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NKG Gravity Data Base and NKG geoid

by

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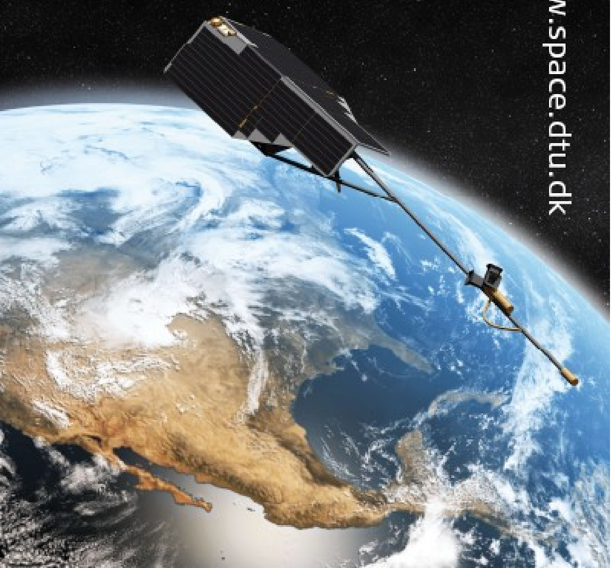

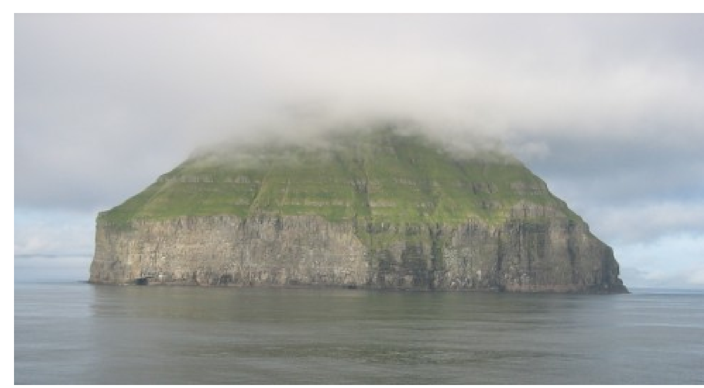

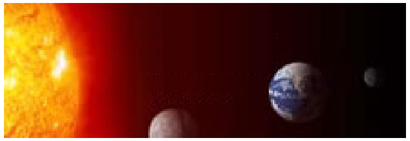

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- NKG gravity data base
- NKG geoid

History: NKG gravity data base and NKG geoid computations

NKG DB started in 1970s at Geodætisk Institut (GI), DK by prof. C.C. Tscherning, University of Copenhagen

1980s Dag Solheim, Statens Kartverk (SK), N visited GI and wrote the present software. Nowdays Sweden also have a copy of the DB (Läntmateriet, LM).

For many years there were two mirror sites, one in Norway (SK) and one in Denmark (GI/KMS/DNSC/DTU Space)

Traditionally, the NKG geoid is computed in Copenhagen using the data from NKG DB.

Over the years most countries in the area contributed to the DB. The data owner can choose their data to be only released by permission.

The existence of the NKG gravity DB makes it straight forward to do the geoid computations without a need to coordinate the data collection first.

Examples of NKG geoid models: NKG96, NKG2004 ... etc

NKG gravity data base

Data stored in binary format (80-char) in geographical tiles Δlat x Δlon: 3   x 6

The reason: quick access to the data and limited storage space

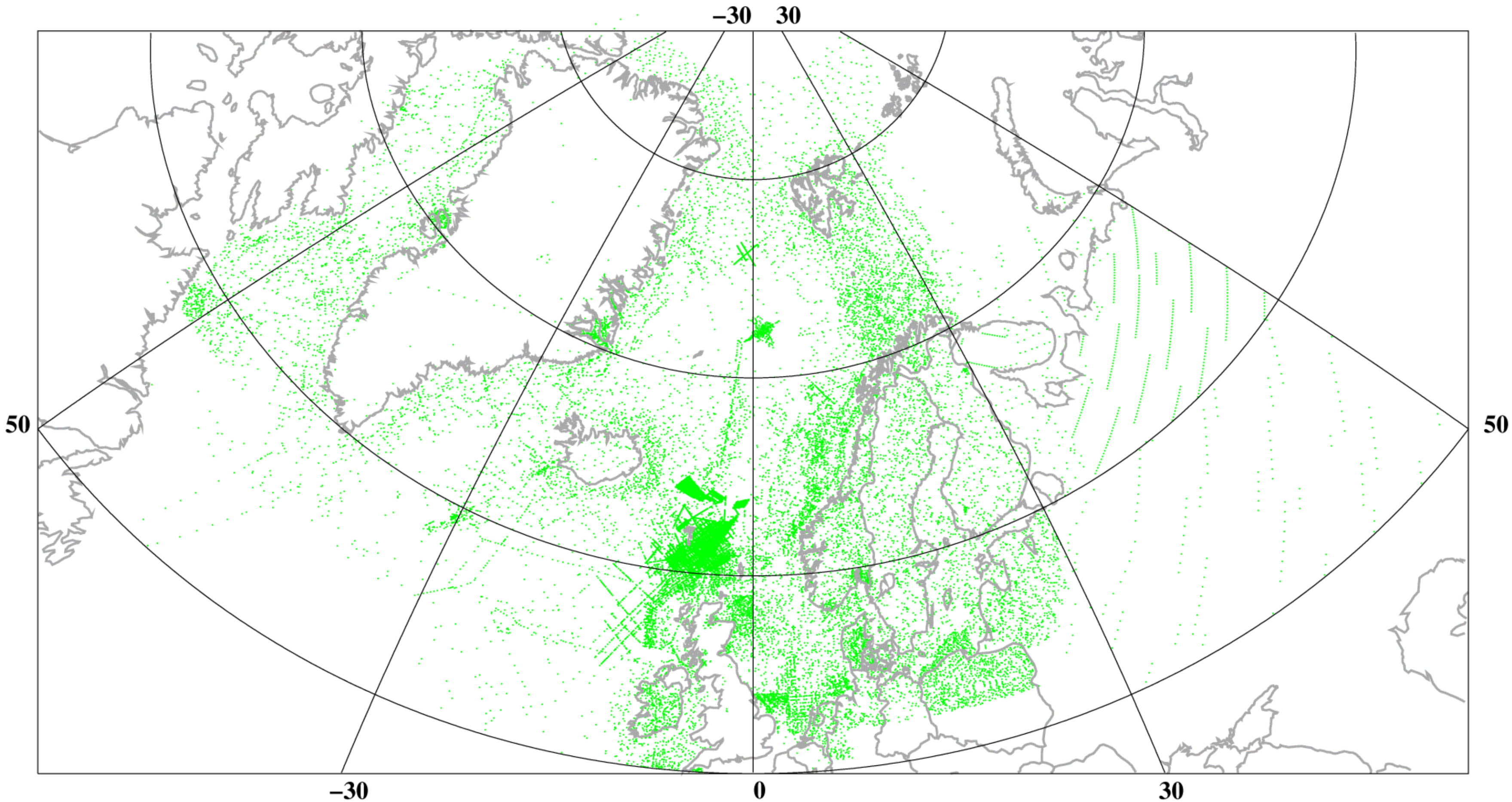
Example:

Simple data format

8065770	62.54833	-3.20800	-694.8	25.17	73.01
8065770	62.54833	-3.20800	-694.8	25.86	73.70
100427	62.54833	-3.07733	-674.4	17.00	63.43

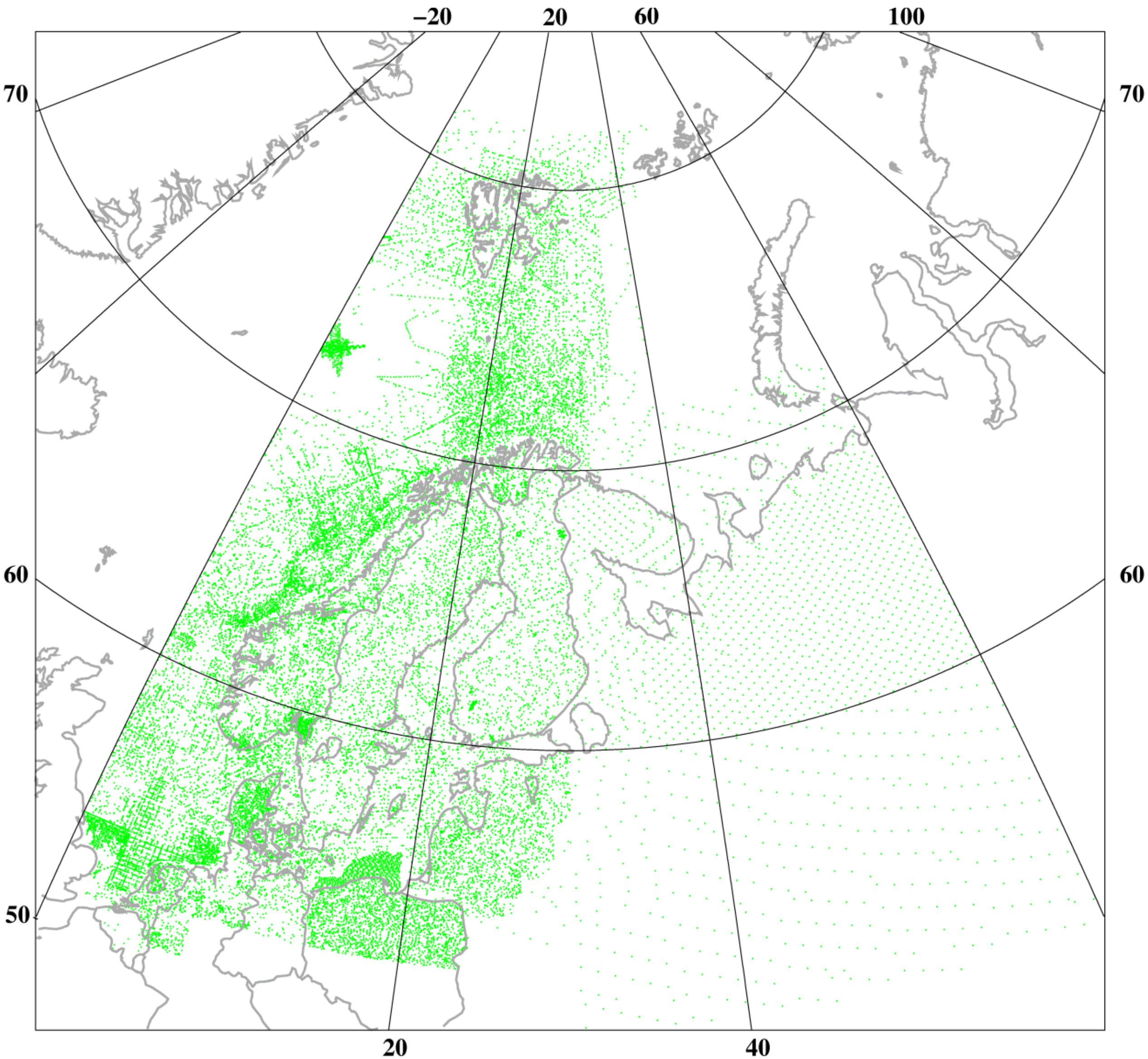
-
80-char format

```
C 6232.90B  -312.48B3 694.80B   0.0X6137.83B  25.17B8758065770  0.0X  3.0  73.01
C 6232.90B  -312.48B3 694.80B   0.0X6138.52B  25.86B8758065770  0.0X  3.0  73.70
C 6232.90B  -304.64B3 674.40B   0.0X6129.66B  17.00B882 100427  0.0X  4.0  63.43
```



Total # of data: 1953441

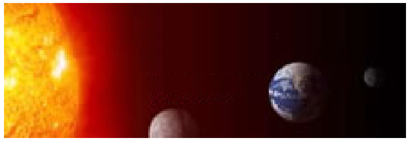
data distribution (except the polar cap) 1923756
(the data thinned by factor 50 on the figure)



The data distribution in the Nordic area

643940 data

(the data thinned by factor 20 on the figure)



Some problems with the data in the DB:

• The DB data are organized by source code. The ”vintage” (epoch) is sometimes unknown.

• The history of correction done to the data is sometimes unknown (e.g. The Potsdam correction, the Gulf of Riga data)

•The national height systems

This could be investigated using an A10/CG5 or an aerogravity survey (GOCINA/OCTAS)

<u>Proposal for the issues related to the NKG gravity DB to be discussed here:</u>

1. A systematic procedure for the quality check of the data.

2. A graphic display of the data.

Geoid determination basics

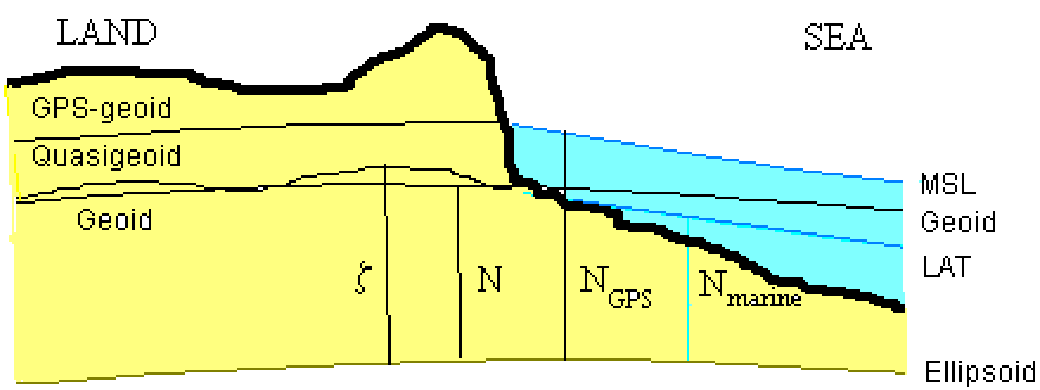
Geoid: Level surface of global undisturbed oceans ..

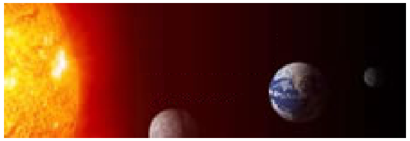
Complications:

”Geoid” is *global* equipotential surface in geopotential ($W = W_0$), but heights refer to *local* reference:

- Mean Sea Level (MSL) – land
- Lowest Astronomic Tide (LAT) – sea

The ”cm-geoid” always refers to a specific vertical datum .. and is not necessarily a ”true” geoid





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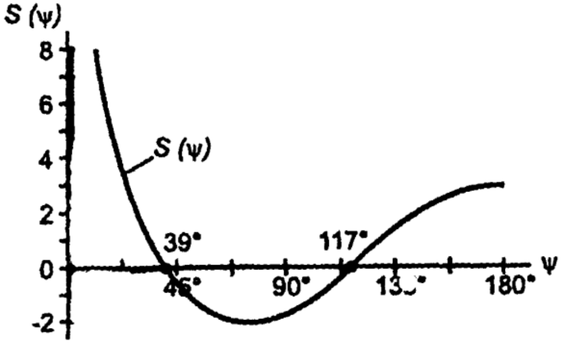
Geoid determination basics

Classical: Stokes formula – geoid from *gravity anomalies* Δg:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta \quad S(\psi)\, d\sigma$$

$$S(\psi \quad = \frac{1}{\sin(\frac{\psi}{2})} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi \quad 3\cos\psi \log(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2})$$



Challenge:

Determining N – "*the gravimetric geoid*" – with sufficient accuracy, aided by

- *New gravity measurement methods (airborne)*
- *New satellite gravity field missions (GRACE, CHAMP, GOCE)*
- *GPS on levelling benchmarks and tide gauges*

Geoid determination basics – global field

For the *global field* (N_{EGM96}) – EGM96 spherical harmonic model (Lemoine et al., 1996), complete to degree and order 360:

$$N = \frac{GM}{R\gamma} \sum_{n=0}^{N} \left(\frac{R}{r}\right)^n \sum_{m=0}^{n} \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin\phi)$$

Global fields determined from:

- Satellite tracking .. laser, doppler, optical ..
- Satellite gravity missions (CHAMP, GRACE, GOCE ..) ..
- Satellite altimetry and local Δg mean values ..

New international global field: EGM07
Complete to degree 2160 (5' resolution – 3 cm geoid)
NGA/NASA (USA) in cooperation with International Association of Geodesy

Gravimetric geoid determination in practice

Anomalous potential T = $W_{physical} - U_{normal}$

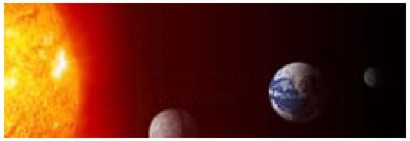
$$N = \frac{T}{\gamma}(on\,geoid);$$

The anomalous gravity potential *T* is split into 3 parts:

$$T = T_{EGM96} + T_{RTM} + T_{res}$$

T_{EGM96} – Global spherical harmonic model (GRACE/EGM96 to 360)
T_{RTM} – residual terrain effect (RTM) .. Computed by *prism integration*
T_{res} – residual (i.e. unmodelled) local gravity effect

Principle used in DNSC gravimetric geoid determination:
*Stokes function implemented by **Fast Fourier Techniques and least-squares collocation gridding***

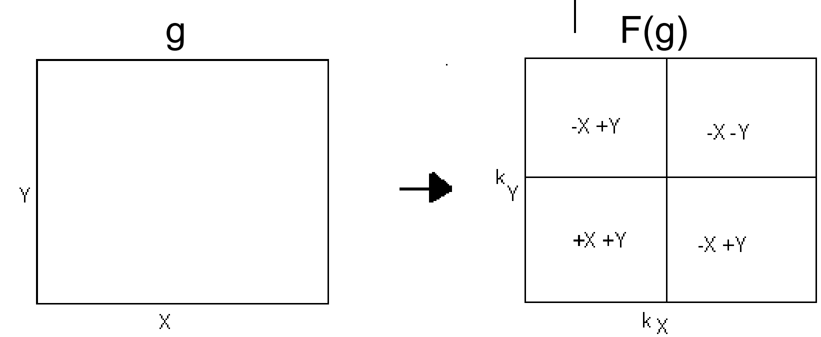


Fourier Transform

- Basic definition of 2-D Fourier transform

$$F(g) = G(k_x, k_y) = \iint g(x,y) e^{-i(k_x x + k_y y)} dxdy$$

$$F^{-1}(G) = g(x,y) = \frac{1}{4\pi^2} \iint G(k_x, k_y) e^{i(k_x x + k_y y)} dk_x$$



- k_x and k_y are called wavenumbers (like frequency in 1 infinite x-y plane

Advantage of Fourier transforms: *convolution theorem*

$$f * g(x,y) = \iint f(x - x', y - y') g(x', y') dx' dy'$$

$$\Rightarrow \quad F(f * g) = F(f) \cdot F(g)$$

• Convolutions must faster in frequency domain than space domain …*many geodetic integrals can be expressed as convolutions*

Fourier transformation and Stokes integral

- Stokes integral can conveniently be evaluated using FFT methods (Strang van Hess, 1990).

$$N = \frac{R}{4\gamma\pi} \iint_{\sigma} \Delta g(\varphi, \lambda) S(\varphi_p, \lambda_p, \varphi, \lambda) \cos\varphi \, d\varphi \, d\lambda$$

- This is convolution form if cosφ is considered constant ("simple spherical FFT") and the sin-formula is used for ψ

$$\sin^2\frac{\psi}{2} = \sin^2\frac{(\varphi_p - \varphi)}{2} + \sin^2\frac{(\lambda_p - \lambda)}{2}\cos\varphi_p \cos\varphi$$

…GRAVSOFT program SPFOUR, multiband-method refined to be exact on sphere

- Stokes formula in planar approximation gives:

$$N(x_p, y_p) = \frac{1}{2\pi\gamma} \iint_{A} \frac{\Delta g(x, y)}{\sqrt{(x_p - x)^2 + (y_p - y)^2}} dxdy = \frac{1}{2\pi\gamma} \Delta g(x_p, y_p) * \frac{1}{s}(x_p, y_p)$$

$$N(x_p, y_p) = \frac{1}{2\pi\gamma} F^{-1}[F(\Delta g(x_p, y_p)) F(\frac{1}{s}(x_p, y_p))]$$

…GRAVSOFT planar FFT program: GEOFOUR

Stokes kernel modification – controlling long-wavelength errors

Stokes' kernel modification (i.e. modified Wong-Gore method without "sharp edges"):

$$S_{\mathrm{mod}}(\psi) = S(\psi) - \sum_{n=2}^{N} \alpha(n) \frac{2n+1}{n-1} P_n(\cos\psi)$$

where

$$\alpha(n) = \begin{cases} 1 & \text{for} \quad 2 \le n \le N_1 \\ \dfrac{N_2 - n}{N_2 - N_1} & \text{for} \quad N_1 \le n \le N_2, \qquad n = 2, \ldots, N \\ 0 & \text{for} \quad N \ge n > N_2 \end{cases}$$

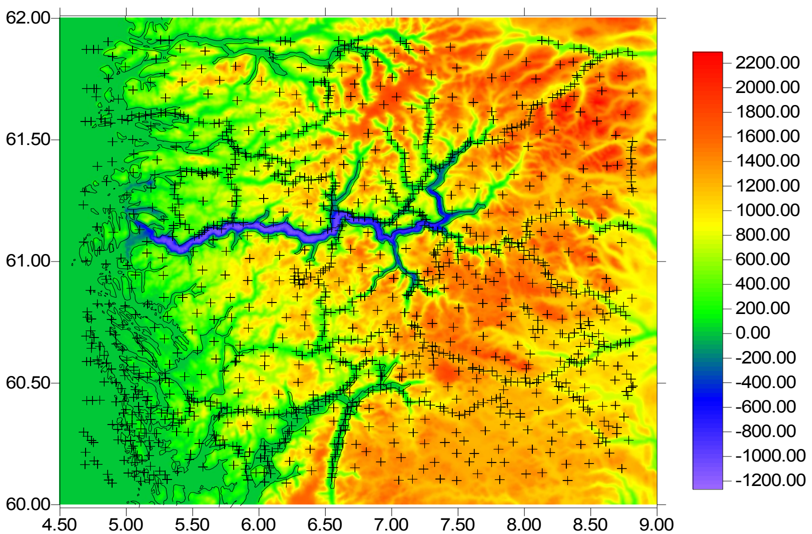
The use of modified kernels means that the satellite information (e.g., from GRACE) is used for the long wavelengths, and the surface gravity data at shorter wavelengths

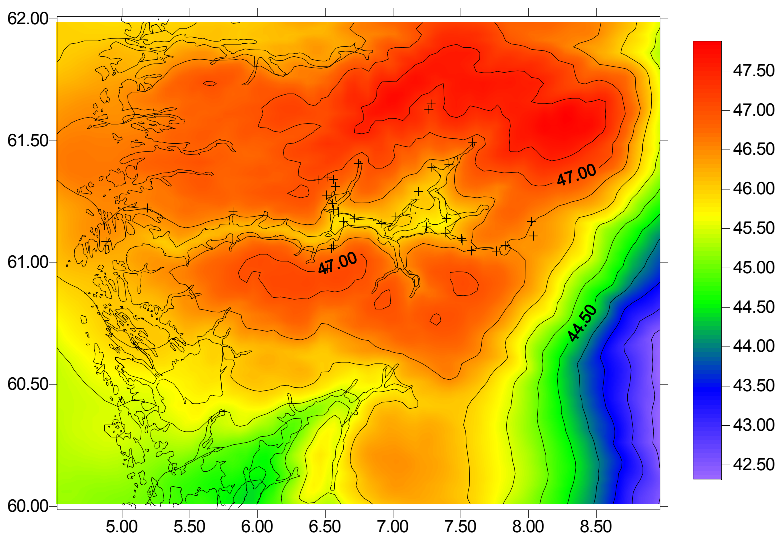
Terrestrial gravity data often biased due to errors (e.g. lacking atmospheric correction), uncertainties in reference systems, aliasing due to terrain …) … major problem for geoid computations ..

Gravimetric geoid determination in mountains

Combining gravity field data and digital terrain models:
Terrain dominates shortwavelength geoid features in mountains

⇒ *terrain effects to be modelled (remove/restore; prisms, FFT ..)*
⇒ *density known only approximatively (but remove-restore methods robust to errors)*
⇒ *Problems with irregular sampling of terrain with gravity stations*





Gravity data and DEM (Sognefjord, Norway) Detailed geoid

Terrain effect computations: the prism building block

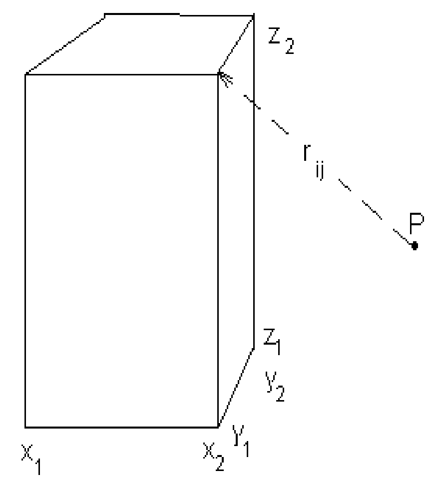
• The rectangular prism of constant density is a useful "building block" for numerical integrations of the basic terrain effects – gravity and geoid formulas:

$$\delta g_m = G\rho \; || \; x \log(y+r) + y \log(x+r) - z \arctan \frac{xy}{zr} \Big|_{x_2}^{x_1} \Big|_{y_2}^{y_1} \Big|_{z_2}^{z_1},$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

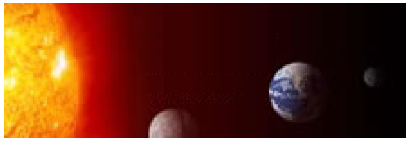
$$T_m = G\rho \; || \; xy \log(z+r) + xz \log(y+r) + yz \log(x+r)$$

$$- \frac{x^2}{2} \arctan \frac{yz}{xr} - \frac{y^2}{2} \arctan \frac{xz}{yr} - \frac{z^2}{2} \arctan \frac{xy}{zr} \Big|_{x_2}^{x_1} \Big|_{y_2}^{y_1} \Big|_{z_2}^{z_1}$$



• Implementation in practice: GRAVSOFT ”TC” program

- *Input of height data: Sequence of DEMs*
- *Speed up prism formulas by approximative formulas at large distance*
- *Supplement ”space domain formulas” with Fourier domain formulas*



Geoid Example #1: NKG geoids

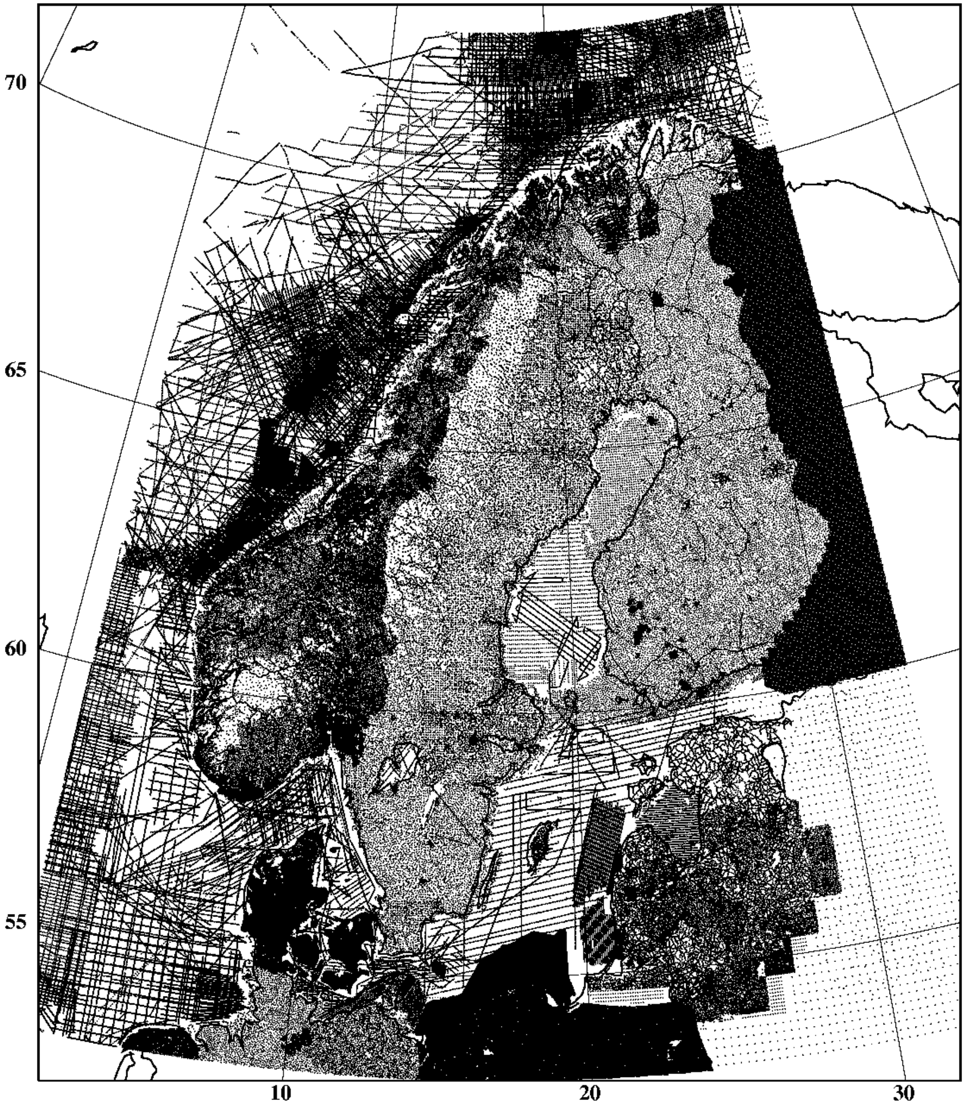
Joint cooperation Sweden+Norway+Finland + Denmark (+ Estonia, Latvia, Lithuania)

11 countries – 6 main height systems
Sweden, Finland … normal heights
Norway, Denmark … orthometric heights

Nordic Commission of Geodesy – NKG,
Joint work since 1986
Latest version: NKG2004 – with GRACE

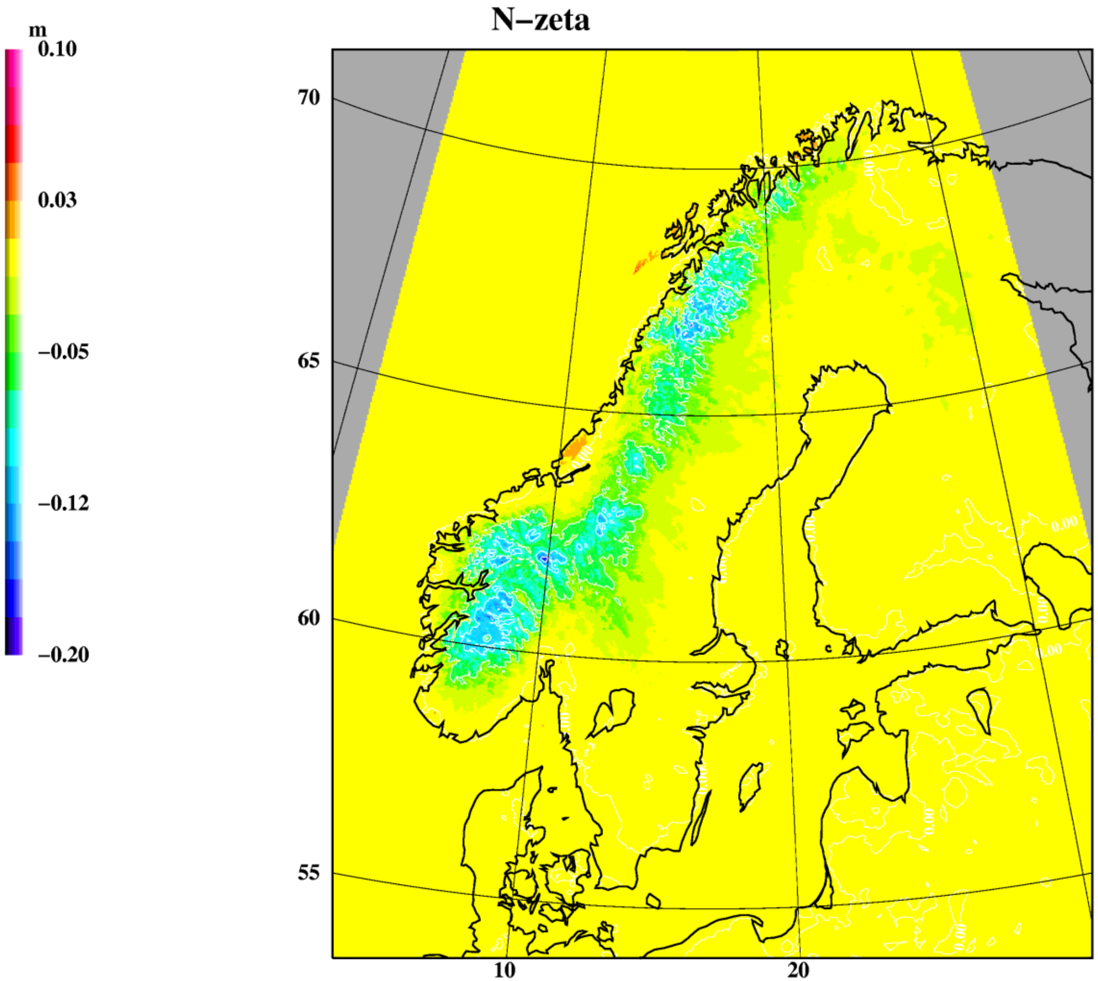
Gravity data from 12 countries
Geodetic marine gravimetry
Oil company data
Airborne gravity surveys

Number of selected points (0.02° resolution):
Surface data: 276241
Airborne data: 7890



Difference between *geoid* and *quasigeoid*

Geoid for orthometric heights, quasigeoid for normal heights of levelling



C: geopotential number

g: physical gravity along plumbline below point

γ : normal gravity below pt

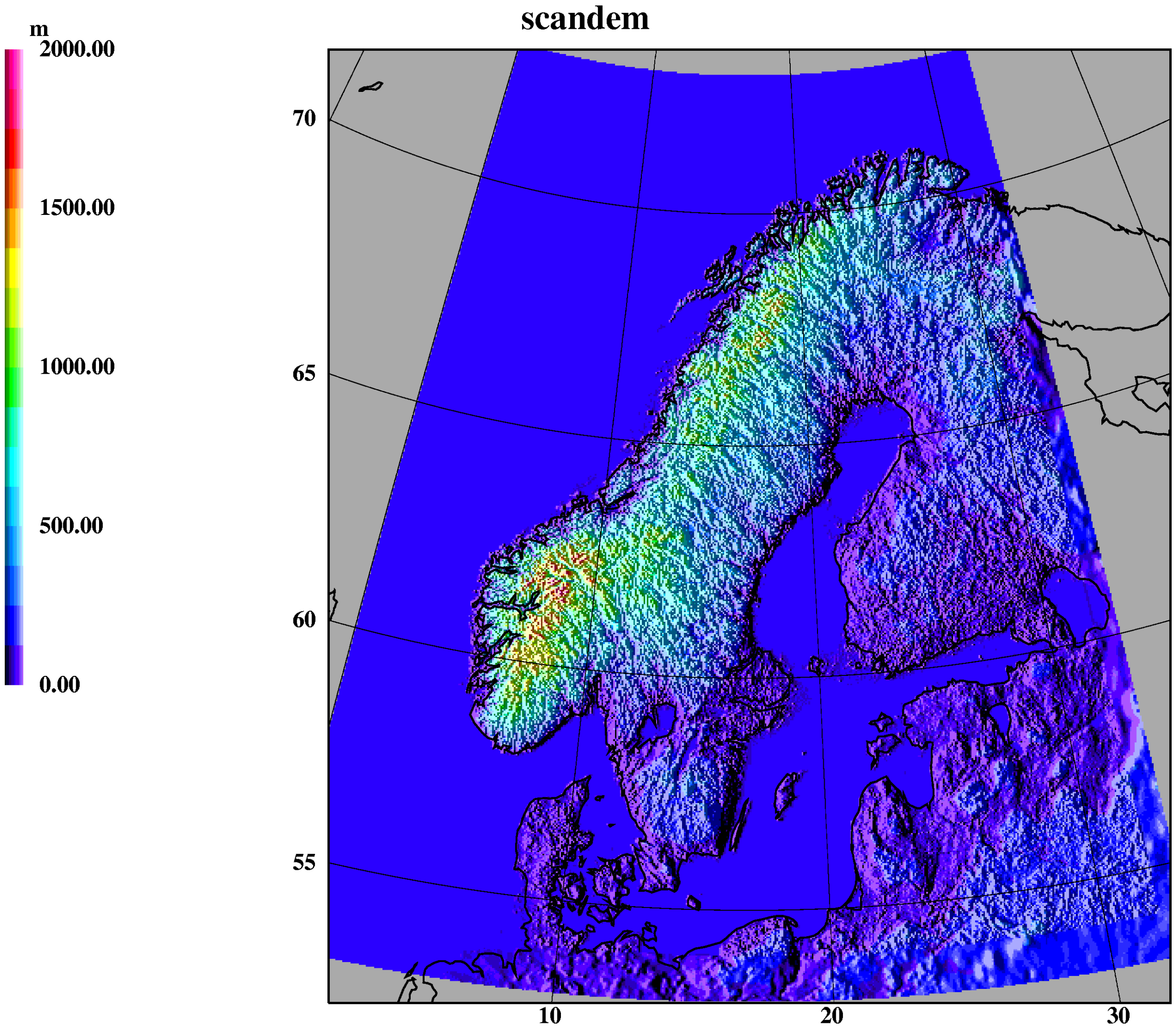
$$N = \frac{C}{g}$$

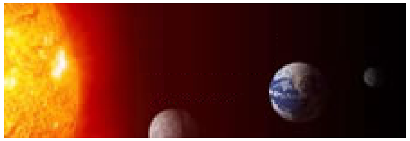
$$\varsigma = \frac{C}{\gamma}$$

Approximative formula for Helmert orthometric heights:

$$N - \cdot = \frac{\Lambda \quad Bouguer}{\gamma}$$

The basic composite DEM





NKG2004 geoid computation scheme

- Make EGM96 spherical harmonic grids (0 and 2 km elevation)
- Terrain reductions by prims integration
- Gridded 801801 grid points by LSC and variable std.dev.(all <8mgal). Corr.length = 25 km, 2nd order Markov cov.fct.
- FFT

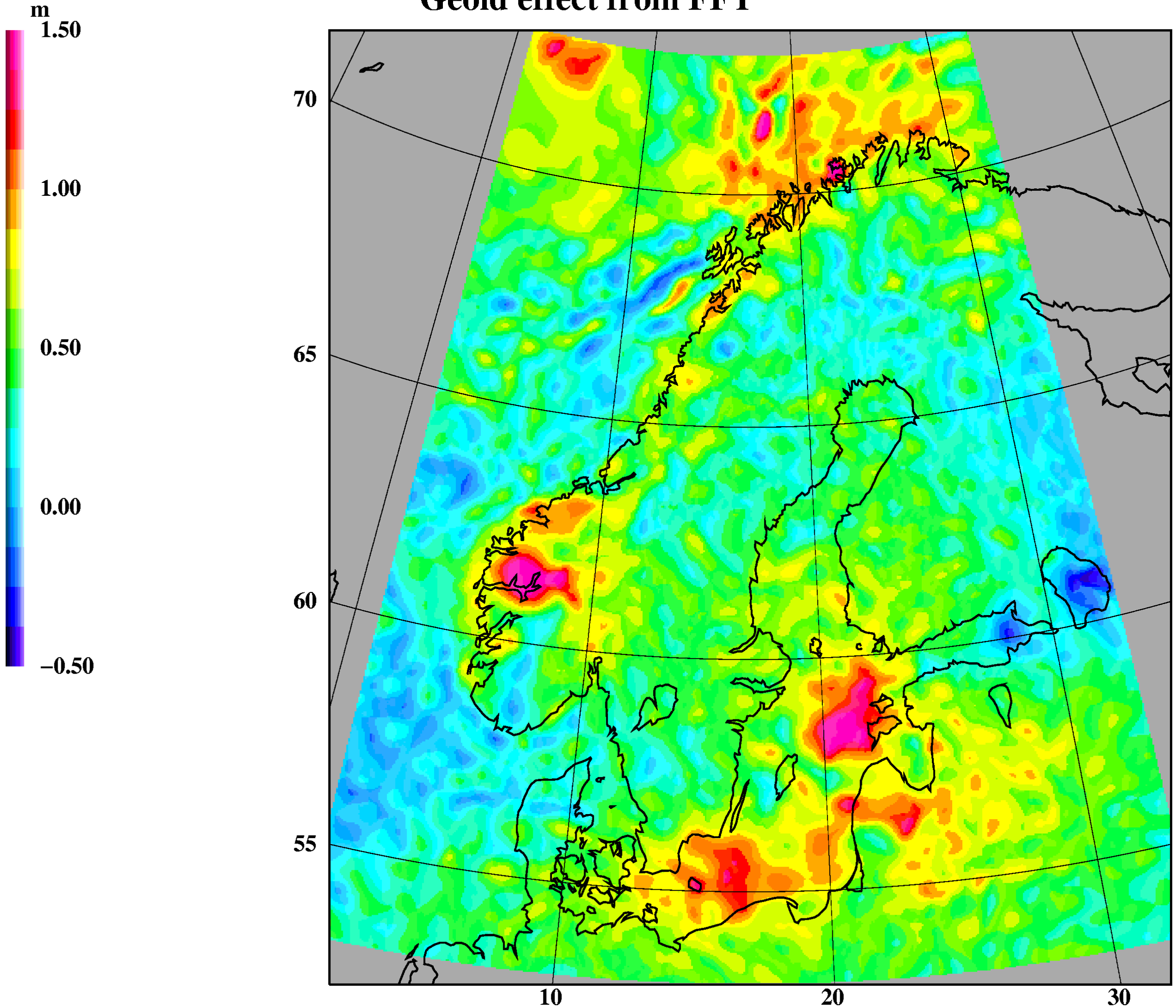
1. *Area: 53N-73N, 1E-33E*
2. *2000×1600 grid (100% zero padding)*
3. *Stokes function without modification. (Data mean not removed)*

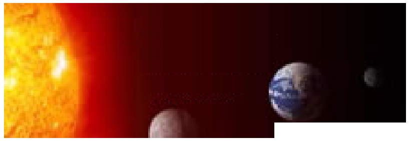
- Restore terrain effects on quasigeoid
- Restore EGM96 effects on quasigeoid
- Convert from quasigeoid to geoid

=> *Gravimetric geoid*

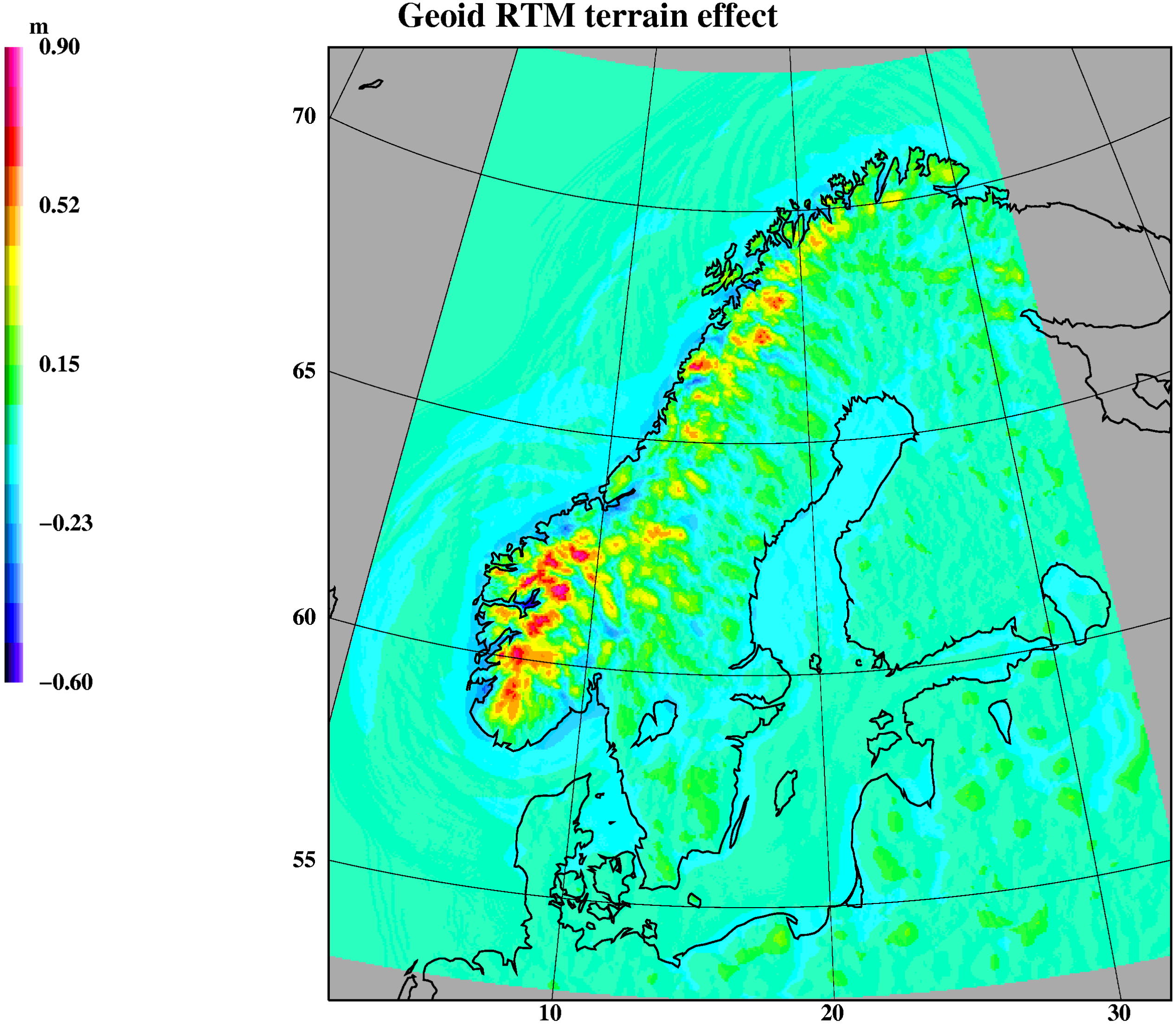
=> *Need for national fitting to GPS-levelling*

Geoid effect from FFT



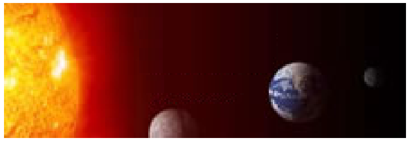


Geoid RTM terrain effect

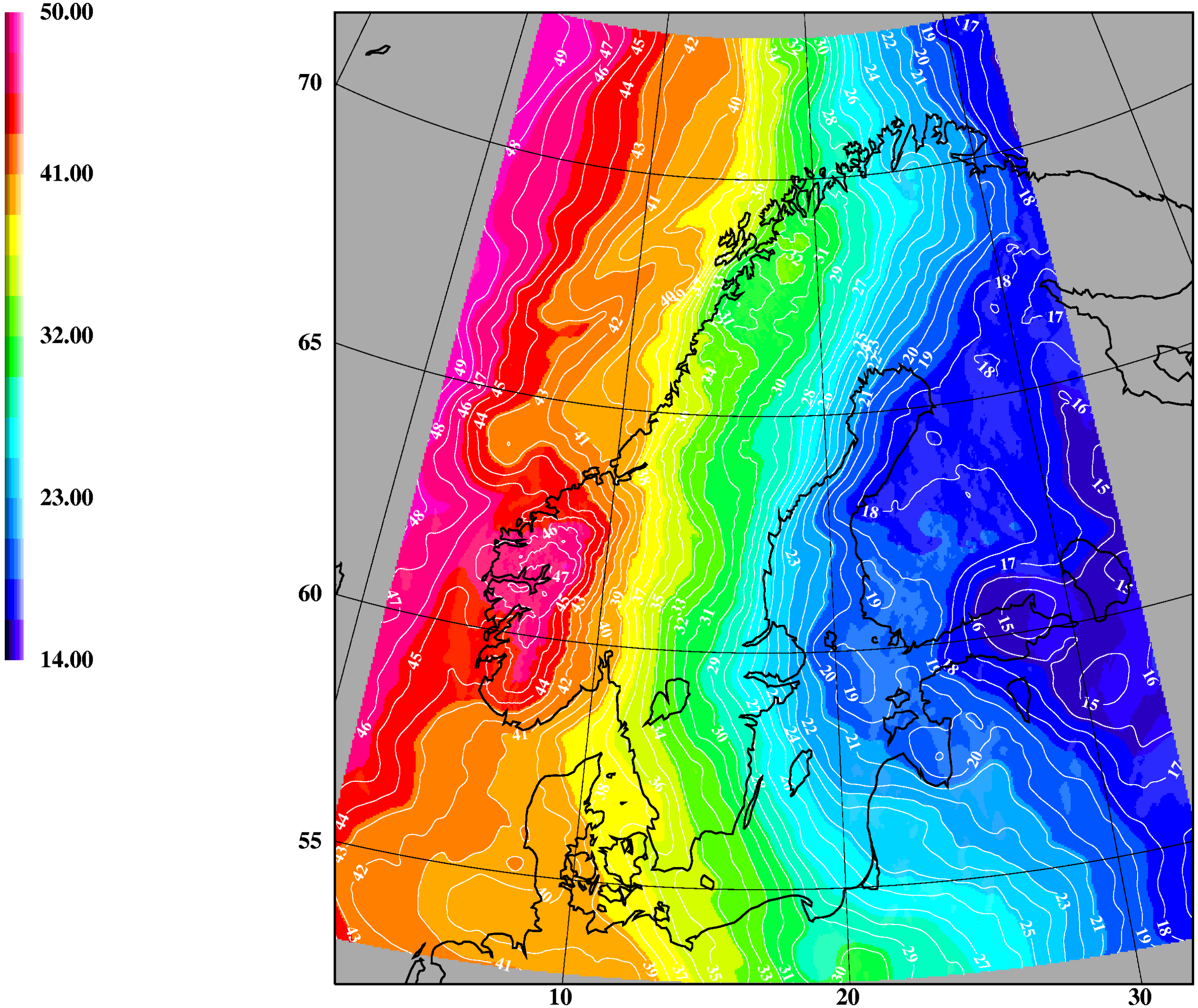


Statistics of data reductions – "remove steps"

unit: mgal	mean	std.dev.	min.	max.
free-air anomalies (surface data)	-0.85	25.15	-141.17	205.58
- EGM96	-1.13	15.48	-178.93	132.34
- EGM96-RTM	0.37	9.68	-104.49	90.01
airborne data (Baltic/Skagerrak)	-14.00	19.82	-80.38	48.89
- EGM96	1.56	9.88	-34.12	48.61
- EGM96-RTM	2.07	9.78	-32.46	50.75

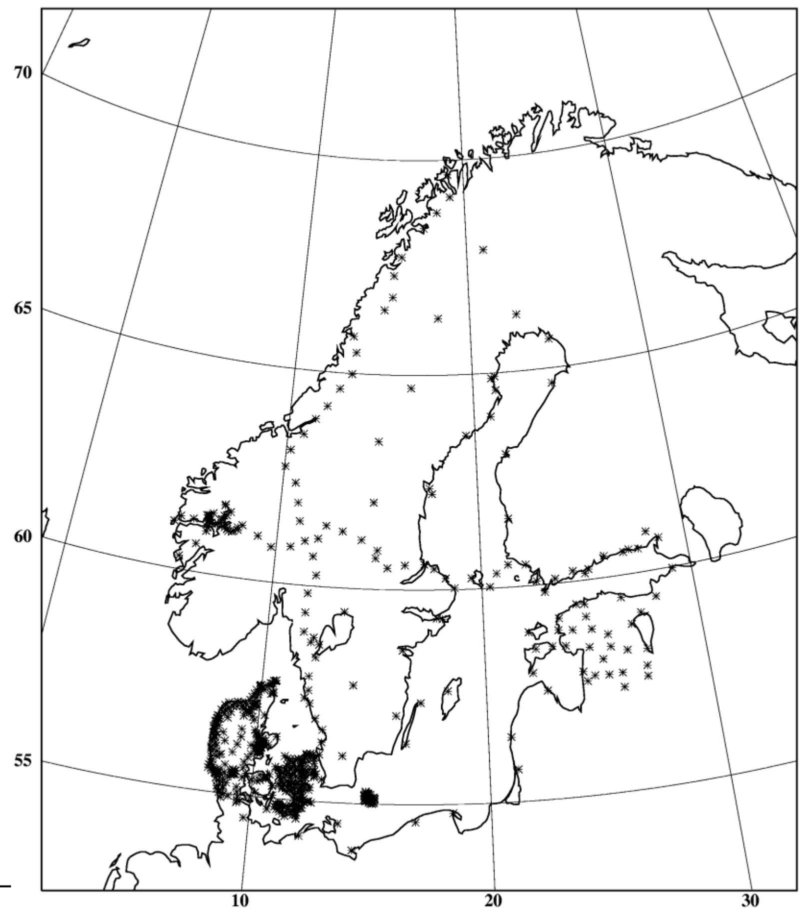


NKG-2004 gravimetric geoid



Fit to GPS data – making a "GPS geoid" fitting data to cm-level

GPS data used for comparisons



Basic principle for geoid fit:

ε = $N_{GPS} - N_{gravimetric}$

Model ε by trend
+ least-squares collocation

$$\varepsilon = \cos\varphi \cos\lambda\, a_1 + \cos\varphi \sin\lambda\, a_2 + \sin\varphi\, a_3 + a_4 + \varepsilon'$$

$$\hat{\varepsilon} = C_{xx}^{-1} C_{sx}\{\varepsilon'\}$$

Fit options:
Correlation length and r.m.s. noise

NKG-2004: Fit to GPS/levelling data

unit: meter	number of points	mean	std.dev.	std.dev. after 4-par fit
”Torge” (N-S line)	46	-0.138	0.117	0.055
”SWET” (E-W line)	34	-0.164	0.114	0.067
SWEREF (Sweden)	21	-0.100	0.079	0.051
Baltic Sea Level Project	27	-0.057	0.125	0.077
DK GPS net (DVR90)	416	-0.110	0.033	0.019
Estonia	31	0.038	0.055	0.047
Sognefjord region (local GPS datum)	42	(-0.531)	0.113	0.070

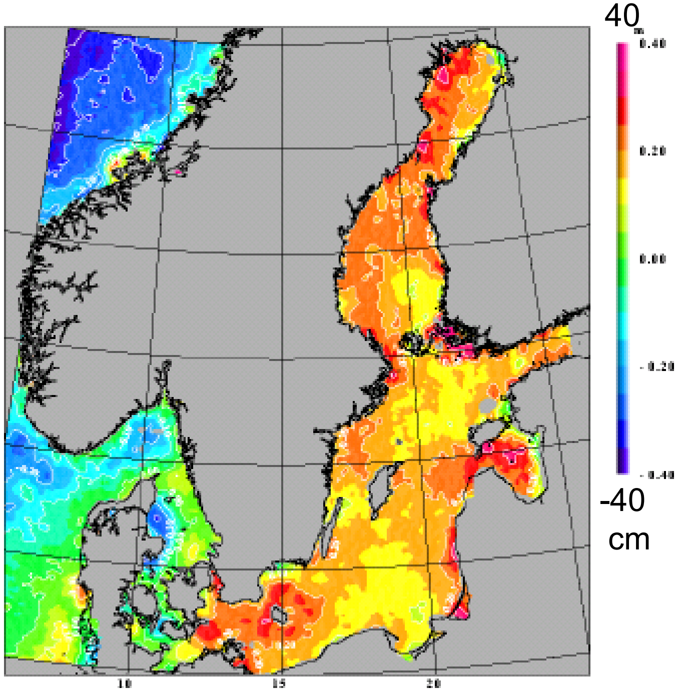
NKG-2004: Fit to sea level in Baltic Sea

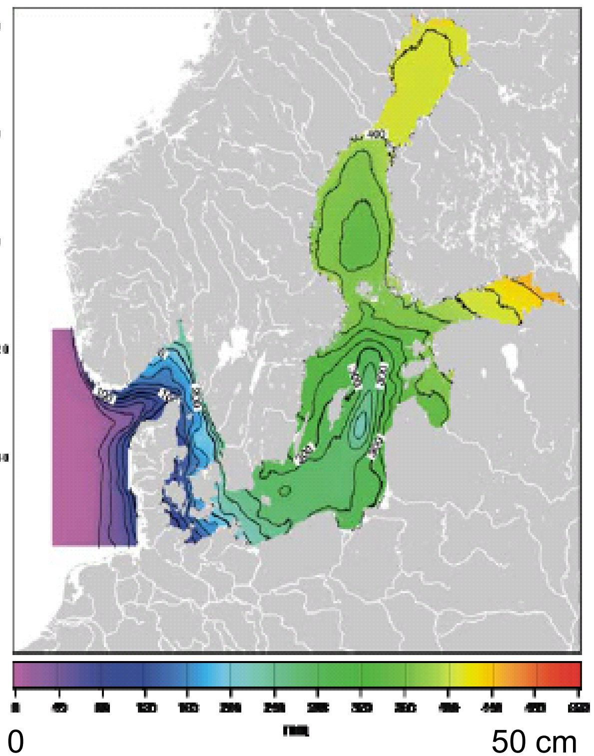
Mean Dynamic Topography: MDT = $h^{ellipsoidal} - N$ … height of ocean above geoid

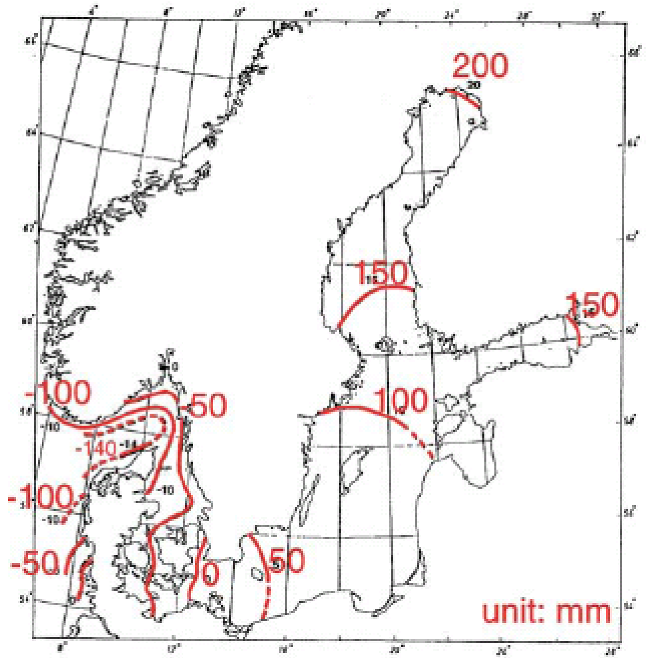
MDT from MSS (ERS altimetry) – Geoid (NKG04)

MDT from oceanographic model

MDT from tidegauges and levelling

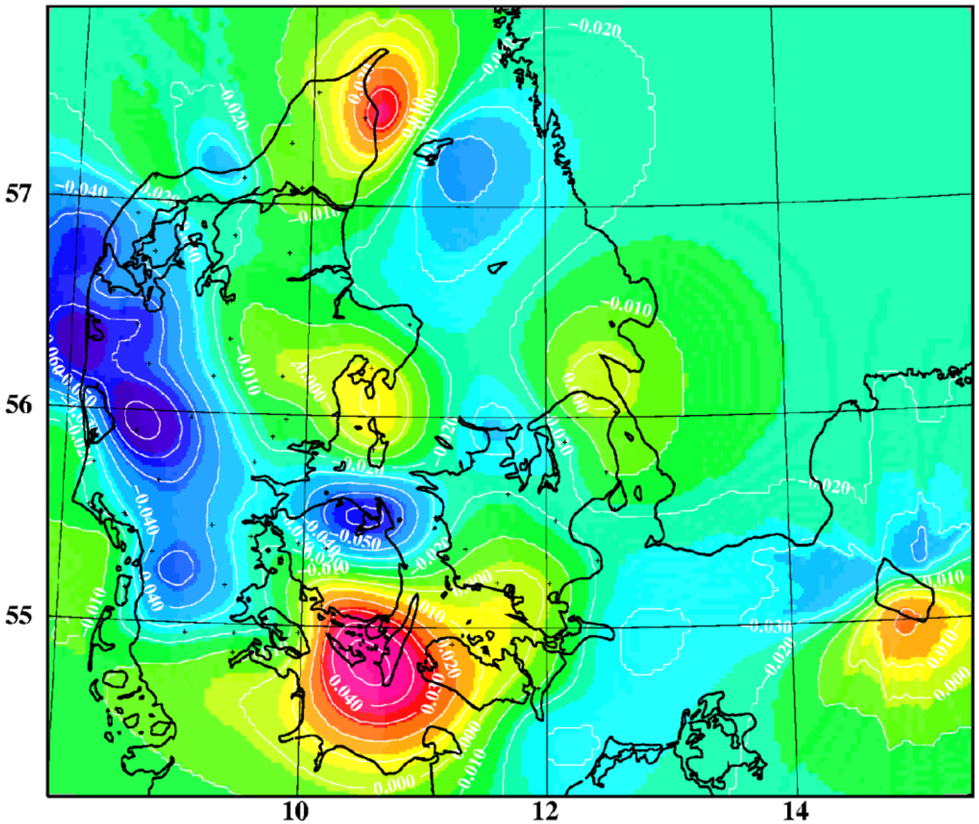






Example of national geoid fit: Denmark – DVR90

GPS differences: meter	mean	std.dev.
Before fit	-0.110	0.033
After collocation fit	0.000	0.008



A-priori standard deviation on
GPS/levelling data: 1cm
All 416 points used in fitting.
Constant offset: 0.11 m