On the need of improved gravity data to compute the next generation of quasigeoid models in Sweden

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Introduction

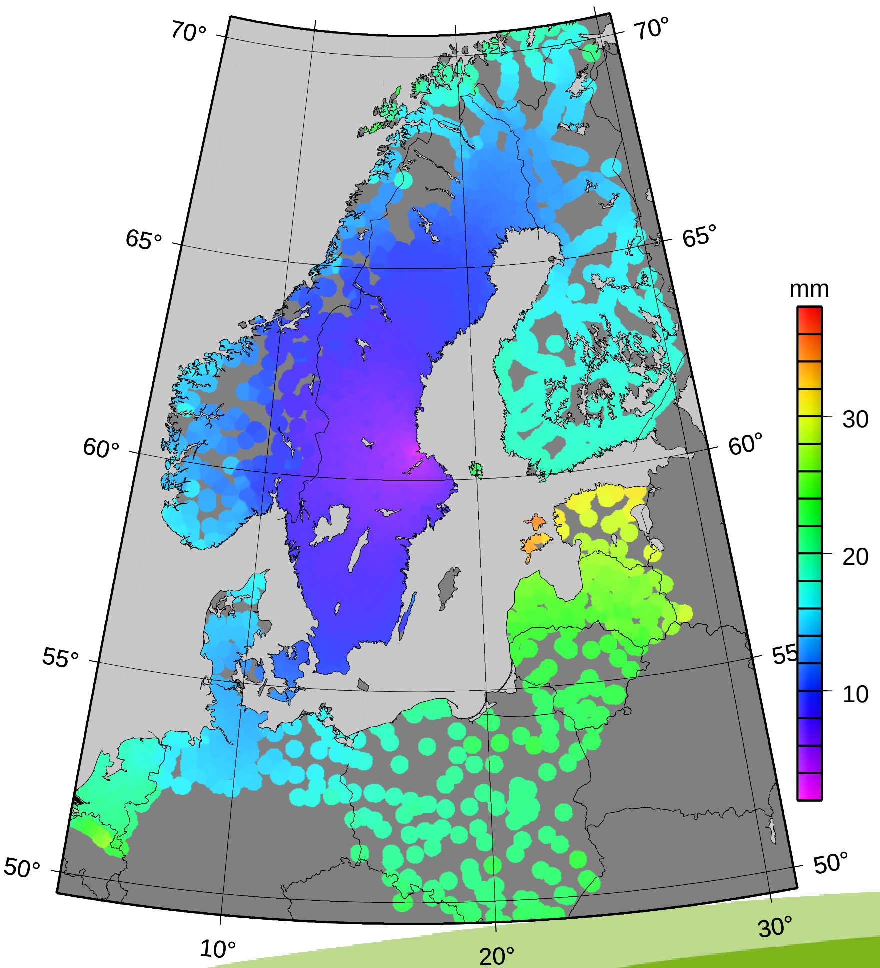
- SWEREF 99 and RH 2000 have now been introduced on a national level and in many municipalities
- A new quasigeoid model SWEN08_RH2000 has been introduced, which is based on the gravimetric model KTH08. Standard error 10-15 mm on the main land.
- GNSS height accuracy will increase in the future. It is not unlikely that a ellipsoidal height standard error as low as 5 mm will soon be obtained fast and easily.
- What is required to obtain a comparable standard error for the quasigeoid model?
- We here use the term “quasigeoid model” in such a way that errors in the reference systems (for GNSS and levelling) and possibly other corrections are included.

Purpose

- The purpose of this talk is to investigate in what way the *Swedish gravity data need to be improved* to make the computation of a gravimetric quasigeoid model with 5 mm standard error *possible* in the future.
- This is investigated by error propagations assuming a GOCE EGM with M = 200 and commission RMS-error of 1 cm.
- The results will be used as a basis for the recommendations in the next Swedish 10 year plan ***Geodesi 2010***.
- Please note that the error propagations do not show that this high accuracy will necessarily be obtained with the assumed gravity accuracy. The geoid height accuracy depends also on other things like modelling errors, weighting, etc. We need to improve *also* the modelling part, etc.
- Improvements are of course also required on the Nordic level. Here we concentrate on Sweden.

Comments on the use of GNSS/levelling

Estimated standard errors for RH 2000 relative to Gävle:



- The height system RH 2000 is realised by the normal heights of the 50000 defining benchmarks
- The GNSS system (SWEREF 99) is realised by the 20 fundamental stations in the SWEPOS network at the reference epoch 1999.5.
- To reach a 5 mm quasigeoid model,
  - errors in the height system RH 2000 need to be separated and modelled.
  - the postglacial land uplift need to be talen care of with sufficient accuracy
  - the more detailed GNSS height observations of *today* are not so useful (too noisy).

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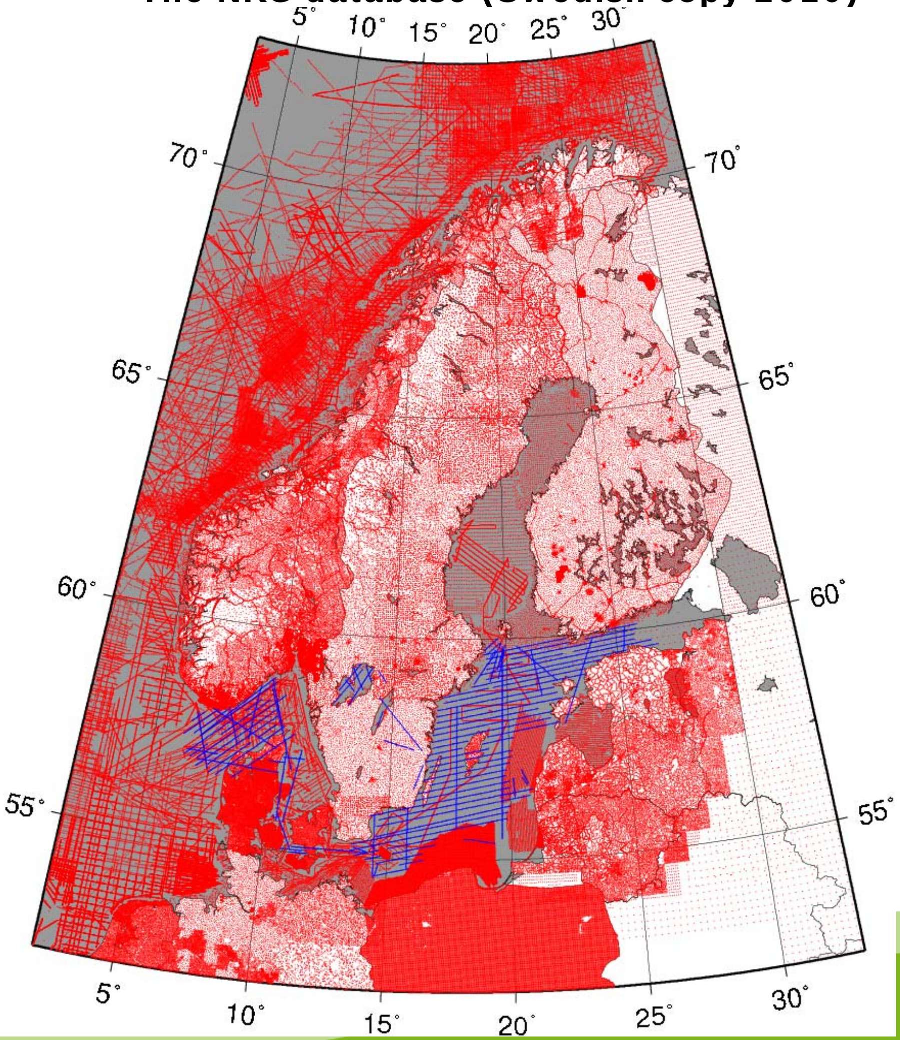
The present Swedish gravity data



- Most of the Swedish gravity data has 5 km resolution.
- The gravity standard errors varies from 0.2 to a few mGal, most around 0.5 mGal.
- Two different gravity systems are in use, RG 62 and RG 82. Some confusion here. The situation needs to be cleaned up.
- The accuracy of the heights vary considerably
- The situation is worse in the big lakes and at Sea. Ship and air data with some data gaps and courser resolution
- The situation is also worse in the high mountains to the north west.
- Agreement along borders?

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Error propagation in the spectral domain

Remove-compute-restore estimator:

$$\tilde{N} = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\left(\Delta\tilde{g} - \Delta g^{DIR} - \sum_{n=2}^{M}\left(\frac{R}{r_p}\right)^{n+2}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right)\right)^{*} d\sigma + \frac{R}{2\gamma}\sum_{n=2}^{M}\frac{2}{n-1}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right) + \delta N_I$$

In spectral form:

$$\tilde{N} = c\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^{*} - Q_n^M\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{\Delta g}\right) + c\sum_{n=2}^{M}\left(Q_n^M + s_n\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{EGM}\right) + \delta N_I$$

The expected global mean square error (assuming the error cov. functions are homogeneous and isotropic):

$$\delta\bar{N}^2 = E\left[\frac{1}{4\pi}\iint_{\sigma}\left(\tilde{N} - N\right)^2 d\sigma\right] = c^2\sum_{n=2}^{M}\left[\left(\frac{2}{n-1} - s_n - Q_n^M\right)^2\sigma_{n,\Delta g}^2 + \left(s_n + Q_n^M\right)^2\sigma_{n,EGM}^2\right] + c^2\sum_{n=M+1}^{\infty}\left[\left(\frac{2}{n-1} - Q_n^M\right)^2\sigma_{n,\Delta g}^2 + \left(Q_n^M\right)^2 c_n^{red}\right]$$

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Spectral analysis of the gravity field in Sweden

- Cf. Forsberg (1986). The spectral analysis below is more or less only a confirmation of his results with modern, improved data.
- We assume a remove-compute-restore (r-c-r) estimator
  - Sjöberg's combined method need to utilise the r-c-r strategy in the gridding phase in order to be efficient. The results are therefore relevant also for his case.
- Study Swedish gravity anomalies from the KTH08 computation reduced for
  - The EGM effect with M = 360 (GGM02C to 200 + EGM 96 above that)
  - The Residual Terrain Model (RTM) effect computed using GEOGRID (rect. prisms) with a 100 m x 100 m DEM (averaged from the Swedish photogrammetric DEM, now old).

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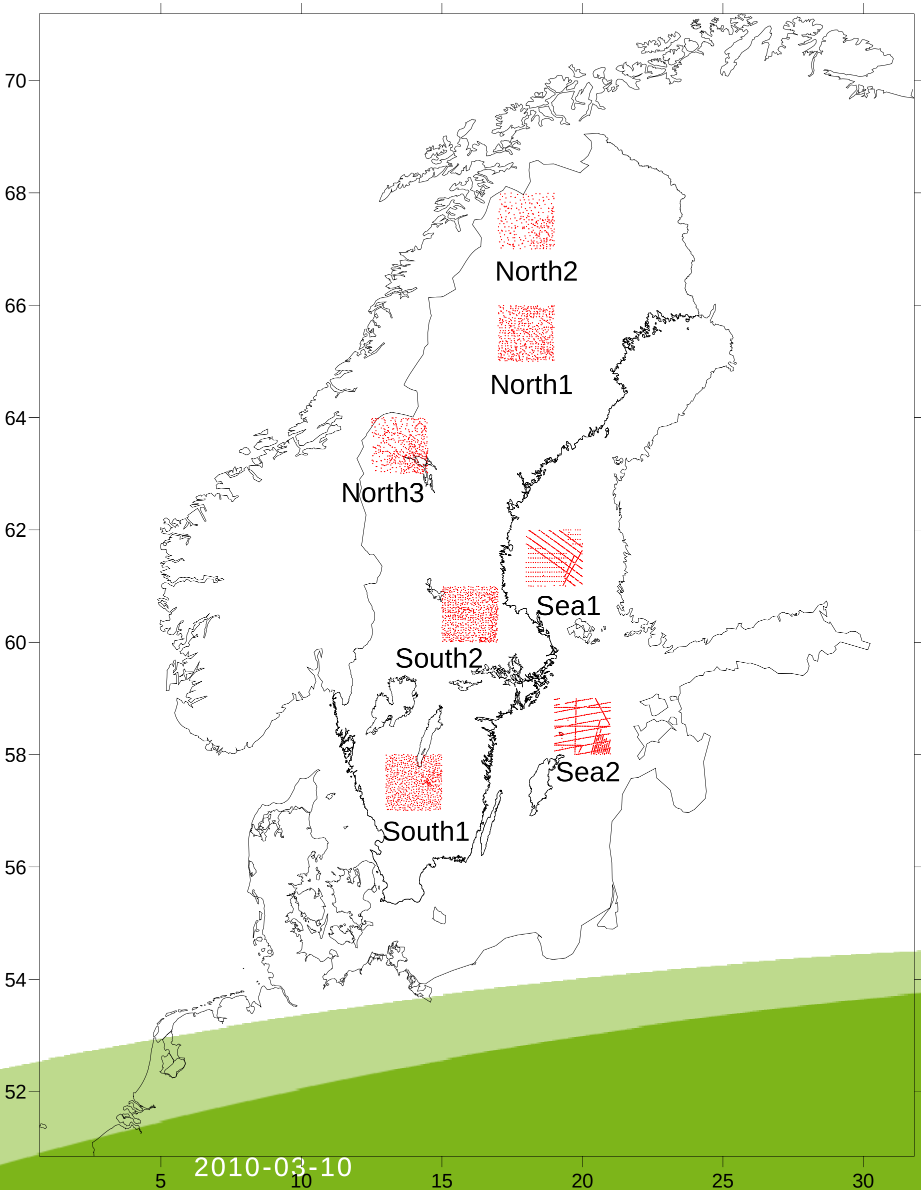
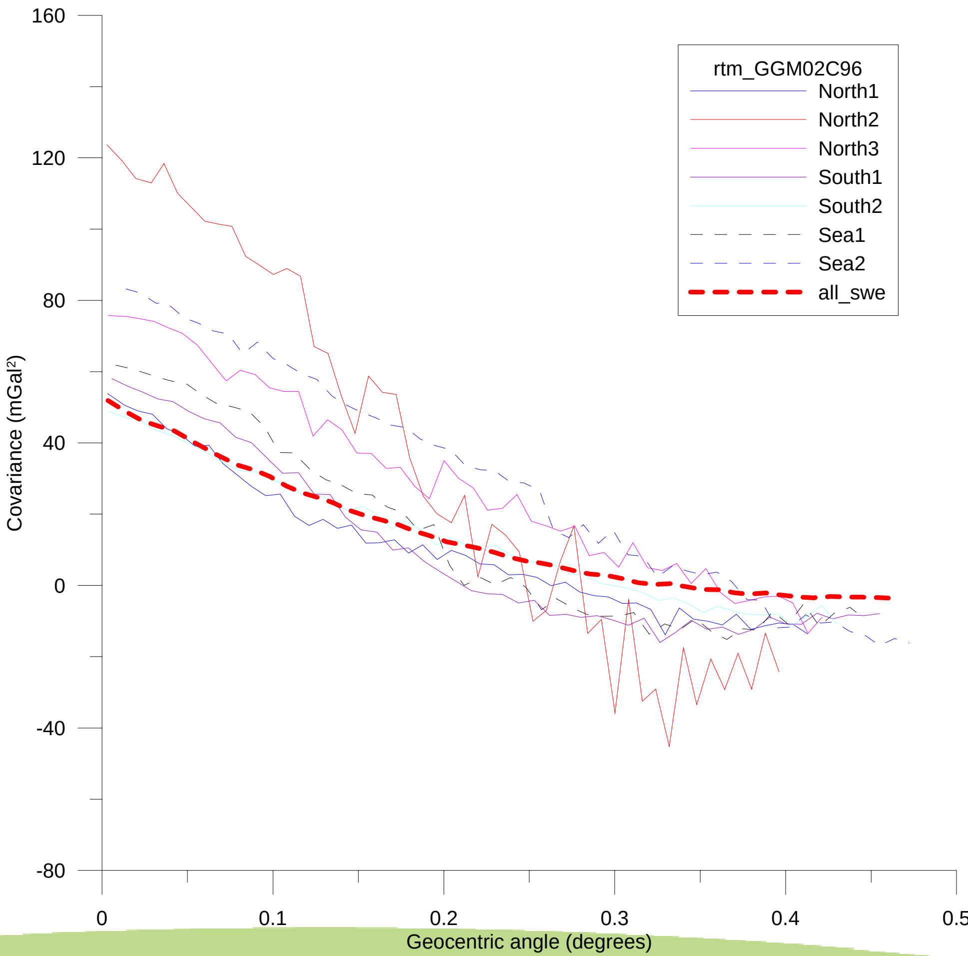
Computed covariance functions





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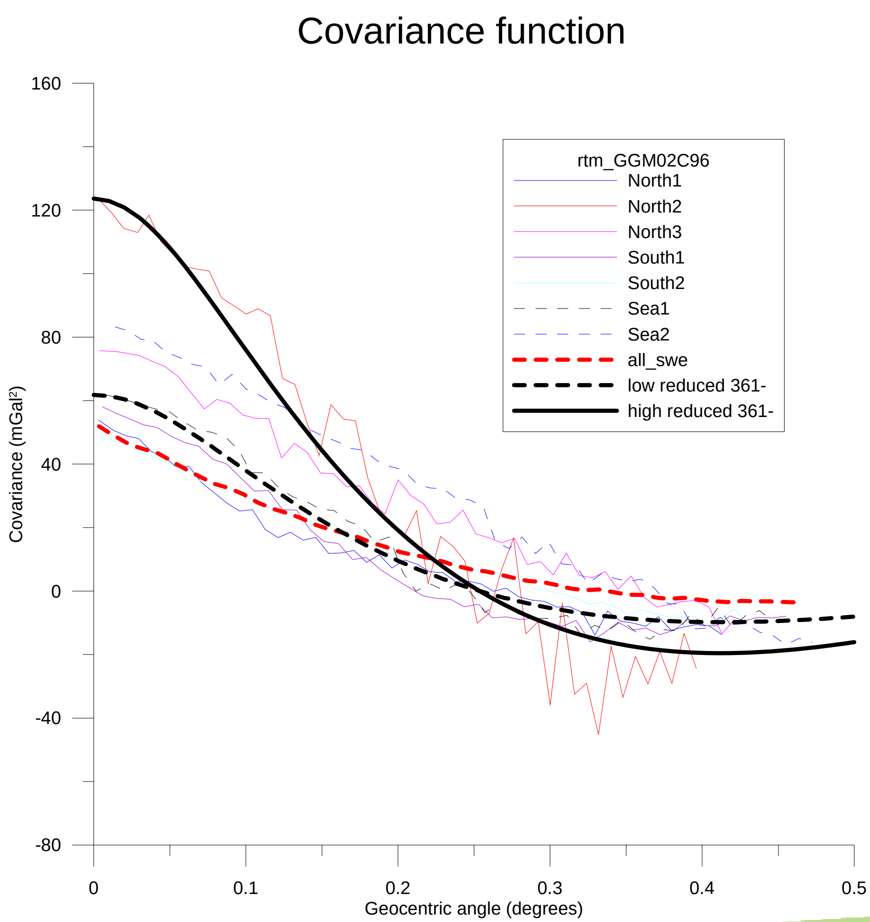
Degree variance models



Tscherning and Rapp (1974) type of model:

$$c_n = \alpha \frac{(n-1)}{(n-2)(n+4)} \left(\frac{(R-D)^2}{R^2} \right)^{n+2} \qquad n > 360$$

High reduced model:

α = 256 mGal^2 D = 5.0 km

Low reduced model:

α = 128 mGal^2 D = 5.0 km

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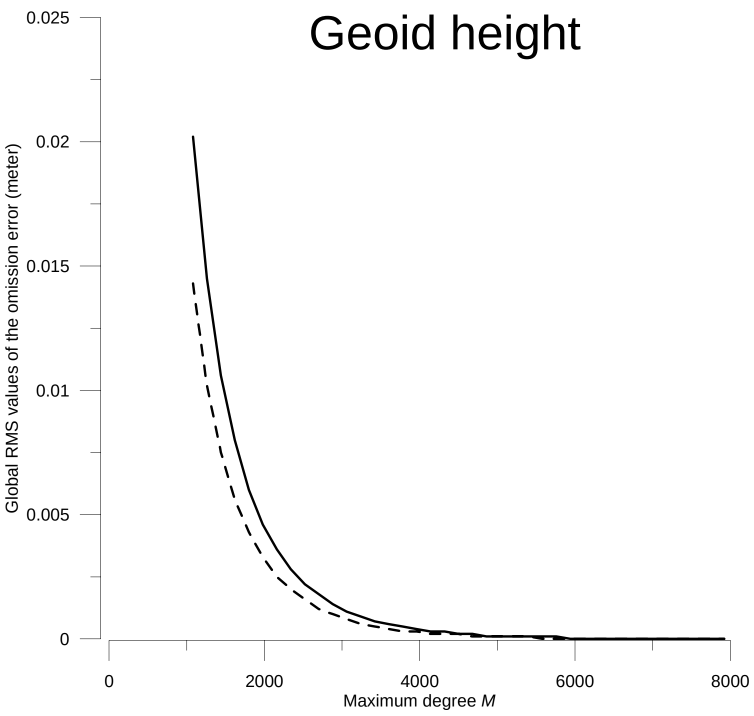
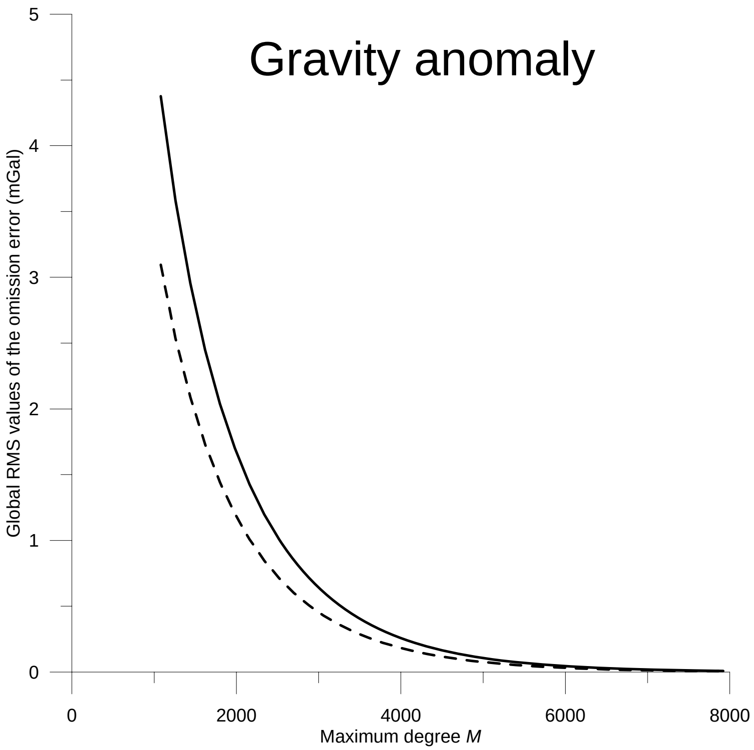
Global RMS omission errors





- It seems that 5 km resolution (corresponding to the Nyquist degree 3960) is sufficient for all of Sweden.

Error propagation

Remove-compute-restore estimator:

$$\tilde{N} = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\left(\Delta\tilde{g} - \Delta g^{DIR} - \sum_{n=2}^{M}\left(\frac{R}{r_p}\right)^{n+2}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right)\right)^{*} d\sigma$$

$$+\frac{R}{2\gamma}\sum_{n=2}^{M}\frac{2}{n-1}\left(\Delta\tilde{g}_n^{EGM} - \Delta g_n^{DIR}\right) + \delta N_I$$

In spectral form:

$$\tilde{N} = c\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^{*} - Q_n^M\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{\Delta g}\right)$$

$$+c\sum_{n=2}^{M}\left(Q_n^M + s_n\right)\left(\Delta g_n - \Delta g_n^{DIR} + \varepsilon_n^{EGM}\right) + \delta N_I$$

The expected global mean square error:

$$\delta\bar{N}^2 = E\left[\frac{1}{4\pi}\iint_{\sigma}\left(\tilde{N} - N\right)^2 d\sigma\right] = c^2\sum_{n=2}^{M}\left[\left(\frac{2}{n-1} - s_n - Q_n^M\right)^2\sigma_{n,\Delta g}^2 + \left(s_n + Q_n^M\right)^2\sigma_{n,EGM}^2\right]$$

$$+c^2\sum_{n=M+1}^{\infty}\left[\left(\frac{2}{n-1} - Q_n^M\right)^2\sigma_{n,\Delta g}^2 + \left(Q_n^M\right)^2 c_n\right]$$

We assume that

- a GOCE EGM with M = 200 and 1 cm commission RMS-error is used; see Ågren (2004) p. 31.
- either the high or the low reduced signal degree variance models are correct.
- the resolution 5 km is used for the gravity anomalies.
- the error degree variances for the gravity anomalies are either
  - bandlimited white noise or
  - following the reciprocal distance model (Moritz 1980) with the correlation length 0.25 degrees.
  - Above the Nyquist degree, the signal degree variances are utilised.
  - Remember that the error covariance function is assumed to be **homogeneous** and **isotropic**.

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Propagated geoid height RMS errors

- The high and low reduced signal degree variance models (very similar results)
- The LSMM-method with correct weighting. ψ_0 = 3 degrees. Perfect weighting.

Gravity anomaly noise model	Standard error (mGal)	Correlation length (deg)	Nyquist degree	Expected global RMS error (mm)
White	2	-	3960	9
White	1	-	3960	5
White	0.5	-	3960	3
White	0.2	-	3960	2
Reciprocal dist.	2	0.25	3960	31
Reciprocal dist.	1	0.25	3960	16
Reciprocal dist.	0.5	0.25	3960	9
Reciprocal dist.	0.2	0.25	3960	4

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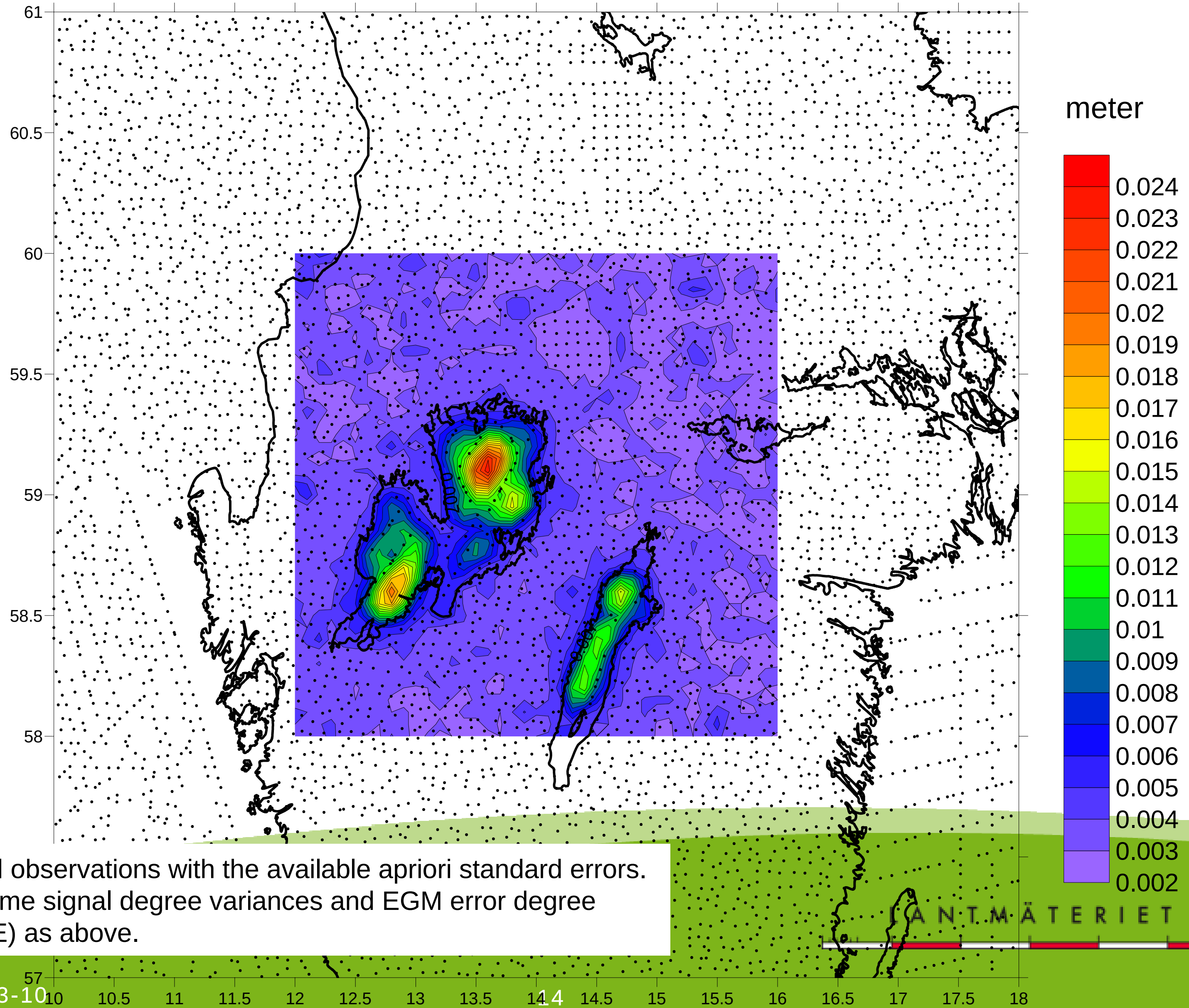
Discussion

- Gravity data with 5 km resolution is clearly sufficient to obtain a quasigeoid with 5 mm standard error over Sweden.
- It is very important that the gravity data are as uncorrelated as possible. Systematic errors (in space) magnifies the standard error in a very significant way.
- Please note that the error propagations show the standard error possible to achieve considering that no other modelling errors are added, e.g. errors in
  - The topographic correction
  - The DEM
  - Downward Continuation (DWC)
  - weighting of the terrestrial gravity anomalies (correct weighting is assumed above).
- The purpose here is only to study the how the quality of the gravity anomalies propagate to the quasigeoid height.
- Note that the gravity data are assumed to be homogeneous. How does non-homogeneous gravity data affect the situation? (Realistic sampling, data gaps, varying standard errors.)

Error propagation by Least Squares Collocation



Uncorrelated real observations with the available apriori standard errors. Otherwise the same signal degree variances and EGM error degree variances (GOCE) as above.

Conclusions

- 5 km resolution is sufficient for gravity data to make it possible to determine a 5 mm quasigeoid model over Sweden.
- To make gravity data become as uncorrelated as possible,
  - a new gravity system should be introduced (RG 2000), based on the Swedish stations in Nordic Absolute Gravity Project.
  - the old gravity networks and data should then be properly connected. This will require much work and many new measurements.
  - control measurements should be made with A10.
- The standard error for the 5 km gravity data on the main land should be better than 0.2 (- 0.5?) mGal in gravity and 0.2 m in height.
- The data gaps in the big lakes and at sea should be filled, especially near the coasts. (Air gravity, ship gravity and measurements on the ice.)
- Significant methodological improvements are also required to reach 5 mm.
- What is required on the Nordic level? Control measurements across the borders.
- This work is still not finished.