

# The Development of Physical Geodesy during 1984-2014 - A personal view



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# My background experiences



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- **Ph.D. training 1971-1975**
  - “The three Swedish kings of Geodesy” (L. Asplund, A. Bjerhammar and E. Tengström)
  - Bjerhammar’s book “Theory of Errors and Generalized Matrix Inverses”, 1973
  - Molodensky’s new theory on physical geodesy:
    - a) surface gravity anomaly b) The quasigeoid
  - T. Krarup’s (1969) foundations on ph. geodesy

# My background experiences (cont.)



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- **With R. H. Rapp at OSU 1977-1978 (15 months):**
  - OSU EGMs; satellite altimetry; collocation, etc.
  - 5 personal OSU reports
- **H. Moritz (1980): Advanced Physical Geodesy**
- **The Int. Symposium on Satellite Geodesy, Cannes, 1980: "Spectral combination"**
- **Spring 1983: AvH-Fellow, Stuttgart (VC component estimation)**

# 1984-2014

## SOME EVENTS:

- **1982-2002: presidium member of NKG**
- **GPS is realized: 1) calls for accurate geoid models; 2) GPS/levelling**
- **Goals for geoid accuracy:**  
1970: 1-2 m; 1980: 2-5 dm; 2000: 1 cm; 2020: 5 mm?



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# 1984-2014 (cont.)

## DATA DEVELOPMENT

- **Earth Gravitational Model developments:**

$N_{\max}$  36 - 180 - 360 – 720 - 1800 -- 2160

Models: OSU- EGM96 -- Wenzel –EGM2008



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- **Satellite altimetry**
- From 2000: **dedicated satellite gravity missions** (CHAMP, GRACE, GOCE)
- **Airborne gravimetry**
- **Height data:** regional – global data sets, e.g. Space shuttle experiment, DTM2006, etc, ...

# Theoretical developments

- Combinations of EGM + Stokes  
(long+ short wavelengths)
- Collocation (+generalized interpolation; - covariances; - huge systems)
- RCR techniques (+works with residual gravity anomalies)
- Modified Stokes' methods
- 1984: Least Squares Modification of Stokes' Formula (first two reports) (+simplified, rigorous corrections)



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# The RCR - technique

$$\hat{N} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\psi) \Delta \tilde{g}^M d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M \frac{2}{n-1} \Delta \tilde{g}_n^{EGM} + \delta N_{dir,M}^T + \delta N_I + \delta N_I^a + \delta N_I^e$$



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$$\Delta \tilde{g}^M = \Delta \tilde{g} - \sum_{n=2}^M \Delta \tilde{g}_n^{EGM} + \delta \Delta g_{dir}^T + \delta \Delta g_{dwc} + \delta \Delta g_{dir}^a + \delta \Delta g_{dir}^e$$

- The IAG Geoid School practices (only) this method; in the past with  $L=0$ , and some corrections are ignored or simplified
- The modification by Vanicek-Kleusberg uses  $L = M$  and Molodensky type of modification parameters (i.e. minimum truncation error bound)

# Least Squares Modification with Additive Corrections (LSMSA)



- Uses the rigorous rcr formula above, but direct and indirect effects are combined into additive corrections (Sjöberg 2003a).

$$\hat{N} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\psi) \Delta \tilde{g} d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M (Q_n^L + s_n^*) \Delta \tilde{g}_n^{\text{EGM}} \\ + \delta N_{\text{comb}}^T + \delta N_{\text{dwc}} + \delta N_{\text{atm}} + \delta N_{\text{ell}}$$

# NKG GEOID MODELS

- The NKG wg on geoid determination was established 1978:



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- NKG1986                    Collocation                    0.5 m
- NKG1990                    FFT
- NKG1996                    RCR, FFT                    10 cm
- NKG2002-2004                -" -
- NKG2014?                    ?                            1 cm?

# The Geoid – Quasigeoid Separation

- H/M (1967):

$$N - \zeta \approx \frac{\Delta g_p^B}{\bar{g}} H$$

- Fleury/Rummel (2009):

$$N - \zeta \approx \frac{\Delta g_p^B}{\bar{g}} H + \frac{V_g^T}{\gamma_0} - \frac{V_p^T}{\gamma_Q}$$

- Sjöberg (2010, 2012) [arbitrary compensation model]:

$$N - \zeta = \int_0^{H_p} \frac{\Delta g}{\gamma} dh = \frac{\Delta g_p^c}{\bar{g}} H + \frac{dV_g^c}{\gamma_0} - \frac{dV_p^c}{\gamma_Q} + res$$

$$res = \int_0^{H_p} \frac{\Delta g^c}{\gamma} dh - \frac{\Delta g_p^c}{\bar{g}}$$



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# **W<sub>0</sub>= the geoid potential**

- Data: 1) Satellite altimetry (only on oceans)  
2) EGM (T<sub>0</sub> is missing or chosen  
⇒ only relative geoid heights)
- Methods:
  - direct integration over the oceans.
  - geometric methods for ref. ellipsoidal parameters a and b ⇒ U<sub>0</sub>=W<sub>0</sub>.
- Sjöberg (2013) a, b and W<sub>0</sub> jointly



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# The SITE = "the secondary indirect topographic effect"

- The boundary condition:



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$$\Delta \mathbf{g} \approx -\frac{\partial \mathbf{T}}{\partial \mathbf{r}} \cdot 2 \frac{\mathbf{T}}{\mathbf{r}}$$

- The topographic signal:

$$\Delta \mathbf{g}^T \approx -\frac{\partial \mathbf{V}^T}{\partial \mathbf{r}} \cdot 2 \frac{\mathbf{V}^T}{\mathbf{r}}$$

- 

DITE SITE

# THE SITE

- $\Delta g^B$  reduces only for DITE and not for the SITE

Needs for new/rigorous gravity anomalies:



- No-topography gravity anomaly

$$\Delta g^{NT} = \Delta g^B + 2 \frac{V^T}{r}$$

- Rigorous isostatic gravity anomaly

$$\Delta g^{I,new} = \Delta g^I + \frac{2(V^T - V_C)}{r}$$

# Gravity Inversion

- Bouguer vs. no-topography gravity anomaly:  
B. is not a rigorous anomaly - NT. is
- The Bouguer anomaly is not consistent with the Bouguer gravity disturbance
- Traditional vs. rigorous isostatic gravity anomalies
- Vening Meinesz- Moritz (1990) isostatic model
- Gravimetric/Isostatic Moho determination vs. CRUST2/1 seismic models



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# Conclusions

- 1984-2014 saw very exciting developments in physical geodesy:
- GPS stimulates improved modelling of the (quasi)geoid (technical needs)
- Geophysical science pushes satellite gravity missions (science needs)
- Ultra-high-degree EGMs
- EGM+ Stokes towards 5 mm geoids?
- Rigorous gravity anomalies
- Gravity inversion, e.g. VMM Moho models



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# **SOME REMAING PROBLEMS**



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- The SST bias in satellite altimetry?
- The geoid vs. the quasigeoid?
- WO is still to be improved
- Future GNSS/levelling without height fixes?
- The bad economy in physical geodesy training and research



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**THANK YOU FOR YOUR ATTENTION!**