Reanalysing Astronomical Coordinates of Old Fundamental Observatories using Satellite Positioning and Deflections of the Vertical

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Satellite positioning + Deflections of the vertical = Astronomical positioning

Official astronomical latitude determinations:

Stockholm observatory

59°20′31.3″	Wargentin (1759)	59	star observations
59°20′34.8″	Cronstrand (1811)	xx	
59°20′33.8″	Selander (1835)	108	

København observatory (Rundetårn)

55°40′57.0″	Bugge (1779)	56	star observations
55°40′52.6″	Schumacher (1827)	279	

Official astronomical longitude determinations:

Stockholm observatory

18°03’29.8”	Struve (1844) + Fuss & Nyrén (1871)	86 chronometers, 16 voyages Telegraph

København observatory (Rundetårn)

12°34’39.6”	Struve & Struve (1846) + Peters (1884)	42 chronometers, 16 voyages Telegraph

Coordinates based on satellite positioning (ETRS 89):

Kastellholmen → Katarina kyrka → Stockholm observatory

59°20’29.16”
18°03’16.76”

Nikolai tårn → København observatory

55°40’53.06”
12°34’32.79”

... and now comes Jonas

Gravimetric deflections of the vertical

$$\Phi = \phi + \xi$$

$$\Lambda = \lambda + \frac{\eta}{\cos\phi}$$

Φ, Λ = Astronomical latitude and longitude (at the surface)

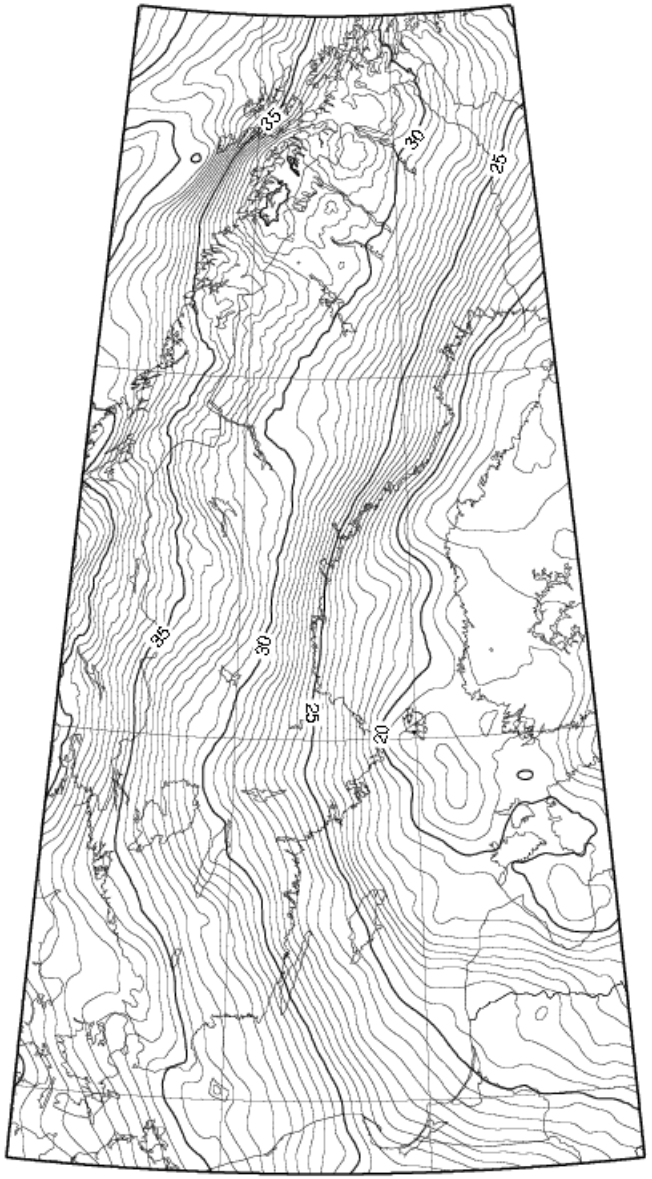
ϕ, λ = Geodetic latitude and longitude, satellite derived

ξ, η = Deflections of the vertical (at the surface)

Relation to the geoid height N

$$\xi = -\frac{\partial N}{\partial x} = -\frac{1}{R}\frac{\partial N}{\partial \phi}$$

$$\eta = -\frac{\partial N}{\partial y} = -\frac{1}{R\cos\phi}\frac{\partial N}{\partial \lambda}$$



Regional model: SWEN08_RH2000

The official Swedish geoid model

The gravimetric model KTH08 (Ågren et al. 2009) adapted to Swedish reference systems using 1570 GNSS/levelling observations

SWEN08_RH2000 = KTH08 + shift
+ land uplift/perm. tide corrections
+ smooth residual surface

Resolution: 0.02° x 0.04°

Estimated standard uncertainty:

Swedish main land: 10 – 15 mm
Outside main land: ≈ 5 -10 cm (?)

Regional model: Computation of deflections of the vertical

Since SWEN08_RH2000 is strictly a quasigeoid model, the deflections of the vertical need to be computed by (Heiskanen and Moritz, 1967, Sec. 8-9):

$$\xi = -\frac{1}{R}\frac{\partial \zeta}{\partial \phi} - \frac{\Delta g}{\gamma}\tan\beta_1 = -\frac{1}{R}\frac{\partial \zeta}{\partial \phi} - \frac{\Delta g}{\gamma}\frac{1}{R}\frac{\partial H}{\partial \phi}$$

$$\eta = -\frac{1}{R\cos\phi}\frac{\partial \zeta}{\partial \lambda} - \frac{\Delta g}{\gamma}\tan\beta_2 = -\frac{1}{R\cos\phi}\frac{\partial \zeta}{\partial \lambda} - \frac{\Delta g}{\gamma}\frac{1}{R\cos\phi}\frac{\partial H}{\partial \lambda}$$

The partial derivatives are obtained by numerical integration

The terrain inclinations are derived using the Swedish photogrammetric DEM

Estimated standard uncertainty: ≈ 0.2″ (corresponds to ≈ 1 mm/km)

Global model: EGM2008

Maximum degree 2160 corresponding to the resolution 0.08°

Pointwise evaluation of the spherical harmonic series using the *harmonic_synth* program of Holmes and Pavlis.

Estimated standard uncertainty ≈ 0.2" – 0.4'' (a little higher than for the regional model)

... and now back to Martin

Astronomical coordinates based on satellite positioning + gravimetric deflections of the vertical:

Stockholm observatory

59°20’33.05”
18°03’29.45”

København observatory

55°40’53.33”
12°34’38.73”

Table 1. Astronomical coordinates for the Stockholm observatory: our values from satellites and gravimetry, the old values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat.	59°20′33.0″	59°20′31.3″	(Wargentin, 1759)	- 1.7
		59°20′34.8″	(Cronstrand, 1811)	+ 1.8
		59°20′33.8″	(Selander, 1835)	+ 0.8
Long.	18°03′29.4″	18°03′29.8″	(Struve, 1844 a.o.)	+ 0.4

Table 2. Astronomical coordinates for the København observatory: our values from satellites and gravimetry, the old values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat.	55°40′53.3″	55°40′57.0″	(Bugge, 1779)	+ 3.7
		55°40′52.6″	(Schumacher, 1827)	- 0.7
Long.	12°34′38.7″	12°34′39.6″	(Struve, 1846 a.o.)	+ 0.9

Table 3. Some astronomical coordinates for the Stockholm and København observatories: our values from satellites and gravimetry, "modern" values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat. St.	59°20′33.0″	59°20′32.9″	(Rosén, 1879)	- 0.1
Long. diff. St - Kø	5°28′50.7″	5°28′50.8″	(Lindhagen et al, 1890)	+ 0.1

Table 4. Astronomical coordinates for the Greenwich observatory (international zero meridian).

	Sat + grav	Stars		Discrepancy
Lat.	51°28′38.0″	-		-
Long.	00°00′00.2″	0°00′00.0″	(Definition)	0.2

Main conclusions:

1. The errors in latitude reached below 1″ with the determinations of Schumacher (1827) and Selander (1835).

2. The errors in longitude reached below 1″ with the sea chronometer expeditions of Struve (1844, 1846), later extended by telegraph connections.

3. The agreement between "modern" star-derived coordinates on one hand and present-day satellite-derived coordinates combined with deflections of the vertical on the other hand is within some 0.2″.