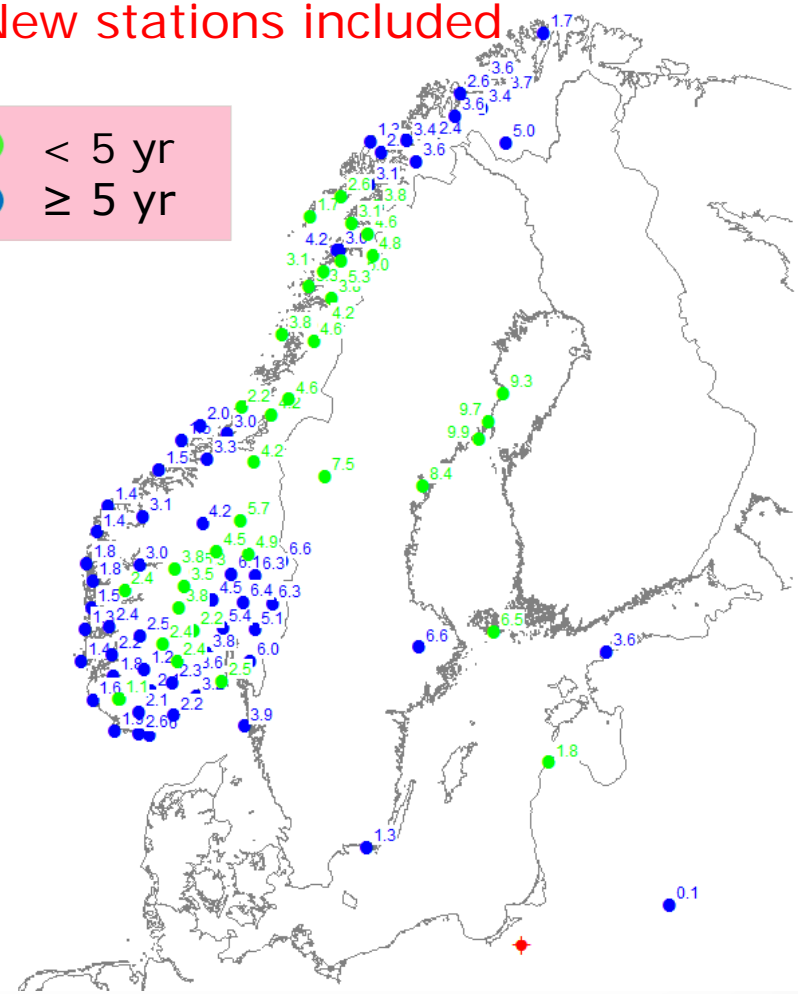
CORS (GPS-stations)

Stations 2005



New stations included

● < 5 yr
● ≥ 5 yr



Kierulf and Lidberg 2013 Gamit-solution

How to handle the new tide gauge rates

The new tide gauges are from 1956 – 2005, (and the Ekman from 1891 – 1990)

| | Diff (Short ÷ Long) | | Diff (Short ÷ Long) |
|---------------|---------------------|----------------------|---------------------|
| Helsinki | -0,155 | Oslo | -0,136 |
| Hanko | -0,187 | Tregde | -0,012 |
| Stockholm | -0,064 | Stavanger | -0,077 |
| Landsort | -0,094 | Bergen | -0,093 |
| Kungholmsfort | -0,074 | Heimsjø | -0,073 |
| København | -0,097 | Narvik | -0,117 |
| Ratan | -0,032 | Smøgen | -0,109 |
| Oulu | -0,077 | Esbjerg | -0,076 |
| Vasa | -0,107 | Århus | -0,021 |
| | | Weighted mean | -0,074 cm/yr |

How to handle the new GPS-rates

The time span vary from 3 to 13 year.

Systematic differences:

Solved for an extra constant

Weighting strategy: (from simple regression)

$$m_b = m_0 \sqrt{\frac{n}{\left(\frac{1}{6}(n^2(n+1)(2n+1)) - \frac{1}{4}(n^2(n+1))\right)}}$$

where $m_0 = 2\text{mm}$

$n =$ number of years

Examples:

5 yr: 0.6 mm/yr

10 yr: 0.2 mm/yr

A summery of the data

Leveling

Relative land uplift values between nodal points.

Reference surface: the rising geoid

Tide gauges

Apparent land uplift values.

Reference surface: the rising mean sea level

GPS-stations

'Absolute' values observed in a geodetic reference frame.

Reference surface: the unstable ellipsoid

Assumptions and simplifications

1. We select the rising mean sea level, in the 1891-1990 period, as reference surface
2. The uplift is linear in time
3. The rise of the geoid is proportional to the land uplift
4. The difference between ellipsoidal and mean sea level related land uplift values can be expressed with constants and a scale

..which implies

The relative sea surface topography is unchanged in the period of interest.

The LS-collocation method

$$\mathbf{l} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{s} + \mathbf{n}$$

Where

- L = Observations
- A = Design matrix
- X = Unknown heights and trend coefficients
- B = Design matrix for the signals
- s = Signals (unknown land uplift.)
- n = Noise

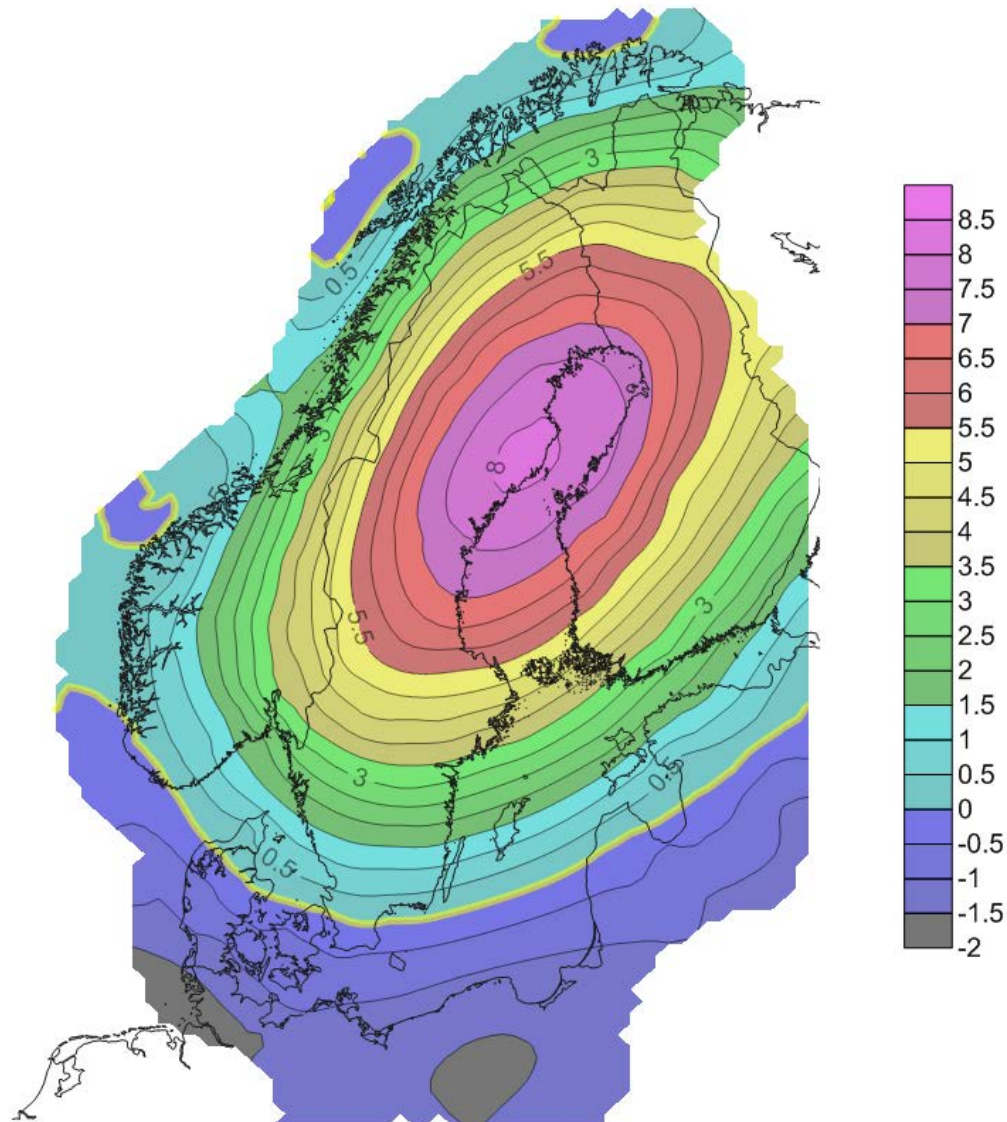
The unknowns

Leveling: Heights
 Polynomial coefficients
 Signals

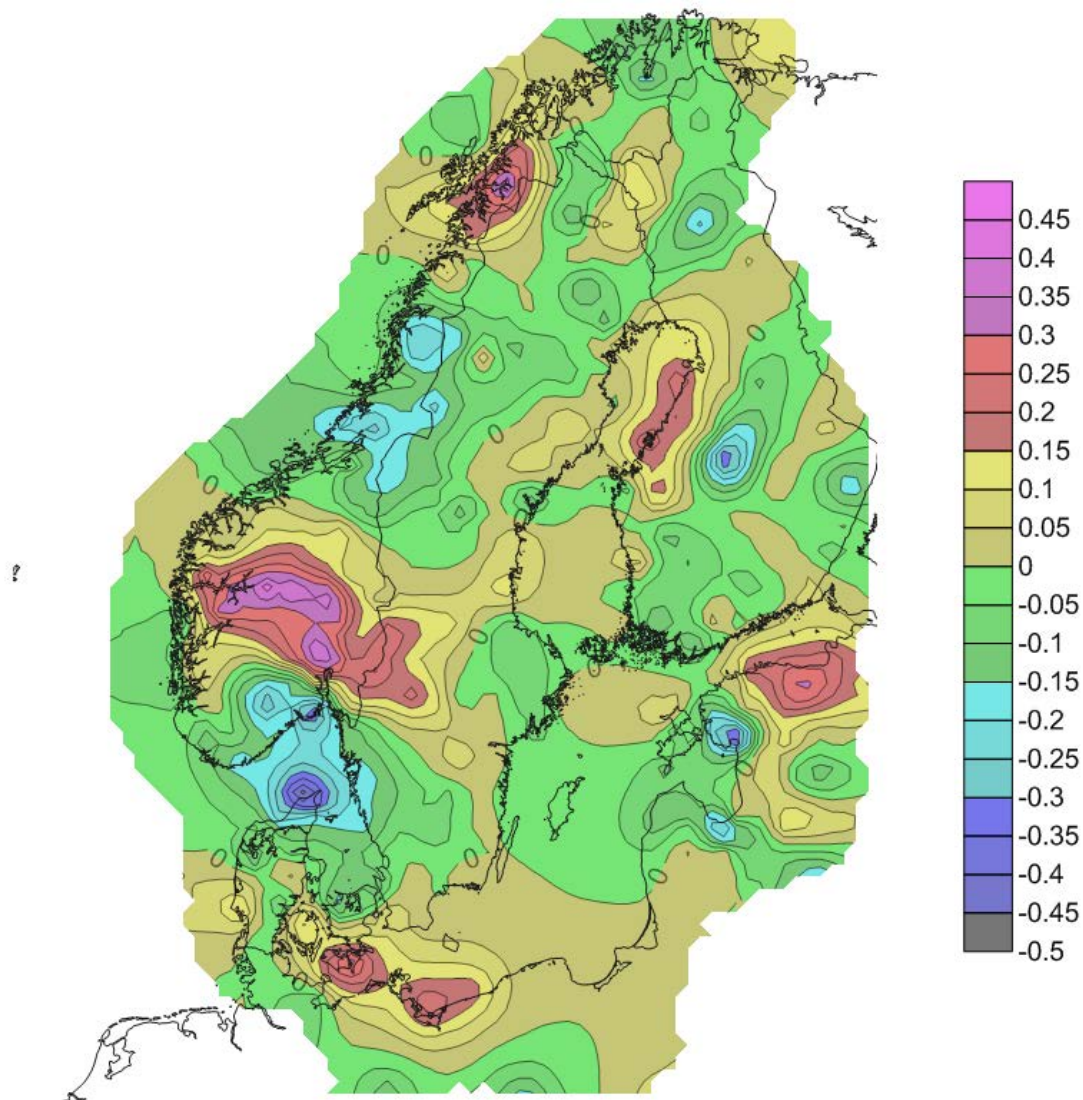
Tide gauges Polynomial coefficients
 Signals

GPS stations Polynomial coefficients
 Signals
 Constant
 Scale

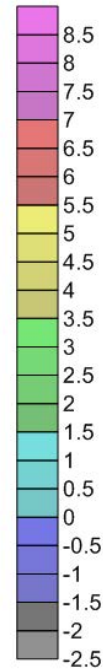
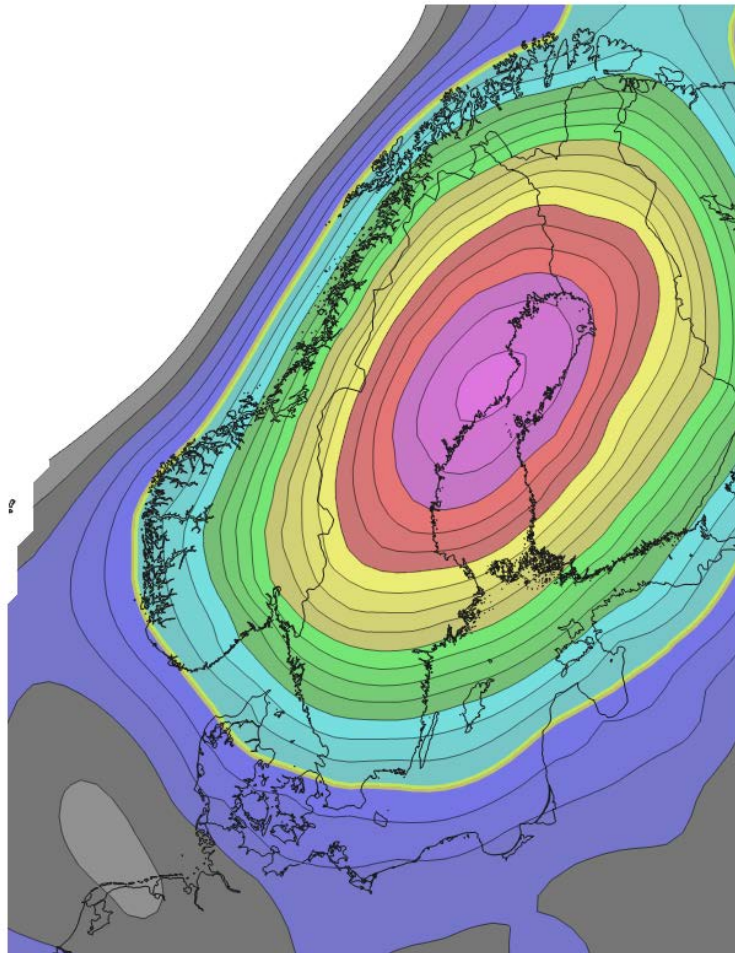
The pure empirical model



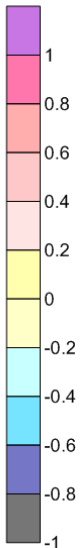
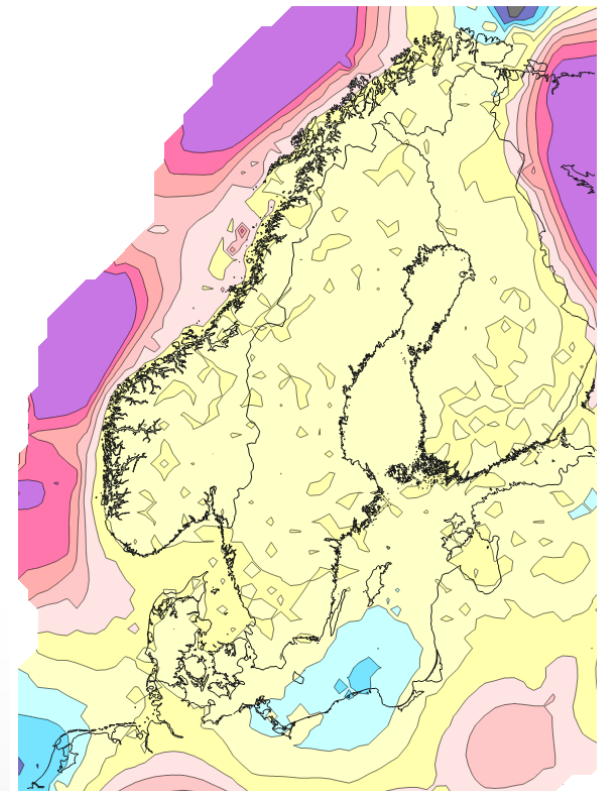
The signal



After combined with GIA (i82_g5102 from Holger Steffen)



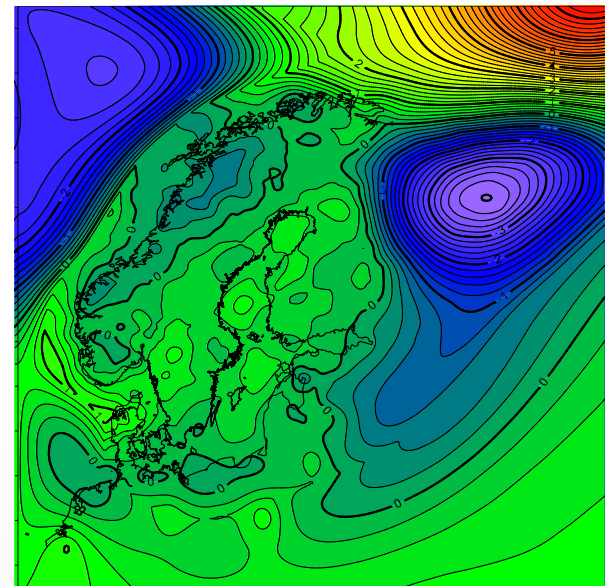
Differences
Pure empirical \div
combined solution



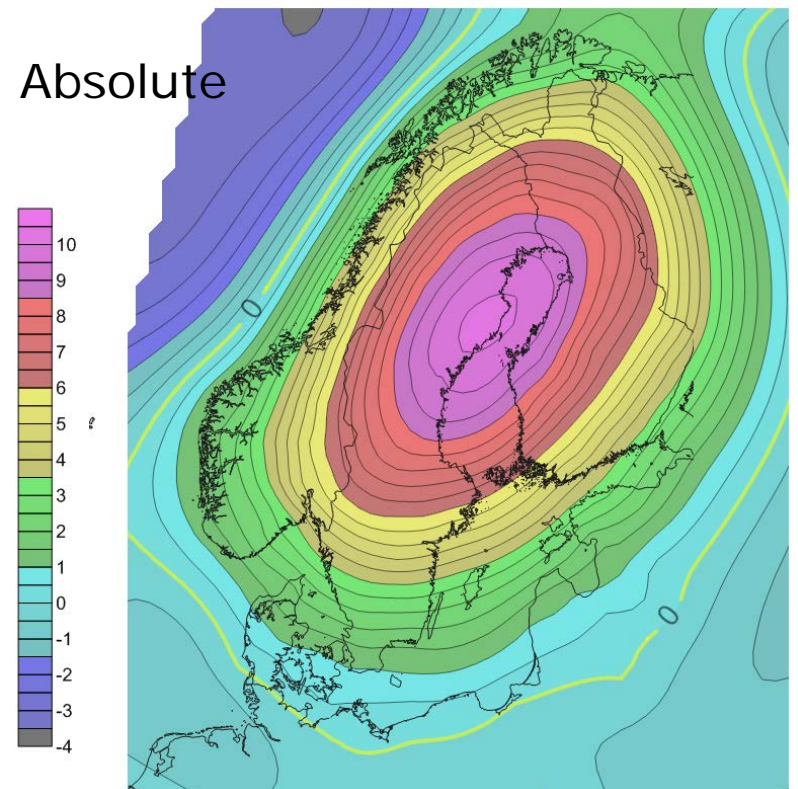
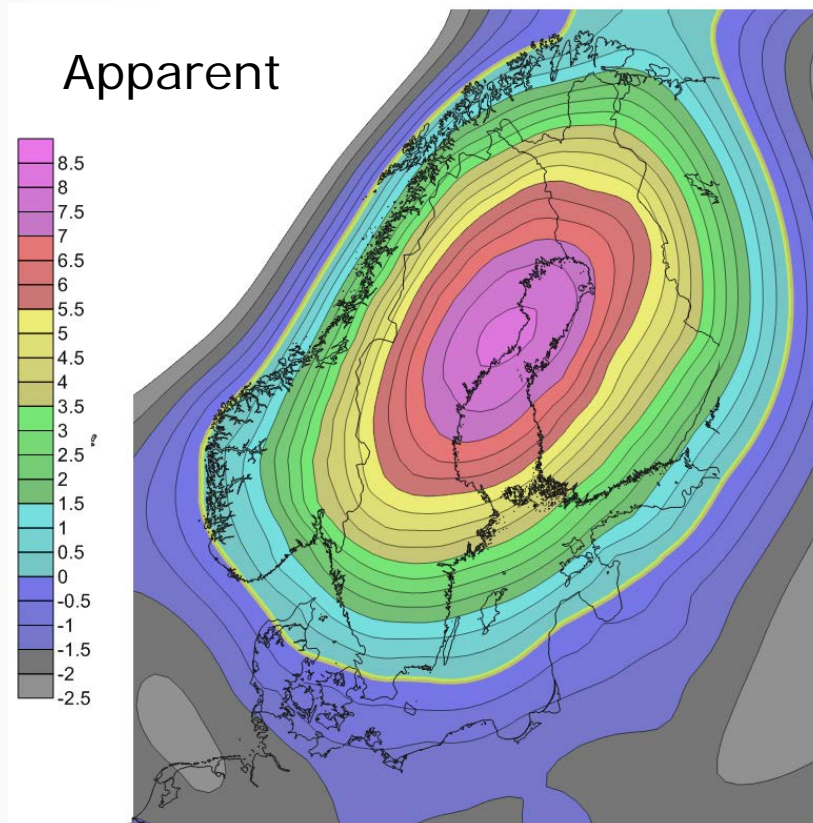
NKG2014LU_test ÷ NKG2005LU

Observation area

For the whole area



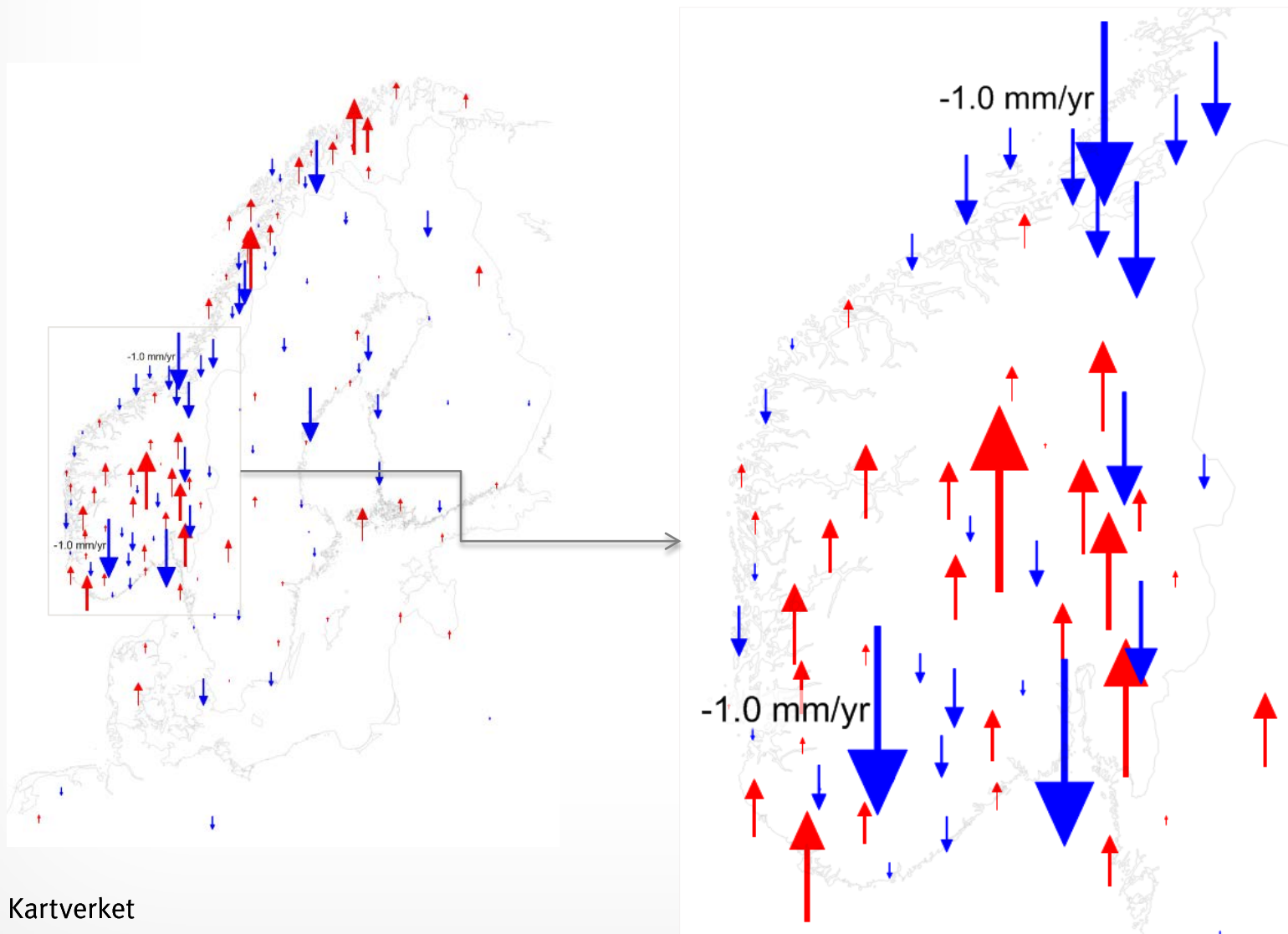
Final results



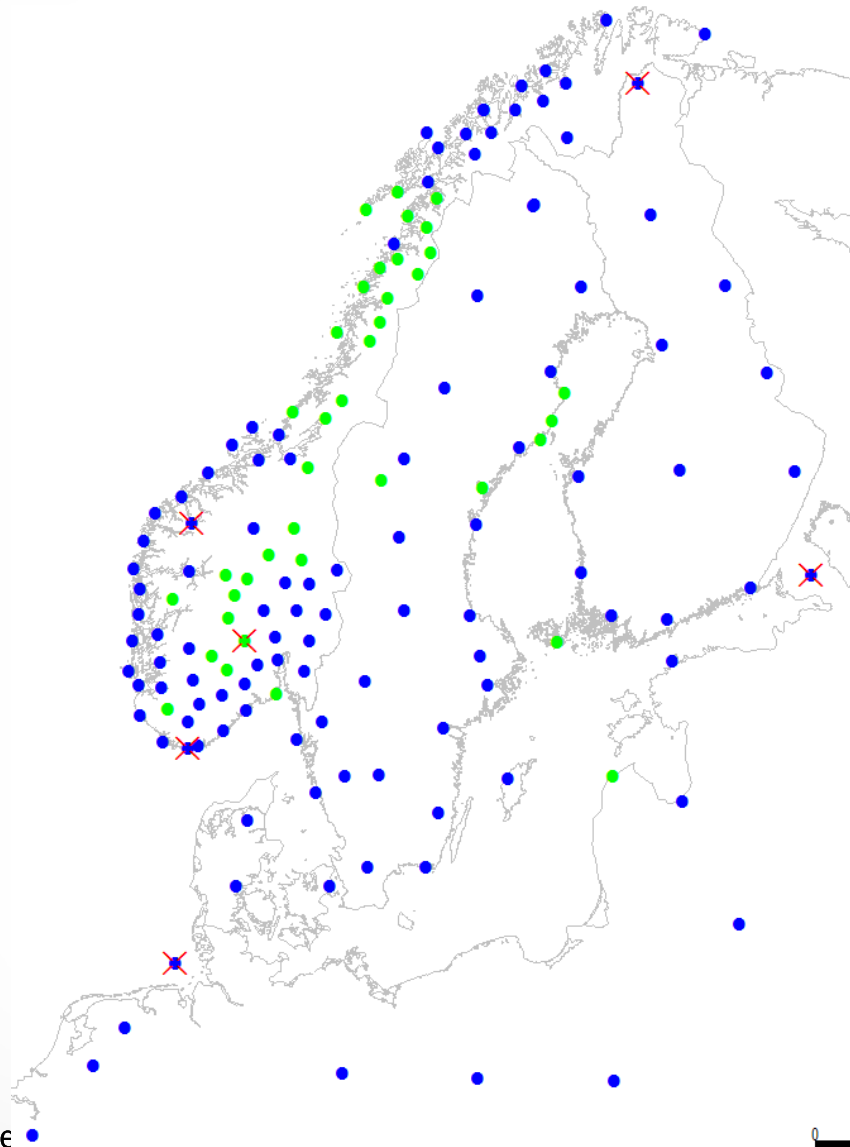
$$ABS = (APP + 1.27) \cdot 1.079$$

If there is still time.....

GPS-rates Residuals



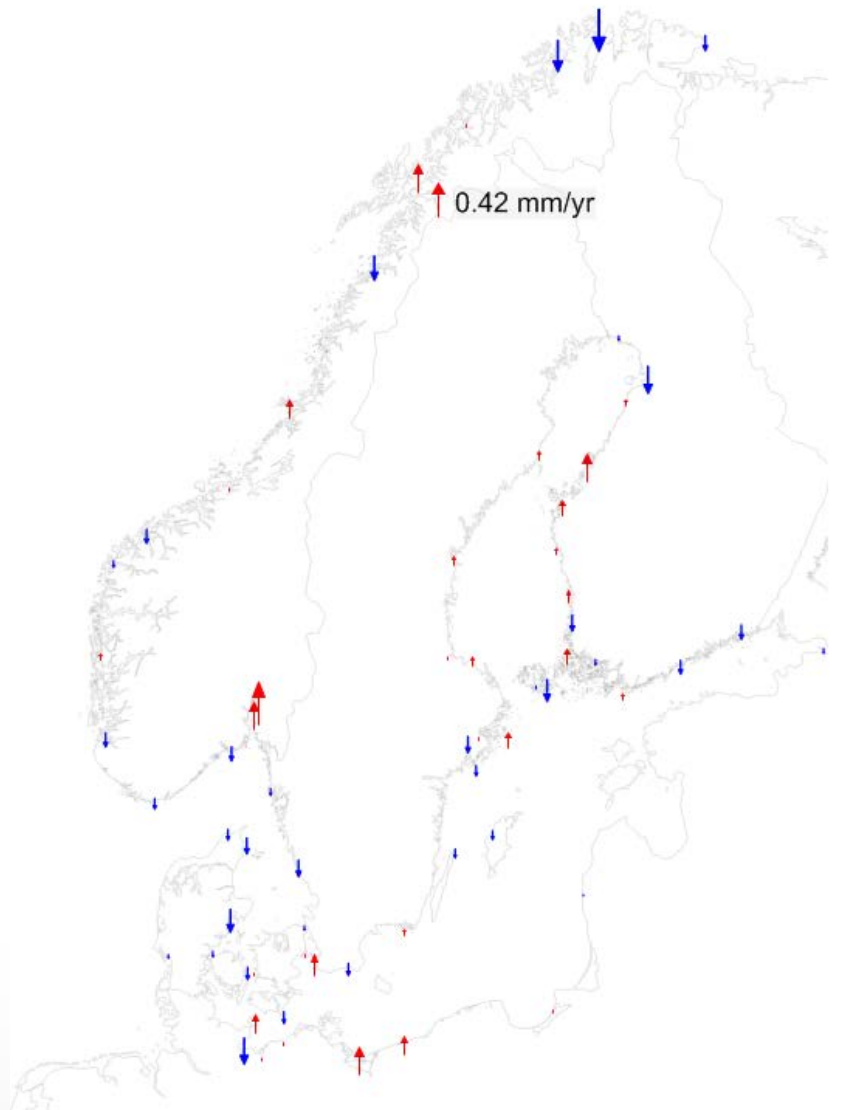
GPS-rates Outliers



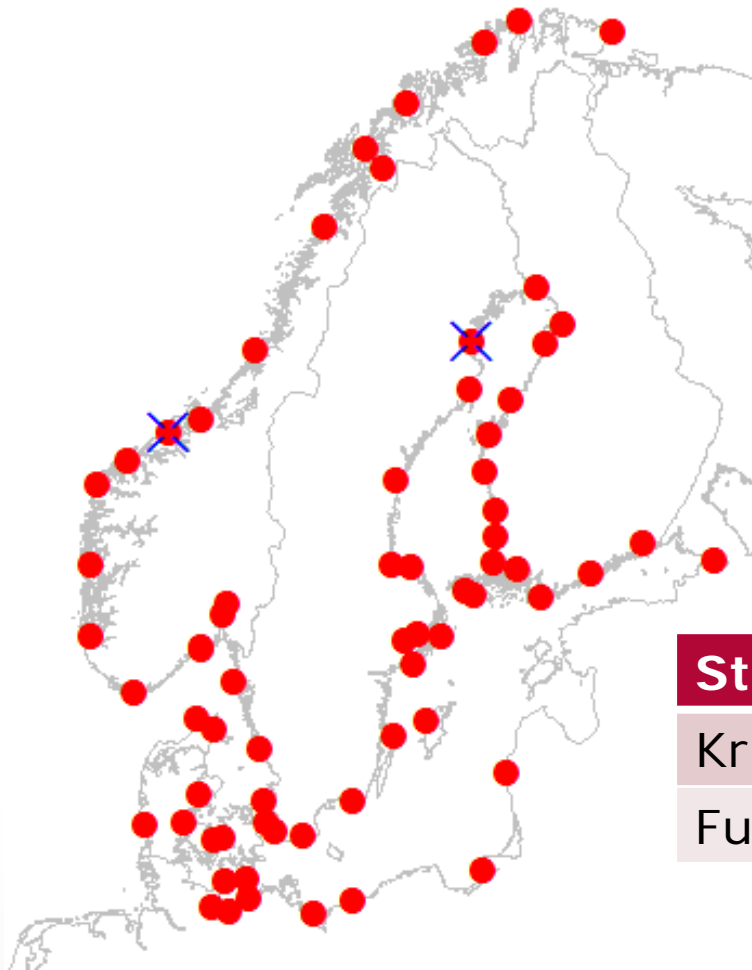
| Station | Outlier mm/yr |
|---------|---------------|
| KEVO | 1.1 |
| HELG | 1.1 |
| SVTL | 1.4 |
| VEGC | -1.6 |
| TGDE | 1.1 |
| HELC | 1.0 |

If there is still time #2

Tide Gauges Residuals



Tide Gauges Outliers

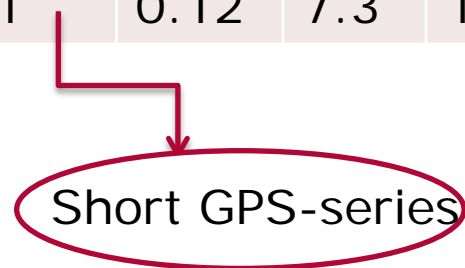


| Station | Outlier mm/yr |
|--------------|---------------|
| Kristiansund | 1.1 |
| Furuøgrund | 0.8 |

If there is still time #3

The fit of the GPS-rates

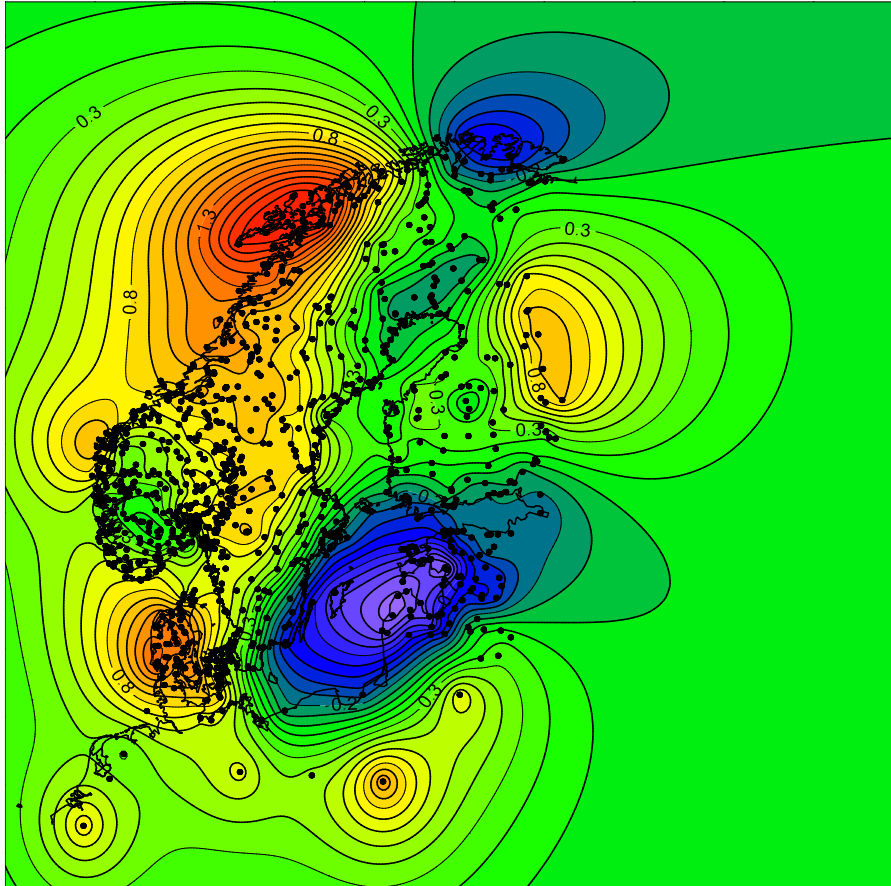
| Model | Const. 1 (mm/yr) | | Const. 2 (mm/yr) | | Scale (%) | |
|----------------|---------------------|----------------|---------------------|----------------|--------------|----------------|
| | Value | $\hat{\sigma}$ | Value | $\hat{\sigma}$ | | $\hat{\sigma}$ |
| NKG2005LU | 1.32 | 0.14 | | | 5.7 | 2.3 |
| NKG2014LU_test | 1.27 | 0.08 | 1.01 | 0.12 | 7.3 | 1.5 |



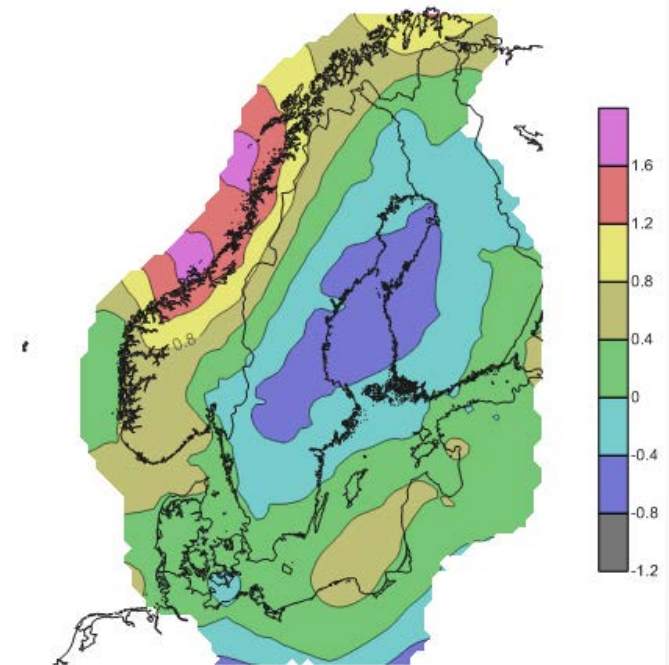
The GPS-rates in NKG2005LU are expressed in ITRF2000, and in ITRF2008 for NKG2014LU_test!

NKG2014LU_test ÷ *GIA*

i82_g5102



1D RSES+ICE-3G



Thank you for your attention!

Observations in NKG2005LU

Leveling observations

From Norway, Sweden and Finland. Geopotential differences between nodal points. From 1890 - 2003

Tide gauges

58 stations. Martin Ekman's values, published in 1996.

GPS-rates

55 stations. Results from Martin Lidberg's Licentiate Thesis in 2004. Reference frame: ITRF2000

Empirical land uplift model vs. GIA-models

GIA (Glacial Isostatic Adjustment):

- Based on Earth Model + Ice history model + Sea Level Equation
- Need observations to tune the parameters

Empirical land uplift models:

- Based on geodetic observations only
- Can be smoothed and extended by a GIA-model